Nucleon Structure Functions
and Form Factors
in semi-perturbative
Quantum Chromodynamics

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1. Introduction of the Dissertation

1.1. Motivation

In modern physics, it is an important challenge to explore the structure of hadrons. Being the constituents of atomic nuclei, the nucleons deserve the main attention in studies about hadrons. Whereas the nucleon composition based on quarks and gluons is acknowledged, the detailed construction is still not completely understood. The theory which describes the interaction between these partons is called Quantum Chromodynamics. An important phenomenon of this theory is called confinement. The consequence of this phenomenon is that the partons cannot exist as free particles. According to this, it is not possible to study the nucleon structure directly.

Applying scattering processes, we can receive information about the desired nucleon structure. In order to get the necessary resolution of the nucleon, it is reasonable to create lepton nucleon collisions with large momentum transfer to the nucleon target. Making assumptions concerning the nucleon structure, one gets theoretical results which can be compared with experimental data in order to verify the assumptions.

The basic process is the elastic scattering between a lepton and a nucleon. The cross section of these processes can be expressed by various nucleon transition probability matrix elements generated by the exchanged gauge bosons. These matrix elements can be represented by nucleon form factors. Every form factor depends on the momentum transfer to the nucleon target. These form factors include the essential information about the nucleon structure. Depending on the region of the momentum transfer, some of the form factors were already calculated and measured.

Another important process is the inclusive inelastic lepton nucleon scattering. In this process, the target nucleon gets fragmented into unknown particles and just the final lepton will be measured. The cross section of these processes can be parameterized by nucleon structure functions. Apart from the momentum transfer to the nucleon target, they depend on the total mass of the unknown created particles. In order to calculate the nucleon structure functions, one must specify a transition process and calculate the cross section for comparison. The main process is the scattering between an electron and a nucleon, the exchange of a single virtual photon, and the creation of an electron and a nucleon together with a pion related to the minimal total mass. The corresponding nucleon to pion nucleon transition probability matrix elements must be studied with several techniques.

In order to evaluate the discussed matrix elements, one has to deal with Quantum Chromodynamics. Being a complicated quantum field theory, one cannot solve this theory exactly. Nevertheless, in some cases it is possible to apply a perturbative approach. Obviously, this requires a small coupling constant as parameter for the perturbative expansion. At large momentum transfer, one gets this behavior which is called asymptotic freedom. Unfortunately, the desired processes always include non-perturbative parts requiring other techniques. In order to deal with this problem, we often have the opportunity to separate the perturbative and non-perturbative parts which is known as factorization.
1. Introduction of the Dissertation

1.2. Outline

Various quantum field theories have been studied and several books were written with different notations and conventions. Following the recommendation at our institute, this work is based on [1]. Furthermore, one can find comprehensive information in [2] and [3]. Let us split our work in three parts where different important topics will be considered.

In the first part of the dissertation, we will study the scattering between an electron and a nucleon target due to exchange of a single virtual photon which carries large momentum. After the scattering process, we receive an electron and a nucleon together with a produced pion with small momentum. This process can be used to calculate the nucleon structure functions in the corresponding region. Therefore, we have to evaluate the required nucleon to pion nucleon transition probability matrix elements. It will be shown that they can be expressed by nucleon to pion nucleon transition form factors and usual nucleon form factors. Reviewing the general structure of the required form factors with the QCD factorization theorem and making reasonable assumptions, we will derive relations between different types of form factors. This approach gives us the opportunity to use experimental data of form factors being already measured in order to get results for form factors which are still not measured. Some theoretical results of the nucleon structure functions will be compared with experimental values. Concerning the other results, various predictions will be presented.

In the second part of the dissertation, we will deal with elastic lepton nucleon scattering. We consider all processes expressible by the exchange of one gauge boson with large momentum. Therefore, we have to study the corresponding nucleon transition probability matrix elements and the representations by nucleon form factors. Expanding these matrix elements at large momentum transfer, one can reduce every decomposition to one dominant term depending on the leading nucleon form factor. Our aim is to calculate these nucleon form factors and to compare the obtained results with experimental data. We will combine QCD perturbation theory with an expansion in nucleon distribution amplitudes to calculate the nucleon form factors. Applying leading twist nucleon distribution amplitudes only, one obtains the desired nucleon form factors. Using different order polynomial expansions and models for the nucleon distribution amplitudes, we will obtain results which can partially be compared with experimental data. The main form factor was already calculated with the QCD factorization theorem by multiple groups. The different results include a discrepancy which will be clarified in this work.

In the third part of the dissertation, we will evaluate the nucleon helicity flip form factor in the one and two photon exchange approximation. Therefore, we have to consider the dominant elastic electron nucleon scattering process generated by the electromagnetic interaction. In the previous part, we calculated the corresponding leading nucleon form factor in the one photon exchange approximation and at the limit of large momentum transfer. Moving to intermediate values of the momentum transfer, one also gets contributions for the sub-leading nucleon form factor which is known as helicity flip form factor. Concerning this form factor, experimental data are available. Moreover, one has discovered a different behavior depending on the type of the experiment. It has been suggested that the two photon exchange contribution can cause this situation. Unfortunately, the calculation of the desired form factor with the QCD factorization theorem is problematic. We will apply the technique specified in the previous part using the combination of leading and sub-leading twist nucleon distribution amplitudes. Using this technique, we will obtain a divergent result for the form factor. Nevertheless, the structure of the divergency can be extracted and we can explain the different behavior in the experiments. Finally, we will discuss the required modifications to avoid the divergency.
Part I.

Contribution of
Hard Near Threshold
Pion Electro Production
to Nucleon Structure Functions
2. Introduction to Part I

In this part of our work, we will study the hard near threshold pion electro production. That means, we consider the scattering between an electron and a nucleon target due to exchange of a single virtual photon which carries large momentum. After the scattering process, we receive an electron and a nucleon together with a produced pion with small momentum. This process can be presented by the following diagram.

![Diagram of electron, photon, nucleon, and pion](image)

The introduced scenario can be identified as an exclusive inelastic scattering process. Nevertheless, the cross section of this process can be related to the cross section of inclusive inelastic electron nucleon scattering. Therefore, one has to consider the produced hadrons as undetected. The final aim of our calculations is to apply this connection in order to derive expressions for the nucleon structure functions.

It is well known that the nucleon structure functions depend on two kinematic variables. Therefore, one can choose the momentum transfer to the nucleon target and the invariant mass of the undetected produced hadrons. Obviously, this mass must be larger than the nucleon mass. The minimal value is given by the nucleon mass plus a pion mass. According to this, the specified process delivers expressions for the nucleon structure functions in the area of large momentum transfer and small invariant masses.

In order to evaluate the cross section of the discussed process, we have to study the required nucleon to pion nucleon transition probability matrix elements. Using soft pion theorems, the matrix elements were evaluated at low virtualities of the exchanged photon. One can find low energy theorems in the approximation of the vanishing pion mass in [4], [5], [6] and for finite pion mass corrections in [7] and [8]. The expansion at small photon virtualities together with negligible pion mass has to be done with care because these limits do not
commute, in general. All these theorems do not work at large virtualities, as discussed in [9].

In the case of asymptotically large virtuality, one can use the QCD factorization theorem, see [10], [11], [12], and [13], [14], [15]. So the representations obtained in [9] can be derived. Applying this technique, we start from asymptotically large virtuality and go to reasonable values systematically. Another approach is studied in [16] for extensive applications. Hereby, the area of $5 - 10 \text{ GeV}^2$ is considered. Starting from the low energy behavior, the desired process is also discussed in [17]. In this work, the region of $1 - 10 \text{ GeV}^2$ is considered.

In the desired region, one can split these matrix elements into an exact threshold part plus a near threshold part. Let us now introduce the graphical representations for both parts. Moreover, we discuss their physical interpretation.

The exact threshold part can be represented by the following diagram which shows the simultaneous production of the nucleon and the pion.

![Diagram of N-pi production](image)

This part can be expressed by nucleon to pion nucleon transition form factors, see [9]. These form factors can be compared with usual nucleon form factors defined by usual nucleon transition probability matrix elements. Applying reasonable assumptions concerning the structure of all required form factors, one can derive relations for the nucleon to pion nucleon transition form factors as functions of usual nucleon form factors.

The near threshold part can be expressed by various soft pion structures. The dominant contribution is generated by the emission of the produced pion from the final nucleon, see [9]. This situation can be represented by the following diagram.

![Diagram of N-pi production](image)

Applying corresponding soft pion theorems, one can create a result of this part which only depends on constants and usual nucleon form factors. Therefore, we can calculate this contribution directly.

Combining the exact threshold and near threshold structures, we will derive results for the nucleon structure functions. Using experimental values for usual nucleon form factors, one gets expressions for nucleon to pion nucleon transition form factors. Afterwards, these expressions can be used in order to obtain representations for the desired nucleon structure functions which can be compared with experimental values.
3. Evaluation of the Cross Section

Let us start with the general representation of the scattering process. Our aim is to study the transition process \( e(l, s) + N_i(P,S) \rightarrow e(l', s') + N_f(P',S') + \pi^0(k) \) which we already discussed in the introduction. The momenta and spins of the particles are introduced in this expression and corresponding masses are denoted by \( m_e, m_N \) and \( m_\pi \). Possible combinations of nucleons and pions are constrained by charge conservation. This leads to the following possible transition processes.

\[
e(l, s) + p(P, S) \quad \rightarrow \quad e(l', s') + p(P', S') + \pi^0(k) \tag{3.1}
\]

\[
e(l, s) + p(P, S) \quad \rightarrow \quad e(l', s') + n(P', S') + \pi^+(k) \tag{3.2}
\]

\[
e(l, s) + n(P, S) \quad \rightarrow \quad e(l', s') + n(P', S') + \pi^0(k) \tag{3.3}
\]

\[
e(l, s) + n(P, S) \quad \rightarrow \quad e(l', s') + p(P', S') + \pi^-(k) \tag{3.4}
\]

Being the dominant interaction, we will work with the exchange of one virtual photon with large virtuality \( Q^2 = -q^2 = -(l - l')^2 \). In order to evaluate the nucleon to pion nucleon transition subprocess, one also needs the invariant mass \( W^2 = (P' + k)^2 = (P + q)^2 \). We will often use the center of mass frame concerning this subprocess, given by \( \vec{P}' + \vec{k} = \vec{P} + \vec{q} = 0 \). One gets \( W^2 = (P'_0 + k_0)^2 = (P_0 + q_0)^2 \) consequently. Moreover, one can derive formulas for all required variables which depend on \( Q^2 \) and \( W^2 \) only.

First of all, we consider the pion production at the exact threshold. In this case, one has to deal with \( \vec{P}' = \vec{k} = 0 \). We obtain the connection \( k = \frac{m_\pi}{m_N} P' \). Applying momentum conservation additionally, one gets \( P' = \frac{m_N}{m_N + m_\pi}(P + q) \). That means, we only have to deal with the momenta of the initial particles concerning the nucleon to pion nucleon transition subprocess. Furthermore, one obtains \( W^2 = (m_N + m_\pi)^2 \). Therefore, we introduce the identification \( W_{th} = m_N + m_\pi \) for the value of \( W \) at the production threshold of the pion.

Moving away from the threshold, the produced pion is no longer at rest. In this case, the kinematics are more complicated and one gets \( W > W_{th} \). We will apply the specified center of mass frame together with momentum conservation in order to derive the desired formulas for all required kinematic variables.

Let us evaluate the zero component of the different momenta at first. We obtain the relations \( 2WP_0 = W^2 + Q^2 + m_N^2 \) and \( 2Wq_0 = W^2 - Q^2 - m_N^2 \). Moreover, one gets the formulas \( 2WP'_0 = W^2 + m_N^2 - m_\pi^2 \) and \( 2Wk_0 = W^2 - m_N^2 + m_\pi^2 \) which are independent of \( Q^2 \). At next, the absolute value concerning the vector component of the momenta will be calculated. We gain the expression \( 4W^2|\vec{P}'|^2 = 4W^2|\vec{q}|^2 = (Q^2 + (W + m_N)^2)(Q^2 + (W - m_N)^2) \). Furthermore, one gets the result \( 4W^2|\vec{P}'|^2 = 4W^2|\vec{k}|^2 = (W^2 - (m_N + m_\pi)^2)(W^2 - (m_N - m_\pi)^2) \) being independent of \( Q^2 \). Let us finish with the necessary scalar products of the momenta. We receive \( 2(P \cdot q) = Q^2 + W^2 - m_N^2 \) and also \( 2(P' \cdot k) = W^2 - m_\pi^2 \) which is independent of \( Q^2 \). Other scalar products are not required.

The discussed scenario is described by a scattering process of two particles in the initial states and three particles in the final states. Therefore, we have to study the following general formula for the cross section.
3. Evaluation of the Cross Section

\[
\frac{d\sigma}{dE_{lab}'} = \frac{\beta(W)}{2(4\pi)^5m_N^2} |\mathcal{M}|^2 
\]

Our aim is to derive an expression for this cross section which is compatible to the cross section representation of inclusive inelastic electron nucleon scattering.

We start with the hadronic integration \(d^3\vec{P}/P_0'\) in combination with the momentum conservation delta function \(\delta^{(4)}(l' + P' + k - l - P)\). In order to solve this three dimensional integration over the four dimensional delta function, one must equalize the dimensions. Therefore, we have to define a modified momentum \(R = (R_0, \vec{P})\). According to this, one gets the relation \(R_0^2 - P_0'^2 = R^2 - m_N^2\) immediately. These constructions must be used together with the universal integration result \(2P_0' \int dR_0 \Theta(R_0)\delta(R_0^2 - P_0'^2) = 1\). Consequently, we obtain the four dimensional integration \(\int d^4R_0 \Theta(R_0)\delta(R_0^2 - m_N^2)\delta^{(3)}(R - (P + q - k))\) for the desired object. This integration delivers the result \(2\Theta(P_0 + q_0 - k_0)\delta((P + q - k)^2 - m_N^2)\) which must be evaluated in combination with another integration.

At next, we have to consider the hadronic integration \(d^3\vec{k}/k_0\). One can apply spherical coordinates in order to evaluate this integration. Therefore, we must use the basic representation \(\int d^3\vec{k} = \int |\vec{k}|^2 d|\vec{k}| d\Omega_\pi\). Hereby, we introduced the scattering angle of the pion \(\Omega_\pi\). Starting with the basic formula \(|\vec{k}|^2 = k_0^2 - m_\pi^2\), one obtains the relation \(\int |\vec{k}|^2 d|\vec{k}| = \int k_0 dk_0\).

When we combine these expressions, we end up with the integration \(\int |\vec{k}| d|\vec{k}| d\Omega_\pi\) for the desired object. The integration \(d\Omega_\pi\) cannot be computed yet, but the integration \(dk_0\) can be calculated together with the previous result.

Let us proceed with the remaining integration \(dk_0\) together with the received theta and delta function. Therefore, we have to apply the center of mass frame. At first, we rewrite the required expression \((P + q - k)^2 - m_N^2 = 2(P + q)(q - k) - q^2 + m_N^2\). Applying the center of mass frame, one obtains the relation \(2(P + q)(q - k) - q^2 + m_N^2 = 2(P_0 + q_0)(q_0 - k_0) - q^2 + m_N^2\). When we use this representation for the desired object, one can integrate over \(k_0\) leading to the result \(\Theta(2(P_0 + q_0)P_0 + q^2 - m_N^2)/(P_0 + q_0)\). Using the center of mass frame again, we obtain the relation \(2(P_0 + q_0)P_0 + q^2 - m_N^2 = (P_0 + q_0)^2 + m_N^2 - m_N^2\). According to this, we gain the final result \(W^{-1}\Theta(W^2 + m_N^2 - m_N^2)\) where we inserted the invariant mass. This result can be reduced to \(W^{-1}\) immediately.

By convention, we introduce a function called \(\beta(W)\) in order to describe the remaining component \(|\vec{k}|\) under the condition \(2|\vec{k}| = W\beta(W)\), noting that \(\beta(W) = 0\).

\[
\beta(W) = \sqrt{1 - \frac{(m_N + m_\pi)^2}{W^2}} \sqrt{1 - \frac{(m_N - m_\pi)^2}{W^2}} \quad (3.6)
\]

We finish with the leptonic integration \(d^3\vec{P}/l_0'\) together with the flux factor. One has to use the laboratory frame for this part which is given by \(\vec{P} = 0\). Furthermore, one can neglect the electron mass in comparison with the other elements. We label the energy of the initial lepton with \(E_{lab}\) and the energy of the final lepton with \(E_{lab}'\) in this frame. The scattering angle of the final lepton is denoted by \(\Omega_{lab}'\). One obtains the integration \(\int dE_{lab}'E_{lab}' d\Omega_{lab}'\) consequently. Moreover, one gets the relation \((P \cdot l)^2 - m_N^2 m_e^2 = m_N^2 E_{lab}^2\) which can be used to simplify the flux factor.

Finally, we insert the received expressions in (3.5). This leads to the following intermediate result for the cross section.

\[
\frac{d\sigma}{dE_{lab}' d\Omega_{lab}'} = \int \left( \frac{E_{lab}'}{E_{lab}} \right) \frac{\beta(W)}{2(4\pi)^5m_N} |\mathcal{M}|^2 
\]

8
The scattering amplitude contribution can be written as $|\mathcal{M}|^2 = ((4\pi\alpha_{em})^2/Q^4)L^{\mu\nu}M_{\mu\nu}$. This representation can already be obtained for elastic electron nucleon scattering. The leptonic tensor $L^{\mu\nu} = (\bar{u}(l', s')\gamma^\mu u(l, s))(\bar{u}(l', s')\gamma^\nu u(l, s))^\dagger$ for this inelastic process is identical to the leptonic tensor of the basic elastic process. In order to derive the expression of the hadronic transition tensor, we must replace the nucleon transition probability matrix element $(N_f(P', S')|j^{em}_{\mu}(0)|N_i(P, S))$, which appears in the hadronic tensor of the basic elastic process, with the nucleon to pion nucleon transition probability matrix element $(N_f(P', S')\pi^{\alpha}(k)|j^{em}_{\mu}(0)|N_i(P, S))$, being required for the discussed inelastic process. One gets $M_{\mu\nu} = ((N_f(P', S')\pi^{\alpha}(k)|j^{em}_{\mu}(0)|N_i(P, S)))((N_f(P', S')\pi^{\alpha}(k)|j^{em}_{\mu}(0)|N_i(P, S)))^\dagger$ as expression for the hadronic transition tensor. This tensor is related to another hadronic transition tensor as follows.

$$W_{\mu\nu} = \int \frac{\beta(W) d\Omega}{2(4\pi)^3 m_N} M_{\mu\nu}$$  \hspace{1cm} (3.8)

Inserting these structures into (3.7), one gets the final result for the cross section.

$$\frac{d\sigma}{dE'_{lab} dW'_{lab}} = \int \left( \frac{E'_{lab}}{E_{lab}} \right)^2 \frac{\alpha^2_{em}}{Q^4} L^{\mu\nu} W_{\mu\nu}$$  \hspace{1cm} (3.9)

The introduced representations for the specified tensors are not complete yet. In order to calculate the cross section, one has to choose the target nucleon. Moreover, the transition states are undetected and so we have to add them. According to this, it is necessary that we have to fix the initial state and to sum over all possible final states for the nucleon to pion nucleon transition. Furthermore, the evaluation of the cross section requires that we have to distinguish between the unpolarized and the polarized case concerning the spins of the initial particles. These configurations can be chosen for the initial electron and nucleon independently. Additionally, we have to sum over the spins of all final particles, because they cannot be detected either. The derived result for the cross section given in (3.9) is identical to the cross section representation of inclusive inelastic electron nucleon scattering and we already know from this process that the hadronic transition tensor $W_{\mu\nu}$ can be parameterized in terms of nucleon structure functions. This important connection will allow us to extract results for the nucleon structure functions finally.

All nucleon structure functions can be presented as functions of the variables $Q^2$ and $W^2$. Additionally, they depend on the nucleon target. Nevertheless, the parametrization can be done for both nucleons simultaneously.

In the case of unpolarized nucleon targets, $W_{\mu\nu}$ can be parameterized by two nucleon structure functions $F_1$ and $F_2$. This parametrization is symmetric under the interchange of $\mu \leftrightarrow \nu$ and according to this, we call this tensor $W^S_{\mu\nu}$.

$$W^S_{\mu\nu} = \frac{1}{m_N} \left[ F_1(W, Q^2)(q_{\mu q_{\nu}}/q^2 - g_{\mu\nu}) + \frac{1}{P \cdot q} F_2(W, Q^2)(P_{\mu} - P \cdot q_{\mu})(P_{\nu} - P \cdot q_{\nu}) \right]$$  \hspace{1cm} (3.10)

In the case of polarized nucleon targets, we have to deal with an antisymmetric parametrization $W^A_{\mu\nu}$, additional to the already introduced symmetric parametrization. This contribution can be expressed by two nucleon structure functions $G_1$ and $G_2$.

$$W^A_{\mu\nu} = i\varepsilon_{\mu\nu\rho\sigma} \left[ \frac{1}{P \cdot q} G_1(W, Q^2) q^\rho S^\sigma + \frac{1}{P \cdot q} G_2(W, Q^2)(q^\rho S^\sigma - S \cdot q q^\rho P^\sigma) \right]$$  \hspace{1cm} (3.11)
3. Evaluation of the Cross Section

Finally, we have to consider the required nucleon to pion nucleon transition probability matrix element given by \( \langle N_f(P', S') \pi^a(k) | J_{\mu}^{em}(0) | N_i(P, S) \rangle \) in order to calculate the hadronic transition tensor. We need a representation of this matrix element at large momentum transfer and near the production threshold of the pion. One can show that the desired matrix element can be presented as sum of an exact threshold part and a near threshold part. The first component is known as S-wave part and the second component is called P-wave part. Let us now derive representations for these objects.

At the exact threshold limit, one can express the specified matrix element by an expansion in form factors. This approach is already known from the evaluation of usual nucleon transition probability matrix elements. We designate the nucleon to pion nucleon transition form factors by \( A(\gamma^* N_i \rightarrow N_f \pi^a) \). Further information on this topic can be taken from [9]. The discussed expression must respect the current conservation relation \( q^\mu \langle N_f(P', S') \pi^a(k) | J_{\mu}^{em}(0) | N_i(P, S) \rangle = 0 \). Current conservation can be realized by the electromagnetic vertex replacement \( \gamma_{\mu} \rightarrow \gamma_{\mu} - (q^2/q^2)q_{\mu} \). Representing the exact threshold kinematics, one obtains the S-wave part of the process.

\[
\text{S-wave} = A(\gamma^* N_i \rightarrow N_f \pi^a)(Q^2)\bar{N}(P', S')(\gamma_{\mu} - \frac{i}{q^2}q_{\mu})\gamma_5 N(P, S) \tag{3.12}
\]

In the near threshold region, we can use soft pion theorems. Additional to the already considered contribution for the exact threshold situation, we get pole contributions known from soft pion theorems. The emission of the pion from the final nucleon generates a large contribution for \( W^2 \) close to \( W_{th}^2 \) and therefore the dominant contribution. The emission of the pion from the initial nucleon is suppressed at large \( Q^2 \) and one can neglect it consequently. Additional to these both nucleon poles, we obtain a pion pole which can also be neglected at large \( Q^2 \). The expression for the dominant contribution was calculated under the constraint that it vanishes at the exact threshold. Moreover, one has to apply the specified current conservation replacement again. According to this, one gets the following expression for the P-wave part of the process:

\[
\text{P-wave} = \frac{ig_{\pi}^2G^N_M(Q^2)}{2f_\pi(W^2 - m_N^2)}\bar{N}(P', S')k\gamma_5(P' + m_N)(\gamma_{\mu} - \frac{i}{q^2}q_{\mu})N(P, S) \tag{3.13}
\]

For further calculations, it is convenient to apply equation of motion in order to reduce the complexity of this structure. We get another representation of the P-wave part:

\[
\text{P-wave} = \frac{ig_{\pi}^2G^N_M(Q^2)}{f_\pi(W^2 - m_N^2)}\bar{N}(P', S')(\gamma_{\mu} - \frac{i}{q^2}q_{\mu})\gamma_5 N(P, S) \tag{3.14}
\]

Using this decomposition of the desired matrix element, the hadronic transition tensor can be calculated. The contraction with the leptonic tensor will ensure that some contributions of the hadronic transition tensor will not contribute to the cross section.

Based on this approach, we will derive expressions for the nucleon structure functions in a comprehensive region. Finally, we will consider the discussed limit of large momentum transfer and small invariant masses, meaning that \( x_B = Q^2/(2(P \cdot q)) = Q^2/(Q^2 + W^2 - m_N^2) \) is close to the threshold value \( x_B = [(1 + (2m_N + m_\pi)m_\pi)/Q^2]^{-1} \). For large \( Q^2 \) one gets \( x_B \rightarrow 1 \) consequently.
4. Determination of the Structure Functions

Our aim is to calculate the leptonic tensor $L_{\mu\nu}$ and also the hadronic transition tensors $M_{\mu\nu}$ and $W_{\mu\nu}$ at the threshold and near the threshold. The result of the leptonic tensor is already known, but we will present its derivation in order to introduce the required technique. The calculation of these tensors requires the summation over the spins of the final particles. Concerning the spins of the initial particles, we have to distinguish between the unpolarized and the polarized case. The unpolarized scenario requires the average over the spins of the initial particles and for the polarized case, we have to express the spins of the initial particles by a corresponding covariant spin vector. We realized that the hadronic transition tensor $W_{\mu\nu}$ can be parameterized in terms of nucleon structure functions and we will determine them finally. For their computation, we have to fix the initial state and to sum over all possible final states. Therefore, we define $X^p = \{p\pi^0, n\pi^+\}$ and $X^n = \{n\pi^0, p\pi^-\}$ as the possible transition states for the proton and the neutron.

4.1. Leptonic Tensor

At first, we calculate the leptonic tensor for unpolarized electrons.

$$L_{\mu\nu} = \frac{1}{2} \sum_{s=\uparrow,\downarrow} \sum_{s'=\uparrow,\downarrow} (\bar{u}(l',s')\gamma^\mu u(l, s))(\bar{u}(l',s')\gamma^\nu u(l, s))^\dagger$$

(4.1)

In order to evaluate this expression, we write out the Lorentz indices of all components. These elements can be arranged to create a trace. Using the required identity given in the appendix about Quantum Chromodynamics, one gets the following representation.

$$L_{\mu\nu} = \frac{1}{2} \text{Tr}[(\not{p} + m_e)\gamma^\mu (\not{p} + m_e)\gamma^\nu]$$

(4.2)

One can see that this trace structure is symmetric under the interchange of $\mu \leftrightarrow \nu$ and according to this, we can write $L_{\mu\nu} = L_{\nu\mu}$. 

$$L^S_{\mu\nu} = 2(\gamma_{\mu} l^\nu + l^\mu \gamma_{\nu}) + q^2 g^{\mu\nu}$$

(4.3)

At next, we calculate the leptonic tensor for polarized electrons.

$$L_{\mu\nu} = \sum_{s'=\uparrow,\downarrow} (\bar{u}(l',s')\gamma^\mu u(l, s))(\bar{u}(l',s')\gamma^\nu u(l, s))^\dagger$$

(4.4)

Like in the previous case, one can write out the Lorentz indices of all components and order them to create a trace. Afterwards, we can apply the required identities summarized in the appendix about Quantum Chromodynamics. This leads to the following expression.

$$L_{\mu\nu} = \text{Tr}[(\not{p} + m_e)\gamma^\mu (\not{p} + m_e)\frac{1 + \gamma^S_5}{2}\gamma^\nu]$$

(4.5)
4. Determination of the Structure Functions

This trace structure is the sum of a symmetric and an antisymmetric part and the symmetric part is identical to $L_{S}^{\mu\nu}$. We can use this property and write $L_{A}^{\mu\nu} = L_{S}^{\mu\nu} + L_{A}^{\mu\nu}$. Consequently, we need to compute the antisymmetric trace structure only.

$$L_{A}^{\mu\nu} = -2i m_{e} \varepsilon^{\mu\nu\rho\sigma} q_{\rho} s_{\sigma}$$ (4.6)

One can expect the same symmetry properties for the hadronic transition tensors as obtained for the leptonic tensor.

4.2. Unpolarized Hadronic Tensor at the Threshold

We have to consider the hadronic transition tensors for unpolarized nucleon targets and at the production threshold of the pion. In this case, one only needs the S-wave part at $W = W_{th}$.

Let us start with the calculation of $M_{\mu\nu}$ expressible as follows.

$$M_{\mu\nu} = \frac{1}{2} \sum_{S=\uparrow, \downarrow} \sum_{S' = \uparrow, \downarrow} |\mathcal{A}(\pi^{+} N_{i} \rightarrow N_{f} \pi^{0})|^{2}$$

$$[\bar{N}(P', S') \langle \gamma_{\mu} - \frac{i}{q^{2} q_{\nu}} \gamma_{5} N(P, S) \rangle [\bar{N}(P, S') \langle \gamma_{\nu} - \frac{i}{q^{2} q_{\nu}} \gamma_{5} N(P, S) \rangle]$$ (4.7)

Applying the required identity given in the appendix about Quantum Chromodynamics, one gets the following trace structure for this representation.

$$M_{\mu\nu} = \frac{1}{2} |\mathcal{A}(\pi^{+} N_{i} \rightarrow N_{f} \pi^{0})|^{2} Tr([P' + m_{N}]_{\mu\nu} - \frac{i}{q^{2} q_{\nu}} (P - m_{N})_{\mu\nu} - \frac{i}{q^{2} q_{\nu}})$$ (4.8)

We realize that this trace structure is symmetric and so one can write $M_{\mu\nu} = M_{\mu\nu}^{S}$. At next, we have to evaluate the obtained expression.

$$M_{\mu\nu}^{S} = 2 |\mathcal{A}(\pi^{+} N_{i} \rightarrow N_{f} \pi^{0})|^{2}$$

$$\left[\left(\frac{P \cdot P'}{m_{N}^{2}} + \frac{q_{\mu} q_{\nu}}{q^{2}} - g_{\mu\nu}\right) + P_{\mu} P'_{\nu} + P'_{\mu} P_{\nu}\right]$$

$$- \frac{P' \cdot q}{q^{2}} (P'_{\mu} q_{\nu} + q_{\mu} P_{\nu}) - \frac{P \cdot q}{q^{2}} (P_{\mu} q_{\nu} + q_{\mu} P'_{\nu}) + \frac{2 (P \cdot q) (P' \cdot q)}{q^{4}} q_{\mu} q_{\nu}$$ (4.9)

Let us proceed with the calculation of the other tensor $W_{\mu\nu}$. One can write $W_{\mu\nu} = W_{\mu\nu}^{S}$, because the considered symmetry is conserved.

$$W_{\mu\nu}^{S} = \int \frac{d \Omega_{\pi}}{(4 \pi)^{2} m_{N}} |\mathcal{A}(\pi^{+} N_{i} \rightarrow N_{f} \pi^{0})|^{2}$$

$$\left[\left(\frac{P \cdot P'}{m_{N}^{2}} + \frac{q_{\mu} q_{\nu}}{q^{2}} - g_{\mu\nu}\right) + P_{\mu} P'_{\nu} + P'_{\mu} P_{\nu}\right]$$

$$- \frac{P' \cdot q}{q^{2}} (P'_{\mu} q_{\nu} + q_{\mu} P_{\nu}) - \frac{P \cdot q}{q^{2}} (P_{\mu} q_{\nu} + q_{\mu} P'_{\nu}) + \frac{2 (P \cdot q) (P' \cdot q)}{q^{4}} q_{\mu} q_{\nu}$$ (4.10)

We must compare this result with (3.10) in order to get expressions for the unpolarized nucleon structure functions. Therefore, we need the same tensor structures. The general expansion in nucleon structure functions does not depend on $P'$ and so one has to eliminate it. Moreover, we have to evaluate the remaining integration. According to this, one can use
4.2. Unpolarized Hadronic Tensor at the Threshold

the relation $P' = \frac{m_N}{m_N + m_p}(P + q)$ which is valid at the production threshold of the pion and the corresponding integration result $\int d\Omega_\pi = 4\pi$.

\[
W_{\mu\nu}^S = \frac{\beta(W)}{(4\pi)^2 m_N + m_\pi} |A(\gamma^* N_i \rightarrow N_f \pi^n)|^2 \]  
\[
\left[ ((P \cdot q) + 2m_N^2 + m_N m_\pi) \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) + 2(P_\mu - \frac{P \cdot q}{q^2} q_\mu)(P_\nu - \frac{P \cdot q}{q^2} q_\nu) \right]
\]  
(4.11)

Let us now derive results for the unpolarized nucleon structure functions.

\[
F_1(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \left( (P \cdot q) + 2m_N + m_N m_\pi \right) |A(\gamma^* N_i \rightarrow N_f \pi^n)|^2
\]  
(4.12)

\[
F_2(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} 2(P \cdot q) |A(\gamma^* N_i \rightarrow N_f \pi^n)|^2
\]  
(4.13)

At $W = W_{th}$, we obtain $2(P \cdot q) = Q^2 + 2m_N m_\pi + m_\pi^2$ leading to these expressions.

\[
F_1(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \left( Q^2 + (2m_N + m_\pi) \right) |A(\gamma^* N_i \rightarrow N_f \pi^n)|^2
\]  
(4.14)

\[
F_2(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \left( Q^2 + (2m_N + m_\pi) m_\pi \right) |A(\gamma^* N_i \rightarrow N_f \pi^n)|^2
\]  
(4.15)

Afterwards, one can create the final results for the proton and neutron structure functions.

\[
F_1^p(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \frac{1}{2} \left( Q^2 + (2m_N + m_\pi) \right) \sum_{X=X_p} |A(\gamma^* p \rightarrow X)|^2
\]  
(4.16)

\[
F_1^n(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \frac{1}{2} \left( Q^2 + (2m_N + m_\pi) \right) \sum_{X=X_n} |A(\gamma^* n \rightarrow X)|^2
\]  
(4.17)

\[
F_2^p(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \left( Q^2 + (2m_N + m_\pi) m_\pi \right) \sum_{X=X_p} |A(\gamma^* p \rightarrow X)|^2
\]  
(4.18)

\[
F_2^n(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \left( Q^2 + (2m_N + m_\pi) m_\pi \right) \sum_{X=X_n} |A(\gamma^* n \rightarrow X)|^2
\]  
(4.19)

Finally, we consider the case when $Q^2$ is large in comparison with all appearing hadron masses. At this limit, one gets the following representations.

\[
F_1^p(W, Q^2) = \frac{Q^2 \beta(W)}{2(4\pi)^2} \sum_{X=X_p} |A(\gamma^* p \rightarrow X)|^2
\]  
(4.20)

\[
F_1^n(W, Q^2) = \frac{Q^2 \beta(W)}{2(4\pi)^2} \sum_{X=X_n} |A(\gamma^* n \rightarrow X)|^2
\]  
(4.21)

\[
F_2^p(W, Q^2) = \frac{Q^2 \beta(W)}{(4\pi)^2} \sum_{X=X_p} |A(\gamma^* p \rightarrow X)|^2
\]  
(4.22)

\[
F_2^n(W, Q^2) = \frac{Q^2 \beta(W)}{(4\pi)^2} \sum_{X=X_n} |A(\gamma^* n \rightarrow X)|^2
\]  
(4.23)

We realize that the result concerning the second unpolarized nucleon structure function for a specified nucleon is precisely a factor of two larger than the result concerning the first unpolarized nucleon structure function for the same nucleon.
4.3. Unpolarized Hadronic Tensor near the Threshold

We proceed with the calculation of the hadronic transition tensors for unpolarized nucleon targets and near the production threshold of the pion. Additional to the S-wave part, one has to deal with the P-wave part now. We write these tensors as sum of three components corresponding to the S-wave contribution, the P-wave contribution and the interference term.

At first, we evaluate the component of $M_{\mu\nu}$ which depends on the S-wave part only.

$$M_{\mu\nu}^{(S)} = \frac{1}{2} \sum_{S=\uparrow,\downarrow} \sum_{S'=\uparrow,\downarrow} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2$$

$$[\bar{N}(P', S')(\gamma_{\mu} - \frac{\not{q}}{q^2} q_{\nu})\gamma_5 N(P, S)] [\bar{N}(P', S')(\gamma_{\nu} - \frac{\not{q}}{q^2} q_{\mu})\gamma_5 N(P, S)]^\dagger$$

(4.24)

The corresponding trace term is already known from the previous section.

$$M_{\mu\nu}^{(S)} = \frac{1}{2} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2 \text{Tr}[(P' + m_N)(\gamma_{\mu} - \frac{\not{q}}{q^2} q_{\mu})(P - m_N)(\gamma_{\nu} - \frac{\not{q}}{q^2} q_{\nu})]$$

(4.25)

We can write $M_{\mu\nu}^{(S)} = m_{\mu\nu}^{(S)}$, because the obtained structure is symmetric. The calculation of this part leads to the following result.

$$M_{\mu\nu}^{(S)} = 2|A(\gamma^* N_i \rightarrow N_f \pi^a)|^2$$

$$\left[(P \cdot P') + m_N^2 \right] (q_{\mu} q_{\nu} q^2 - q_{\mu} q_{\nu}) + P_{\mu} P_{\nu}' + P_{\mu}' P_{\nu}$$

$$- \frac{P' \cdot q}{q^2} (P_{\mu} q_{\nu} + q_{\mu} P_{\nu}) - \frac{P \cdot q}{q^2} (P_{\mu}' q_{\nu} + q_{\mu} P_{\nu}') + \frac{2(P \cdot q)(P' \cdot q)}{q^4} q_{\mu} q_{\nu}$$

(4.26)

Let us now determine the S-wave depending component of $W_{\mu\nu}$. We get the expression $W_{\mu\nu}^{(S)} = W_{\mu\nu}^{(S)}$ because of symmetry conservation.

$$W_{\mu\nu}^{(S)} = \int \frac{\beta(\mathcal{W}) \, d\Omega_{\mathcal{W}}}{(4\pi)^3 m_{\mathcal{N}}} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2$$

$$\left[(P \cdot P') + m_N^2 \right] (q_{\mu} q_{\nu} q^2 - q_{\mu} q_{\nu}) + P_{\mu} P_{\nu}' + P_{\mu}' P_{\nu}$$

$$- \frac{P' \cdot q}{q^2} (P_{\mu} q_{\nu} + q_{\mu} P_{\nu}) - \frac{P \cdot q}{q^2} (P_{\mu}' q_{\nu} + q_{\mu} P_{\nu}') + \frac{2(P \cdot q)(P' \cdot q)}{q^4} q_{\mu} q_{\nu}$$

(4.27)

At next, we evaluate the component of $M_{\mu\nu}$ which depends on the P-wave part only.

$$M_{\mu\nu}^{(P)} = \frac{1}{2} \sum_{S=\uparrow,\downarrow} \sum_{S'=\uparrow,\downarrow} \frac{g_b^2 (\gamma_{\mu}^b)(G_N^b(Q)^2)^2}{f_{\pi}^2 (W^2 - m_N^2)^2}$$

$$\left[(P' \cdot k^2)[\bar{N}(P', S')(\gamma_{\mu} - \frac{\not{q}}{q^2} q_{\nu})\gamma_5 N(P, S)] [\bar{N}(P', S')(\gamma_{\nu} - \frac{\not{q}}{q^2} q_{\mu})\gamma_5 N(P, S)]^\dagger$$

$$- m_N (P' \cdot k)[\bar{N}(P', S')\gamma_{\mu} - \frac{\not{q}}{q^2} q_{\nu})\gamma_5 N(P, S)] [\bar{N}(P', S')\gamma_{\nu} - \frac{\not{q}}{q^2} q_{\mu})\gamma_5 N(P, S)]^\dagger$$

$$- m_N (P' \cdot k)[\bar{N}(P', S')\gamma_{\mu} - \frac{\not{q}}{q^2} q_{\nu})\gamma_5 N(P, S)] [\bar{N}(P', S')\gamma_{\nu} - \frac{\not{q}}{q^2} q_{\mu})\gamma_5 N(P, S)]^\dagger$$

$$+ m_N^2 [\bar{N}(P', S')\gamma_{\mu} - \frac{\not{q}}{q^2} q_{\nu})\gamma_5 N(P, S)] [\bar{N}(P', S')\gamma_{\nu} - \frac{\not{q}}{q^2} q_{\mu})\gamma_5 N(P, S)]^\dagger$$

(4.28)
One can derive different trace structures for these elements as follows.

\[
M^{(P)}_{\mu\nu} = g^2 A (r_\pi^2 (G_M^N(Q^2))^2) \\
\frac{2 f_{\pi}^2 (W^2 - m_N^2)^2}{2 f_{\pi}^2 (W^2 - m_N^2)^2} \\
\left[ (P' \cdot k) \frac{2}{q^2} \text{Tr}[k(P' + m_N)(\gamma_\mu - \frac{i}{q^2} q_\mu)(P - m_N)(\gamma_\nu - \frac{i}{q^2} q_\nu)] \right. \\
- m_N (P' \cdot k) \frac{2}{q^2} \text{Tr}[k(P' + m_N)(\gamma_\mu - \frac{i}{q^2} q_\mu)(P - m_N)(\gamma_\nu - \frac{i}{q^2} q_\nu)] \\
- m_N (P' \cdot k) \frac{2}{q^2} \text{Tr}[k(P' + m_N)(\gamma_\mu - \frac{i}{q^2} q_\mu)(P - m_N)(\gamma_\nu - \frac{i}{q^2} q_\nu)] \\
+ m_N^2 \frac{2}{q^2} \text{Tr}[k(P' + m_N)(\gamma_\mu - \frac{i}{q^2} q_\mu)(P - m_N)(\gamma_\nu - \frac{i}{q^2} q_\nu)] \right] \\
(4.29)
\]

We can write \(M^{(P)}_{\mu\nu} = M^{S(P)}_{\mu\nu}\), because the sum of these structures is symmetric. After evaluation of these structures, one obtains the following result.

\[
M^{S(P)}_{\mu\nu} = ((P' \cdot k)^2 - m_N^2) \frac{2 g^2 A (r_\pi^2 (G_M^N(Q^2))^2)}{2 f_{\pi}^2 (W^2 - m_N^2)^2} \\
\left[ ((P \cdot P') - m_N^2) \frac{2}{q^2} \left( P_\mu P_\nu - g_{\mu\nu} \right) + P_\mu P'_\nu + P'_\mu P_\nu \\
- \frac{P' \cdot q}{q^2} (P_\nu q_\mu + q_\mu P_\nu) \right] \frac{2}{q^2} (P'_\mu q_\nu + q_\nu P'_\mu) + \frac{2(P \cdot q)(P' \cdot q)}{q^4} q_\mu q_\nu \right] \\
(4.30)
\]

For further calculations, we have to apply the center of mass frame where the required relation 4\(((P' \cdot k)^2 - m_N^2)^2\) = \(W^4 \beta^2(W)\) can be derived.

Let us now determine the the P-wave depending component of \(W_{\mu\nu}\). We get the representation \(W^{(P)}_{\mu\nu} = W^{S(P)}_{\mu\nu}\) because of symmetry conservation.

\[
W^{S(P)}_{\mu\nu} = \int \frac{W^4 \beta^2(W) d\Omega_\pi}{(4\pi)^3 m_N} \frac{2 g^2 A (r_\pi^2 (G_M^N(Q^2))^2)}{4 f_{\pi}^2 (W^2 - m_N^2)^2} \\
\left[ ((P \cdot P') - m_N^2) \frac{2}{q^2} \left( P_\mu P_\nu - g_{\mu\nu} \right) + P_\mu P'_\nu + P'_\mu P_\nu \\
- \frac{P' \cdot q}{q^2} (P_\nu q_\mu + q_\mu P_\nu) \right] \frac{2}{q^2} (P'_\mu q_\nu + q_\nu P'_\mu) + \frac{2(P \cdot q)(P' \cdot q)}{q^4} q_\mu q_\nu \right] \\
(4.31)
\]

At last, we consider the contribution to \(M_{\mu\nu}\) depending on the specified interference term.

\[
M^{(I)}_{\mu\nu} = \frac{1}{2} \sum_{S=\uparrow,\downarrow} \sum_{S'=\uparrow,\downarrow} i m_{N_f G_A} r_\pi^2 G_M^N(Q^2) \\
\frac{f_\pi(W^2 - m_N^2)}{f_\pi(W^2 - m_N^2)} A(\gamma^+ N_i \to N_f \pi^0) \left[ \left[ \bar{N}(P', S') (\gamma_\mu - \frac{i}{q^2} q_\mu) \gamma_5 N(P, S) \right] \left[ \bar{N}(P', S') (\gamma_\nu - \frac{i}{q^2} q_\nu) \gamma_5 N(P, S) \right] \right] \\
- \left[ \bar{N}(P', S') (\gamma_\mu - \frac{i}{q^2} q_\mu) \gamma_5 N(P, S) \right] \left[ \bar{N}(P', S') (\gamma_\nu - \frac{i}{q^2} q_\nu) \gamma_5 N(P, S) \right] \right] \\
(4.32)
\]

We get the following trace structures for these components similar to the other cases.
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\[ M^{(I)}_{\mu \nu} = \frac{i m_N G^{N^a}_M(Q^2)}{2 f_\pi} A(\gamma^* N_i \to N_f \pi^a) \]

\[ \text{Tr}[\mathcal{K}(P' + m_N)(\gamma_\mu - \frac{\not{\!P}}{q^2} q_\mu)(P - m_N)(\gamma_\nu - \frac{\not{\!P}}{q^2} q_\nu)] \]

\[ - \text{Tr}[(P' + m_N)\mathcal{K}(\gamma_\mu - \frac{\not{\!P}}{q^2} q_\mu)(P - m_N)(\gamma_\nu - \frac{\not{\!P}}{q^2} q_\nu)] \]  

(4.33)

One can see that this trace structure composition is antisymmetric under the specified interchange. Therefore, we use the notation \( M^{(I)}_{\mu \nu} = M^{A(I)}_{\mu \nu} \).

\[ M^{A(I)}_{\mu \nu} = \frac{4i m_N^2 G^{N^a}_M(Q^2)}{f_\pi(W^2 - m_N^2)} A(\gamma^* N_i \to N_f \pi^a) \]

\[ \left[ \frac{P' \cdot q}{q^2} (q_\mu k_\nu - k_\mu q_\nu) + \frac{q \cdot k}{q^2} (P'_\mu q_\nu - q_\mu P'_\nu - P'_\mu k_\nu + k_\mu P'_\nu) \right] \]  

(4.34)

Let us determine the contribution to \( W_{\mu \nu} \) which depends on the interference term. We use the notation \( W^{(I)}_{\mu \nu} = W^{A(I)}_{\mu \nu} \), because the considered symmetry is conserved.

\[ W^{A(I)}_{\mu \nu} = \int \frac{\beta(W) \, d\Omega_\pi}{(4\pi)^3} \frac{2i m_N G^{N^a}_M(Q^2)}{f_\pi(W^2 - m_N^2)} A(\gamma^* N_i \to N_f \pi^a) \]

\[ \left[ \frac{P' \cdot q}{q^2} (q_\mu k_\nu - k_\mu q_\nu) + \frac{q \cdot k}{q^2} (P'_\mu q_\nu - q_\mu P'_\nu - P'_\mu k_\nu + k_\mu P'_\nu) \right] \]  

(4.35)

This part should not contribute to the cross section, because we consider an unpolarized nucleon target. To prove this, we contract with the leptonic tensor and evaluate the remaining integration. The contraction with the symmetric leptonic tensor is obviously zero, but the contraction with the antisymmetric leptonic tensor delivers a non-vanishing result. We change from the four dimensional representation to the three dimensional representation by applying the center of mass frame.

\[ L^\mu_{\mu} W^{A(I)}_{\nu \nu} = \int W \beta(W) \, d\Omega_\pi \frac{8m_N m_e G^{N^a}_M(Q^2)}{f_\pi(W^2 - m_N^2)} A(\gamma^* N_i \to N_f \pi^a) \overline{k} \cdot (q \times s) \]  

(4.36)

The remaining integration delivers \( \int \, d\Omega_\pi \overline{k} \cdot (q \times s) = 0 \), leading to \( L^\mu_{\mu} W^{A(I)}_{\nu \nu} = 0 \).

The contributing parts of \( W_{\mu \nu} \) are all symmetric, namely \( W^{(S)}_{\mu \nu} \) and \( W^{(P)}_{\mu \nu} \). Like in the exact threshold scenario before, we must compare these results with (3.10) in order to get expressions for the unpolarized nucleon structure functions. According to this, we need the same tensor structures for all components. In order to obtain the required tensor structures, we have to eliminate the dependence on \( P' \) in a special way. Therefore, we have to apply the center of mass frame. In this case, one can evaluate the remaining integration and obtain for \( P_0 \) the correlation \( 2W^2 P'_\mu = (W^2 + m_N^2 - m_\pi^2)(P + q) \). Through this procedure, we can derive the required tensor structures for both components.

\[ W^{(S)}_{\mu \nu} = \frac{\beta(W)}{(4\pi)^2 m_N} \left| A(\gamma^* N_i \to N_f \pi^a) \right|^2 \]

\[ \left[ \frac{W^2 + m_N^2 - m_\pi^2}{W^2} ((P \cdot q) + m_N^2 + m_\pi^2) \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu \nu} \right) \right. \]

\[ + \left. \frac{W^2 + m_N^2 - m_\pi^2}{W^2} (P_\mu - \frac{P \cdot q}{q^2} q_\mu)(P_\nu - \frac{P \cdot q}{q^2} q_\nu) \right] \]  

(4.37)
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\[ W^{S(P)}_{\mu\nu} = \frac{W^4\beta^3(W)}{\Lambda^2} \frac{g_A^2(\tau^a_p)^2(G_M^N(Q^2))^2}{4f_\pi^2(W^2 - m_N^2)^2} \]

\[ \left[ \frac{W^2 + m_N^2 - m_p^2}{2W^2} \left( (P \cdot q) + m_N^2 \right) \frac{g_{\mu\nu} - g_{\mu\nu}}{q^2} \right] \]

\[ + \frac{W^2 + m_N^2 - m_p^2}{W^2} (P_\mu - \frac{P \cdot q}{q^2} q_\mu) (P_\nu - \frac{P \cdot q}{q^2} q_\nu) \]  

Let us now derive expressions for the unpolarized nucleon structure functions.

\[ F_1(W, Q^2) = \]

\[ + \left( (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 - m_N^2) + 2W^2 m_N^2 \right) \frac{\beta(W)}{2(4\pi)^2 W^2} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2 \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 - m_N^2) \frac{g_A^2(\tau^a_p)^2(G_M^N(Q^2))^2W^2\beta^3(W)}{8(4\pi)^2 f_\pi^2(W^2 - m_N^2)^2} \]

\[ F_2(W, Q^2) = \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 - m_N^2) \frac{\beta(W)}{2(4\pi)^2 W^2} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2 \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 - m_N^2) \frac{g_A^2(\tau^a_p)^2(G_M^N(Q^2))^2W^2\beta^3(W)}{16(4\pi)^2 f_\pi^2(W^2 - m_N^2)^2} \]

Applying the kinematic relation \( 2(P \cdot q) = Q^2 + W^2 - m_N^2 \), one gets the following results.

\[ F_1(W, Q^2) = \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 + m_N^2) + 4W^2 m_N^2 \frac{\beta(W)}{4(4\pi)^2 W^2} \sum_{X = X_p} |A(\gamma^* p \rightarrow X)|^2 \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 + m_N^2) \frac{g_A^2(\tau^a_p)^2(G_M^N(Q^2))^2W^2\beta^3(W)}{16(4\pi)^2 f_\pi^2(W^2 - m_N^2)^2} \]

\[ F_2(W, Q^2) = \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 - m_N^2) \frac{\beta(W)}{2(4\pi)^2 W^2} \sum_{X = X_n} |A(\gamma^* n \rightarrow X)|^2 \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 - m_N^2) \frac{g_A^2(\tau^a_p)^2(G_M^N(Q^2))^2W^2\beta^3(W)}{8(4\pi)^2 f_\pi^2(W^2 - m_N^2)^2} \]

At next, we can create the final results for the proton and neutron structure functions.

\[ F_1^p(W, Q^2) = \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 + m_N^2) + 4W^2 m_N^2 \frac{\beta(W)}{4(4\pi)^2 W^2} \sum_{X = X_p} |A(\gamma^* p \rightarrow X)|^2 \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 + m_N^2) \frac{3g_A^2(G_M^N(Q^2))^2W^2\beta^3(W)}{16(4\pi)^2 f_\pi^2(W^2 - m_N^2)^2} \]

\[ F_1^n(W, Q^2) = \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 + m_N^2) + 4W^2 m_N^2 \frac{\beta(W)}{4(4\pi)^2 W^2} \sum_{X = X_n} |A(\gamma^* n \rightarrow X)|^2 \]

\[ + (W^2 + m_N^2 - m_p^2)(Q^2 + W^2 + m_N^2) \frac{3g_A^2(G_M^N(Q^2))^2W^2\beta^3(W)}{16(4\pi)^2 f_\pi^2(W^2 - m_N^2)^2} \]
4. Determination of the Structure Functions

\[ F_2^p(W, Q^2) = \]
\[ + (W^2 + m_N^2 - m_H^2)(Q^2 + W^2 - m_N^2) \frac{\beta(W)}{(2\pi)^2 W^2} \sum_{x=x_F} |\mathcal{A}(\gamma^* p \to X)|^2 \]
\[ + (W^2 + m_N^2 - m_H^2)(Q^2 + W^2 - m_N^2) \frac{3g_A^2(G_M^p(Q^2))^2 W^2 \beta^2(W)}{8(4\pi)^2 f_H^2(W^2 - m_N^2)^2} \]
\[ F_2^n(W, Q^2) = \]
\[ + (W^2 + m_N^2 - m_H^2)(Q^2 + W^2 - m_N^2) \frac{\beta(W)}{(2\pi)^2 W^2} \sum_{x=x_F} |\mathcal{A}(\gamma^* n \to X)|^2 \]
\[ + (W^2 + m_N^2 - m_H^2)(Q^2 + W^2 - m_N^2) \frac{3g_A^2(G_M^n(Q^2))^2 W^2 \beta^2(W)}{8(4\pi)^2 f_H^2(W^2 - m_N^2)^2} \]

Finally, we consider the case when \( Q^2 \) is large in comparison with all appearing hadron masses and \( W^2 \) is close to \( W_{th}^2 \). At this limit, one gets the following representations.

\[ F_2^p(W, Q^2) = \frac{Q^2 \beta(W)}{(2\pi)^2} \left[ \sum_{x=x_F} |\mathcal{A}(\gamma^* p \to X)|^2 + \frac{3g_A^2(G_M^p(Q^2))^2 W^2 \beta^2(W)}{4f_H^2(W^2 - m_N^2)^2} \right] \]  
\[ F_1^n(W, Q^2) = \frac{Q^2 \beta(W)}{(2\pi)^2} \left[ \sum_{x=x_F} |\mathcal{A}(\gamma^* n \to X)|^2 + \frac{3g_A^2(G_M^n(Q^2))^2 W^2 \beta^2(W)}{4f_H^2(W^2 - m_N^2)^2} \right] \]  
\[ F_2^n(W, Q^2) = \frac{Q^2 \beta(W)}{(2\pi)^2} \left[ \sum_{x=x_F} |\mathcal{A}(\gamma^* n \to X)|^2 + \frac{3g_A^2(G_M^n(Q^2))^2 W^2 \beta^2(W)}{4f_H^2(W^2 - m_N^2)^2} \right] \]  
\[ F_2^p(W, Q^2) = \frac{Q^2 \beta(W)}{(2\pi)^2} \left[ \sum_{x=x_F} |\mathcal{A}(\gamma^* p \to X)|^2 + \frac{3g_A^2(G_M^p(Q^2))^2 W^2 \beta^2(W)}{4f_H^2(W^2 - m_N^2)^2} \right] \]

The important remark here is that the result for the unpolarized nucleon structure function \( F_2(W, Q^2) \) for a specified nucleon is precisely a factor of two larger than the result for the unpolarized nucleon structure function \( F_1(W, Q^2) \) for the same nucleon, which is in agreement with the parton model prediction \( F_2 = 2x_B F_1 \) applying the limit \( x_B \to 1 \).

4.4. Polarized Hadronic Tensor at the Threshold

We have to consider the hadronic transition tensors for polarized nucleon targets and at the production threshold of the pion. In this case, one just needs the S-wave part at \( W = W_{th} \).

Let us begin with the calculation of \( M_{\mu\nu} \) representable as follows.

\[ M_{\mu\nu} = \sum_{S_{Z}=\uparrow,\downarrow} |\mathcal{A}(\gamma^* N_i \to N_f \pi^o)|^2 \]
\[ [\bar{N}(P', S')(\gamma_{\mu} - \frac{\not{q}}{q} q_{\nu})\gamma_5 N(P, S)][\bar{N}(P', S')(\gamma_{\nu} - \frac{\not{q}}{q} q_{\nu})\gamma_5 N(P, S)]^{\dagger} \]

(4.51)

Applying the required identities summarized in the appendix about Quantum Chromodynamics, one obtains the following trace structure for this representation.

\[ M_{\mu\nu} = |\mathcal{A}(\gamma^* N_i \to N_f \pi^o)|^2 \text{Tr}[(P' + m_N)(\gamma_{\mu} - \frac{\not{q}}{q} q_{\mu})(P - m_N)\frac{1 + g_5 q_{\nu}}{2}(\gamma_{\nu} - \frac{\not{q}}{q} q_{\nu})] \]  

(4.52)
4.4. Polarized Hadronic Tensor at the Threshold

This trace structure can be presented as sum of the known symmetric part and a new antisymmetric part. Using this property, we write \( M_{\mu\nu} = M_{\mu\nu}^S + M_{\mu\nu}^A \) and calculate the antisymmetric part only. Based on the Schouten identity, we get this expression.

\[
M_{\mu\nu}^A = 2i m_N \varepsilon_{\mu\nu\rho\sigma} |\mathcal{A}(\gamma^* N_i \to N_f \pi^a)|^2 \\
\left[ \frac{P' \cdot q}{q^2} q^\rho S^\sigma + \frac{P \cdot q}{q^2} q^\rho S^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma \right]
\]  
(4.53)

Let us continue with the calculation of \( W_{\mu\nu} \). Symmetry conservation leads to the representation \( W_{\mu\nu} = W_{\mu\nu}^S + W_{\mu\nu}^A \) including the known result for the symmetric contribution.

\[
W_{\mu\nu}^A = i \varepsilon_{\mu\nu\rho\sigma} \int \frac{\beta(W) d\Omega_\pi}{(4\pi)^3} |\mathcal{A}(\gamma^* N_i \to N_f \pi^a)|^2 \\
\left[ \frac{P' \cdot q}{q^2} q^\rho S^\sigma + \frac{P \cdot q}{q^2} q^\rho S^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma \right]
\]  
(4.54)

We must compare this result with (3.11) in order to derive expressions for the polarized nucleon structure functions. Therefore, we require the introduced exact threshold procedure.

\[
W_{\mu\nu}^A = i \varepsilon_{\mu\nu\rho\sigma} \frac{\beta(W)}{(4\pi)^3 (m_N + m_\pi)} |\mathcal{A}(\gamma^* N_i \to N_f \pi^a)|^2 \\
\left[ m_N q^\rho S^\sigma + (2m_N + m_\pi) \frac{P \cdot q}{q^2} (q^\rho S^\sigma - \frac{S \cdot q}{P \cdot q} q^\rho P^\sigma) \right]
\]  
(4.55)

At next, we can extract results for the polarized nucleon structure functions.

\[
G_1(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} (P \cdot q) |\mathcal{A}(\gamma^* N_i \to N_f \pi^a)|^2
\]  
(4.56)

\[
G_2(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{2m_N + m_\pi}{m_N + m_\pi} \left( \frac{P \cdot q}{q^2} \right)^2 |\mathcal{A}(\gamma^* N_i \to N_f \pi^a)|^2
\]  
(4.57)

We obtain \( 2(P \cdot q) = Q^2 + 2m_N m_\pi + m_\pi^2 \) at \( W = W_{th} \), leading to these expressions.

\[
G_1(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \frac{1}{2} (Q^2 + (2m_N + m_\pi) m_\pi) |\mathcal{A}(\gamma^* N_i \to N_f \pi^a)|^2
\]  
(4.58)

\[
G_2(W, Q^2) = -\frac{\beta(W)}{(4\pi)^2} \frac{2m_N + m_\pi}{m_N + m_\pi} \frac{1}{4Q^2} (Q^2 + (2m_N + m_\pi) m_\pi)^2 |\mathcal{A}(\gamma^* N_i \to N_f \pi^a)|^2
\]  
(4.59)

Afterwards, one can create the final results for the proton and neutron structure functions.

\[
G_1^p(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \frac{1}{2} (Q^2 + (2m_N + m_\pi) m_\pi) \sum_{X = X^p} |\mathcal{A}(\gamma^p p \to X)|^2
\]  
(4.60)

\[
G_1^n(W, Q^2) = \frac{\beta(W)}{(4\pi)^2} \frac{m_N}{m_N + m_\pi} \frac{1}{2} (Q^2 + (2m_N + m_\pi) m_\pi) \sum_{X = X^n} |\mathcal{A}(\gamma^n n \to X)|^2
\]  
(4.61)

\[
G_2^p(W, Q^2) = -\frac{\beta(W)}{(4\pi)^2} \frac{2m_N + m_\pi}{m_N + m_\pi} \frac{1}{4Q^2} (Q^2 + (2m_N + m_\pi) m_\pi)^2 \sum_{X = X^p} |\mathcal{A}(\gamma^p p \to X)|^2
\]  
(4.62)

\[
G_2^n(W, Q^2) = -\frac{\beta(W)}{(4\pi)^2} \frac{2m_N + m_\pi}{m_N + m_\pi} \frac{1}{4Q^2} (Q^2 + (2m_N + m_\pi) m_\pi)^2 \sum_{X = X^n} |\mathcal{A}(\gamma^n n \to X)|^2
\]  
(4.63)
4. Determination of the Structure Functions

Finally, we consider the case when \( Q^2 \) is large in comparison with all appearing hadron masses. At this limit, one gets the following representations:

\[
G_1^p(W, Q^2) = \frac{Q^2 \beta(W)}{2(4\pi)^2} \sum_{X=X^p} |A(\gamma^* p \rightarrow X)|^2
\]

(4.64)

\[
G_1^n(W, Q^2) = \frac{Q^2 \beta(W)}{2(4\pi)^2} \sum_{X=X^n} |A(\gamma^* n \rightarrow X)|^2
\]

(4.65)

\[
G_2^p(W, Q^2) = -\frac{Q^2 \beta(W)}{2(4\pi)^2} \sum_{X=X^p} |A(\gamma^* p \rightarrow X)|^2
\]

(4.66)

\[
G_2^n(W, Q^2) = -\frac{Q^2 \beta(W)}{2(4\pi)^2} \sum_{X=X^n} |A(\gamma^* n \rightarrow X)|^2
\]

(4.67)

We realize that the only difference in the results concerning the polarized nucleon structure functions for a specified nucleon is the opposite sign.

4.5. Polarized Hadronic Tensor near the Threshold

We continue with the evaluation of the hadronic transition tensors for polarized nucleon targets and near the production threshold of the pion. Additional to the S-wave part, one has to deal with the P-wave part. Therefore, we use the introduced tensor splitting again.

We start with the calculation concerning the S-wave depending component of \( M_{\mu\nu} \).

\[
M_{\mu\nu}^{(S)} = \sum_{S' = \uparrow, \downarrow} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2
\]

\[
[N(P', S')(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu)\gamma_5 N(P, S))[N(P', S')(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu)\gamma_5 N(P, S)]^\dagger
\]

(4.68)

The corresponding trace structure is already known from the previous section.

\[
M_{\mu\nu}^{(S)} = |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2 \text{Tr}[(P' + m_N)(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu)(P - m_N) 1 + \frac{S\gamma_5}{2}(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu)]
\]

(4.69)

We obtain the formula \( M_{\mu\nu}^{(S)} = M_{\mu\nu}^{(S)} + M_{\mu\nu}^{(A)} \) because of the symmetry property. Consequently, one only needs to evaluate the antisymmetric part.

\[
M_{\mu\nu}^{(A)} = 2im_N \varepsilon_{\mu\nu\rho\sigma} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2
\]

\[
\left[\frac{P' \cdot q}{q^2} q^\rho S^\sigma + \frac{P \cdot q}{q^2} q^\rho S^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma\right]
\]

(4.70)

The S-wave depending component of \( W_{\mu\nu} \) can be expressed as \( W_{\mu\nu}^{(S)} = W_{\mu\nu}^{(S)} + W_{\mu\nu}^{(A)} \).

\[
W_{\mu\nu}^{(A)} = i\varepsilon_{\mu\nu\rho\sigma} \int \frac{\beta(W) d\Omega_\pi}{(4\pi)^3} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2
\]

\[
\left[\frac{P' \cdot q}{q^2} q^\rho S^\sigma + \frac{P \cdot q}{q^2} q^\rho S^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma\right]
\]

(4.71)

We proceed with the calculation concerning the P-wave depending component of \( M_{\mu\nu} \).
4.5. Polarized Hadronic Tensor near the Threshold

\[
M^{(P)}_{\mu \nu} = \sum_{S' = 1, \perp} \frac{g_A^2 (\tau_f^P)^2 (G_M^N(Q^2))^2}{f_\pi^2 (W^2 - m_N^2)^2} \left[ (P' \cdot k)^2 \tilde{N}(P', S')(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu) \gamma_5 N(P, S) [\tilde{N}(P', S')(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu) \gamma_5 N(P, S)] \right] \\
- m_N (P' \cdot k) [\tilde{N}(P', S')(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu) \gamma_5 N(P, S)] [\tilde{N}(P', S')(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu) \gamma_5 N(P, S)] \right) (4.72) \\
- m_N (P' \cdot k) [\tilde{N}(P', S')(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu) \gamma_5 N(P, S)] [\tilde{N}(P', S')(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu) \gamma_5 N(P, S)] \right] \\
+ m_N^2 [\tilde{N}(P', S')(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu) \gamma_5 N(P, S)] [\tilde{N}(P', S')(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu) \gamma_5 N(P, S)] \right]
\]

One can derive different trace structures for these components as follows.

\[
M^{(P)}_{\mu \nu} = \frac{g_A^2 (\tau_f^P)^2 (G_M^N(Q^2))^2}{f_\pi^2 (W^2 - m_N^2)^2} \left[ (P' \cdot k)^2 Tr[(P' + m_N)(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu)(P - m_N)^{1 + \frac{2}{\gamma_5}}(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu)] \\
- m_N (P' \cdot k) Tr[\not{k}(P' + m_N)(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu)(P - m_N)^{1 + \frac{2}{\gamma_5}}(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu)] \\
- m_N (P' \cdot k) Tr[(P' + m_N)\not{k}(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu)(P - m_N)^{1 + \frac{2}{\gamma_5}}(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu)] \\
+ m_N^2 Tr[\not{k}(P' + m_N)(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu)(P - m_N)^{1 + \frac{2}{\gamma_5}}(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu)] \right]
\]

We gain the formula \( M^{(P)}_{\mu \nu} = M^{S(\rho)}_{\mu \nu} + M^{A(\rho)}_{\mu \nu} \) for the sum of these structures. Therefore, it is only necessary to calculate the antisymmetric part.

\[
M^{A(\rho)}_{\mu \nu} = i m_N \varepsilon_{\mu \nu \rho \sigma} (P' \cdot k)^2 - m_N^2 k^2) \frac{2 g_A^2 (\tau_f^P)^2 (G_M^N(Q^2))^2}{f_\pi^2 (W^2 - m_N^2)^2} \left[ \frac{P' \cdot q}{q^2} q^\rho S^\sigma - \frac{P \cdot q}{q^2} q^\rho S^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma + \frac{S \cdot q}{q^2} q^\rho P^\sigma \right]
\]

One requires the center of mass frame relation \( 4((P' \cdot k)^2 - m_N^2 k^2) = W^2 \beta^2(W) \) at next. The P-wave depending component of \( W_{\mu \nu} \) is given by \( W^{(P)}_{\mu \nu} = W^{S(\rho)}_{\mu \nu} + W^{A(\rho)}_{\mu \nu} \).

\[
W^{A(\rho)}_{\mu \nu} = i m_N \varepsilon_{\mu \nu \rho \sigma} \int \frac{W^4 \beta^2(W) d\Omega_\rho}{(4\pi)^3} g_A^2 (\tau_f^P)^2 (G_M^N(Q^2))^2 \\
\left[ \frac{P' \cdot q}{q^2} q^\rho S^\sigma - \frac{P \cdot q}{q^2} q^\rho S^\sigma - \frac{S \cdot q}{q^2} q^\rho P^\sigma + \frac{S \cdot q}{q^2} q^\rho P^\sigma \right]
\]

We finish with the contribution to \( M_{\mu \nu} \) depending on the specified interference term.

\[
M^{(I)}_{\mu \nu} = \sum_{S' = 1, \perp} \frac{i m_N g_A^2 (\tau_f^P)^2 G_M^N(Q^2)}{f_\pi (W^2 - m_N^2)} A(\gamma^* N_i \rightarrow N f \pi^0) \left[ \tilde{N}(P', S')(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu) \gamma_5 N(P, S) [\tilde{N}(P', S')(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu) \gamma_5 N(P, S)] \right] \\
- [\tilde{N}(P', S')(\gamma_\mu - \frac{\not{q}}{q^2} q_\mu) \gamma_5 N(P, S)] [\tilde{N}(P', S')(\gamma_\nu - \frac{\not{q}}{q^2} q_\nu) \gamma_5 N(P, S)] \right]
\]

\( (4.76) \)
4. Determination of the Structure Functions

One gets the following trace structures for these elements similar to the other cases.

\[
M_{\mu}^{I} = \frac{i m_{NG} G_{N}^{a}}{f_{\pi}(W^{2} - m_{N}^{2})} A(\gamma N_{i} \to N_{f}^{a}) [\text{Tr}[\mathcal{M}(\mathbf{P}', m_{N})(\gamma_{\mu} - \frac{g_{F}^{2}}{q^{2}} q_{\mu})(\mathbf{P} - m_{N})] 1 + \frac{S_{\gamma_{5}}}{2} (\gamma_{\nu} - \frac{g_{F}^{2}}{q^{2}} q_{\nu})] (4.77)
\]

This trace structure composition can be written as \( M_{\mu}^{I} = M_{\mu}^{\Delta(I)} + M_{\mu}^{S(I)} \). Applying the Schouten identity frequently, we receive a representation for the required symmetric part.

\[
M_{\mu}^{S(I)} = \frac{4 m_{N} \alpha_{G} G_{M}^{a}}{f_{\pi}(W^{2} - m_{N}^{2})} A(\gamma N_{i} \to N_{f}^{a}) [\left( \frac{2(P \cdot q)}{q^{2}} q_{\mu} q_{\nu} - \frac{1}{q^{2}} (P_{\mu} q_{\mu} + q_{\mu} P_{\nu} - q_{\mu} q_{\nu}) - g_{\mu} q_{\nu} \right) \epsilon_{\alpha \beta \gamma \delta} P^{\alpha} k^{\beta} P^{\gamma} S^{\delta}
- \left( P \cdot q \right) q_{\mu} q_{\nu} - P_{\mu} \epsilon_{\nu \beta \gamma \delta} k^{\beta} P^{\gamma} S^{\delta} - \left( P \cdot q \right) q_{\mu} q_{\nu} - P_{\mu} \epsilon_{\mu \beta \gamma \delta} k^{\beta} P^{\gamma} S^{\delta}
+ \left( \frac{q \cdot k}{q^{2}} q_{\mu} - k_{\mu} \right) \epsilon_{\nu \beta \gamma \delta} P^{\gamma} S^{\delta} + \left( \frac{q \cdot k}{q^{2}} q_{\nu} - k_{\nu} \right) \epsilon_{\mu \beta \gamma \delta} P^{\gamma} S^{\delta}] (4.78)
\]

The component of \( W_{\mu}^{S} \) which depends on the interference term can be expressed by the formula \( W_{\mu}^{S(I)} = W_{\mu}^{\Delta(I)} + W_{\mu}^{S(I)} \) consequentially.

\[
W_{\mu}^{S(I)} = \int \frac{\beta(W) d\Omega_{\pi} 2 q^{2}}{(4\pi)^{3}} \frac{2 m_{N} \alpha_{G} G_{M}^{a}}{f_{\pi}(W^{2} - m_{N}^{2})} A(\gamma N_{i} \to N_{f}^{a}) [\left( \frac{2(P \cdot q)}{q^{2}} q_{\mu} q_{\nu} - \frac{1}{q^{2}} (P_{\mu} q_{\mu} + q_{\mu} P_{\nu} - q_{\mu} q_{\nu}) - g_{\mu} q_{\nu} \right) \epsilon_{\alpha \beta \gamma \delta} P^{\alpha} k^{\beta} P^{\gamma} S^{\delta}
- \left( P \cdot q \right) q_{\mu} q_{\nu} - P_{\mu} \epsilon_{\nu \beta \gamma \delta} k^{\beta} P^{\gamma} S^{\delta} - \left( P \cdot q \right) q_{\mu} q_{\nu} - P_{\mu} \epsilon_{\mu \beta \gamma \delta} k^{\beta} P^{\gamma} S^{\delta}
+ \left( \frac{q \cdot k}{q^{2}} q_{\mu} - k_{\mu} \right) \epsilon_{\nu \beta \gamma \delta} P^{\gamma} S^{\delta} + \left( \frac{q \cdot k}{q^{2}} q_{\nu} - k_{\nu} \right) \epsilon_{\mu \beta \gamma \delta} P^{\gamma} S^{\delta}] (4.79)
\]

This part should not contribute to the cross section, because we consider a polarized nucleon target. The difference compared to the unpolarized case is that we get a vanishing result for the contraction with the antisymmetric leptonic tensor and a non-vanishing result for the contraction with the symmetric leptonic tensor.

\[
L_{S}^{\mu} W_{\mu}^{S(I)} = \int \frac{\beta(W) d\Omega_{\pi} 2 q^{2}}{(4\pi)^{3}} \frac{8 m_{N} \alpha_{G} G_{M}^{a}}{f_{\pi}(W^{2} - m_{N}^{2})} A(\gamma N_{i} \to N_{f}^{a}) [\left( \frac{1}{q^{2}} (P \cdot q)(q \cdot l') + (P \cdot l)(q \cdot l') + (P \cdot l')(q \cdot l) - (q \cdot l)(q \cdot l') \right)
- \frac{2}{q^{2}} (P \cdot q)(q \cdot l)(q \cdot l') - P \cdot l' + l \cdot l' + \frac{1}{4} q^{2} (P_{0} + k_{0})(\vec{k} \cdot (\vec{P} \times \vec{S}))
- l_{0}(P \cdot (l + l'))(\vec{k} \cdot (\vec{P} \times \vec{S})) + P_{0}(P \cdot (l + l'))(\vec{k} \cdot (\vec{l} \times \vec{S}))
+ (P_{0} + k_{0})(\vec{k} \cdot (l + \vec{l}))(\vec{l} \cdot (\vec{P} \times \vec{S})]) (4.80)
\]
4.5. Polarized Hadronic Tensor near the Threshold

The remaining integration delivers the same type of integrals as in the unpolarized case and we receive \( L_S^{\mu\nu} W^{S(l)}_{\mu\nu} = 0 \) according to this.

Every contributing structure of \( W_{\mu\nu} \) can be written as sum of an already known symmetric part and a new antisymmetric part and we only have to focus on the antisymmetric parts, namely \( W^{A(S)}_{\mu\nu} \) and \( W^{A(P)}_{\mu\nu} \). We must compare these results with (3.11) in order to derive expressions for the polarized nucleon structure functions again. Therefore, we require the introduced near threshold procedure.

\[
W^{A(S)}_{\mu\nu} = i \varepsilon_{\mu\nu\rho\sigma} \frac{\beta(W)}{(4\pi)^2} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2
\]

\[
\left[ \frac{W^2 + m_i^2 - m_j^2}{2W^2} q^\rho S^{\sigma} + \frac{3W^2 + m_i^2 - m_j^2}{2W^2} P \cdot q \frac{q^\rho S^{\sigma}}{q^2} - \frac{S \cdot q}{P \cdot q} q^\rho P^{\sigma} \right] (4.81)
\]

\[
W^{A(P)}_{\mu\nu} = i \varepsilon_{\mu\nu\rho\sigma} \frac{W^4 \beta^3(W) g_A^2(\tau^a_{fi})^2 (G_M^N(Q^2))^2}{4 f_{\pi}^2 (W^2 - m_N^2)^2}
\]

\[
\left[ \frac{W^2 + m_i^2 - m_j^2}{2W^2} q^\rho S^{\sigma} - \frac{-W^2 + m_i^2 - m_j^2}{2W^2} P \cdot q \frac{q^\rho S^{\sigma}}{q^2} - \frac{S \cdot q}{P \cdot q} q^\rho P^{\sigma} \right] (4.82)
\]

At next, we can extract results for the polarized nucleon structure functions.

\[
G_1(W, Q^2) =
\]

\[
+ (W^2 + m_i^2 - m_j^2)(P \cdot q)^2 \frac{\beta(W)}{2(4\pi)^2 W^2} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2
\]

\[
+ (W^2 + m_i^2 - m_j^2)(P \cdot q)^2 \frac{g_A^2(\tau^a_{fi})^2 (G_M^N(Q^2))^2 W^2 \beta^3(W)}{8(4\pi)^2 f_{\pi}^2 (W^2 - m_N^2)^2} \quad (4.83)
\]

\[
G_2(W, Q^2) =
\]

\[
+ (3W^2 + m_i^2 - m_j^2)(P \cdot q)^2 \frac{\beta(W)}{q^2} \frac{2(4\pi)^2 W^2}{4(4\pi)^2 W^2} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2
\]

\[
+ (-W^2 + m_i^2 - m_j^2)(P \cdot q)^2 \frac{g_A^2(\tau^a_{fi})^2 (G_M^N(Q^2))^2 W^2 \beta^3(W)}{8(4\pi)^2 f_{\pi}^2 (W^2 - m_N^2)^2} \quad (4.84)
\]

Applying the kinematic relation \( 2(P \cdot q) = Q^2 + W^2 - m_N^2 \), one obtains the following results.

\[
G_1(W, Q^2) =
\]

\[
+ (W^2 + m_i^2 - m_j^2)(Q^2 + W^2 - m_N^2) \frac{\beta(W)}{4(4\pi)^2 W^2} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2
\]

\[
+ (W^2 + m_i^2 - m_j^2)(Q^2 + W^2 - m_N^2) \frac{g_A^2(\tau^a_{fi})^2 (G_M^N(Q^2))^2 W^2 \beta^3(W)}{16(4\pi)^2 f_{\pi}^2 (W^2 - m_N^2)^2} \quad (4.85)
\]

\[
G_2(W, Q^2) =
\]

\[
- (3W^2 + m_i^2 - m_j^2)(Q^2 + W^2 - m_N^2) \frac{\beta(W)}{8(4\pi)^2 W^2 Q^2} |A(\gamma^* N_i \rightarrow N_f \pi^a)|^2
\]

\[
- (-W^2 + m_i^2 - m_j^2)(Q^2 + W^2 - m_N^2) \frac{g_A^2(\tau^a_{fi})^2 (G_M^N(Q^2))^2 W^2 \beta^3(W)}{32(4\pi)^2 f_{\pi}^2 (W^2 - m_N^2)^2 Q^2} \quad (4.86)
\]

At next, we can create the final results for the proton and neutron structure functions.
4. Determination of the Structure Functions

\[ G_1^p(W, Q^2) = \]
\[ + (W^2 + m_N^2 - m^2_p)(Q^2 + W^2 - m_N^2) \frac{\beta(W)}{4(4\pi)^2 W^2} \sum_{X=X^p} |\mathcal{A}(\gamma^* p \to X)|^2 \]
\[ + (W^2 + m_N^2 - m^2_p)(Q^2 + W^2 - m_N^2) \frac{3g_A^2(G_M^p(Q^2))^2 W^2 \beta^2(W)}{16(4\pi)^2 f^2_\pi(W^2 - m_N^2)^2} \quad (4.87) \]

\[ G_1^n(W, Q^2) = \]
\[ + (W^2 + m_N^2 - m^2_p)(Q^2 + W^2 - m_N^2) \frac{\beta(W)}{4(4\pi)^2 W^2} \sum_{X=X^n} |\mathcal{A}(\gamma^* n \to X)|^2 \]
\[ + (W^2 + m_N^2 - m^2_p)(Q^2 + W^2 - m_N^2) \frac{3g_A^2(G_M^n(Q^2))^2 W^2 \beta^2(W)}{16(4\pi)^2 f^2_\pi(W^2 - m_N^2)^2} \quad (4.88) \]

\[ G_2^p(W, Q^2) = \]
\[ - (3W^2 + m_N^2 - m^2_p)(Q^2 + W^2 - m_N^2)^2 \frac{\beta(W)}{8(4\pi)^2 W^2 Q^2} \sum_{X=X^p} |\mathcal{A}(\gamma^* p \to X)|^2 \]
\[ - (-W^2 + m_N^2 - m^2_p)(Q^2 + W^2 - m_N^2)^2 \frac{3g_A^2(G_M^p(Q^2))^2 W^2 \beta^2(W)}{32(4\pi)^2 f^2_\pi(W^2 - m_N^2)^2 Q^2} \quad (4.89) \]

\[ G_2^n(W, Q^2) = \]
\[ - (3W^2 + m_N^2 - m^2_p)(Q^2 + W^2 - m_N^2)^2 \frac{\beta(W)}{8(4\pi)^2 W^2 Q^2} \sum_{X=X^n} |\mathcal{A}(\gamma^* n \to X)|^2 \]
\[ - (-W^2 + m_N^2 - m^2_p)(Q^2 + W^2 - m_N^2)^2 \frac{3g_A^2(G_M^n(Q^2))^2 W^2 \beta^2(W)}{32(4\pi)^2 f^2_\pi(W^2 - m_N^2)^2 Q^2} \quad (4.90) \]

Finally, we consider the case when \( Q^2 \) is large in comparison with all appearing hadron masses and \( W^2 \) is close to \( W_{th}^2 \). At this limit, one gets the following representations.

\[ G_1^p(W, Q^2) = \frac{Q^2 \beta(W)}{2(4\pi)^2} \left[ \sum_{X=X^p} |\mathcal{A}(\gamma^* p \to X)|^2 \right] + \frac{3g_A^2(G_M^p(Q^2))^2 W^4 \beta^2(W)}{4f^2_\pi(W^2 - m_N^2)^2} \quad (4.91) \]

\[ G_1^n(W, Q^2) = \frac{Q^2 \beta(W)}{2(4\pi)^2} \left[ \sum_{X=X^n} |\mathcal{A}(\gamma^* n \to X)|^2 \right] + \frac{3g_A^2(G_M^n(Q^2))^2 W^4 \beta^2(W)}{4f^2_\pi(W^2 - m_N^2)^2} \quad (4.92) \]

\[ G_2^p(W, Q^2) = -\frac{Q^2 \beta(W)}{2(4\pi)^2} \left[ \sum_{X=X^p} |\mathcal{A}(\gamma^* p \to X)|^2 \right] \quad (4.93) \]

\[ G_2^n(W, Q^2) = -\frac{Q^2 \beta(W)}{2(4\pi)^2} \left[ \sum_{X=X^n} |\mathcal{A}(\gamma^* n \to X)|^2 \right] \quad (4.94) \]

The important conclusion here is that we receive no P-wave contribution for \( G_2(W, Q^2) \). We mention that this contribution is not exactly zero, but it is suppressed at the considered limit. Despite from that, we notice the opposite sign in the final results.
5. Transition Form Factor Expansion

We have to derive expressions for the nucleon to pion nucleon transition form factors as functions of usual nucleon form factors at large $Q^2$. It should be clear that we need additional assumptions in order to obtain these connections. According to this, we have to expand the form factors in terms of comparable components. The main tool is the QCD factorization theorem allowing us to split the form factors in one hard part and two soft parts which can be calculated separately. It is recommended to have a look on [10], [11], [12], and also [13], [14], [15] for more details. By convention, we work with a spin configuration of $\uparrow \downarrow \uparrow$ for the quarks in a corresponding nucleon with spin $\uparrow$ and apply the notation $f(x) = f(x_i)$ concerning the dependence on the quark momentum fractions.

We will introduce the expressions of the hard parts now. Their representations depend on three functions $T_i(x,y)$. The number of a function is related to the number of the quark interacting with the chosen current. It can be shown that these three functions are identical for every required current. We do not need the explicit results for these functions, however we have to use that $T_1$ and $T_3$ get identical by exchanging $x_1 \leftrightarrow x_3$ and $y_1 \leftrightarrow y_3$ in one of them. The corresponding quarks cannot be distinguished.

At first, we introduce the hard part expression generated by the electromagnetic current. This current can be matched between two proton states or two neutron states. The same hard part can be applied in both cases and the only difference is generated by the electromagnetic charge operator. Information concerning the derivation of these expressions can be taken from [11] and [12]. Further information can be seen in [18]. At next, we consider the isovector axial-vector current. This current can be matched between all combinations of nucleon states. We will use both transition processes for the desired expressions. The hard part depends on the chosen nucleon combination. Let us start with the proton to neutron transition and finish with the neutron to proton transition. Information about the derivation can be taken from [19] and further applications can be seen in [20]. At last, we consider the isoscalar axial-vector current. This current can be matched between identical nucleon states only. The hard part does not depend on the chosen nucleons. Information about this can be taken from [21].

$$T_{H^m}^{(x,y,Q^2)} = \frac{16}{9} \frac{(4\pi \alpha_s)^2}{Q^4} \sum_{i=1}^{3} e_i [T_i(x, y) + T_i(y, x)]$$  \hspace{1cm} (5.1)$$

$$T_{H5}^{(V-)}(x, y, Q^2) = \frac{16}{9} \frac{(4\pi \alpha_s)^2}{Q^4} \sum_{i=1}^{3} \text{sign}(h_i)[I_{i-}T_i(x, y) + I_{i-}T_i(y, x)]$$  \hspace{1cm} (5.2)$$

$$T_{H5}^{(V+)}(x, y, Q^2) = \frac{16}{9} \frac{(4\pi \alpha_s)^2}{Q^4} \sum_{i=1}^{3} \text{sign}(h_i)[I_{i+}T_i(x, y) + I_{i+}T_i(y, x)]$$  \hspace{1cm} (5.3)$$

$$T_{H5}^{(S)}(x, y, Q^2) = \frac{16}{9} \frac{(4\pi \alpha_s)^2}{Q^4} \sum_{i=1}^{3} \text{sign}(h_i)[T_i(x, y) + T_i(y, x)]$$  \hspace{1cm} (5.4)$$

The electromagnetic charge operator $e_i$ determines the charge of quark $i$ and so we get $e_i|u\rangle = e_u|u\rangle = \frac{2}{3}|u\rangle$ and $e_i|d\rangle = e_d|d\rangle = -\frac{1}{3}|d\rangle$. The operator $I_{i-}$ denotes the isospin
5. Transition Form Factor Expansion

lowering operator of quark $i$ leading to $I_{i+}|u⟩ = I_{i-}|u⟩ = |d⟩$ and $I_{i-}|d⟩ = I_{i+}|d⟩ = |0⟩$. The operator $I_{i+}$ denotes the isospin raising operator of quark $i$ leading to $I_{i+}|d⟩ = I_{i+}|u⟩$ and $I_{i+}|u⟩ = I_{i+}|u⟩ = |0⟩$. The sign of the helicity of quark $i$ is generated by the sign operator sign$(h_i)$ leading to sign$(h_i)|↑⟩ = + |↑⟩$ and sign$(h_i)|↓⟩ = - |↓⟩$.

At next, we will summarize the representations of the soft parts. The required leading twist nucleon distribution amplitude will be split in distribution functions $ϕ_S(x)$ and $ϕ_A(x)$ which are symmetric and antisymmetric respectively under the exchange of $x_1 ↔ x_3$. These components are related to quarks with parallel helicities, see [9]. We obtain the following list of functions related to the corresponding hadronic states.

\[ ϕ_p(x) = + \frac{1}{\sqrt{6}} φ_S(x) |2u_1 d_1 u_↑ - u_↑ u_1 d_↑ - d_↑ u_1 u_↑⟩ + \frac{1}{\sqrt{2}} φ_A(x) |u_1 u_↓ d_↑ - d_↑ u_1 u_↑⟩ \]  
(5.5)

\[ ϕ_n(x) = - \frac{1}{\sqrt{6}} φ_S(x) |2d_1 u_1 u_↑ - d_↑ u_1 u_↑ - u_↑ d_1 u_↑⟩ - \frac{1}{\sqrt{2}} φ_A(x) |d_1 u_1 u_↑ - u_↑ d_1 u_↑⟩ \]  
(5.6)

\[ ϕ_{π^0}(x) = + \frac{1}{2\sqrt{6}f_π} φ_S(x) |6u_1 d_1 u_↑ + u_1 u_1 d_↑ + d_↑ u_1 u_↑⟩ - \frac{1}{2\sqrt{2}f_π} φ_A(x) |u_1 u_↓ d_↑ - d_↑ u_1 u_↑⟩ \]  
(5.7)

\[ ϕ_{π^0}(x) = + \frac{1}{2\sqrt{6}f_π} φ_S(x) |6d_1 u_1 d_↑ + d_↑ d_1 u_↑ + u_1 d_1 u_↑⟩ - \frac{1}{2\sqrt{2}f_π} φ_A(x) |d_1 u_1 u_↑ - u_↑ d_1 u_↑⟩ \]  
(5.8)

\[ ϕ_{π^+}(x) = + \frac{1}{\sqrt{12}f_π} φ_S(x) |2u_1 d_1 u_↑ - 3u_1 d_1 u_↑ - 3d_↑ u_1 u_↑⟩ - \frac{1}{2f_π} φ_A(x) |u_1 u_↓ d_↑ - d_↑ u_1 u_↑⟩ \]  
(5.9)

\[ ϕ_{π^-}(x) = - \frac{1}{\sqrt{12}f_π} φ_S(x) |2d_1 u_1 d_↑ - 3d_↑ d_1 u_↑ - 3u_1 d_1 u_↑⟩ + \frac{1}{2f_π} φ_A(x) |d_1 u_1 u_↑ - u_↑ d_1 u_↑⟩ \]  
(5.10)

Let us now combine the hard and soft parts in order to obtain expressions for the usual nucleon form factors. For convenience, we will omit the dependence on the quark momentum fractions if possible. Moreover, we introduce the notations: $ϕ_{SS} = ϕ_Sϕ_S$, $ϕ_{AA} = ϕ_Aϕ_A$ and $ϕ_{AS} = (ϕ_Aϕ_S + ϕ_Sϕ_A)$. We will expand the proton magnetic form factor, the neutron magnetic form factor, the isovector axial-vector form factor and the isoscalar axial-vector form factor.
\[ G_M^p(Q^2) = \int [dx][dy] \phi_\rho^*(y) T_H^m(x, y, Q^2) \phi_\rho(x) \]
\[ = \frac{16}{9} \frac{(4\pi\alpha_s)^2}{Q^4} \int [dx][dy]\left\{ 2T_1 \phi_{SS} - \frac{2}{\sqrt{3}} T_1 \phi_{AS} + \frac{2}{3} [T_1 + 2T_2] \phi_{AA} \right\} \quad (5.11) \]

\[ G_M^n(Q^2) = \int [dx][dy] \phi_\rho^*(y) T_H^m(x, y, Q^2) \phi_n(x) \]
\[ = \frac{16}{9} \frac{(4\pi\alpha_s)^2}{Q^4} \int [dx][dy]\left\{ \frac{2}{3} [T_2 - T_1] \phi_{SS} + \frac{2}{\sqrt{3}} T_1 \phi_{AS} + \frac{2}{3} [T_1 - T_2] \phi_{AA} \right\} \quad (5.12) \]

\[ G_A^p(Q^2) = \int [dx][dy] \phi_\rho^*(y) T_H^{(V-)}(x, y, Q^2) \phi_\rho(x) \]
\[ = \int [dx][dy] \phi_\rho^*(y) T_H^{(V+)}(x, y, Q^2) \phi_n(x) \]
\[ = \frac{16}{9} \frac{(4\pi\alpha_s)^2}{Q^4} \int [dx][dy]\left\{ \frac{4}{3} [4T_1 + 2T_2] \phi_{SS} - \frac{4}{\sqrt{3}} T_1 \phi_{AS} - 2T_2 \phi_{AA} \right\} \quad (5.13) \]

\[ G_A^n(Q^2) = \int [dx][dy] \phi_\rho^*(y) T_H^{(S)}(x, y, Q^2) \phi_\rho(x) \]
\[ = \frac{16}{9} \frac{(4\pi\alpha_s)^2}{Q^4} \int [dx][dy]\left\{ 2[2T_1 - T_2] \phi_{SS} + 2[2T_1 - T_2] \phi_{AA} \right\} \quad (5.14) \]

Finally, we will combine the hard and soft parts in order to obtain expressions for the nucleon to pion nucleon transition form factors. We will receive expansions for all transition form factors similar to nucleon form factors.

\[ A(\gamma^* p \rightarrow p\pi^0) (Q^2) = \int [dx][dy] \phi_\rho^*(y) T_H^m(x, y, Q^2) \phi_\rho(x) = \]
\[ \frac{16}{9} \frac{(4\pi\alpha_s)^2}{Q^4} \frac{1}{f_\pi} \int [dx][dy]\left\{ \frac{1}{9} [23T_1 - 8T_2] \phi_{SS} + \frac{1}{\sqrt{3}} T_1 \phi_{AS} - \frac{1}{3} [T_1 + 2T_2] \phi_{AA} \right\} \quad (5.15) \]

\[ A(\gamma^* p \rightarrow n\pi^0) (Q^2) = \int [dx][dy] \phi_\rho^*(y) T_H^m(x, y, Q^2) \phi_\rho(x) = \]
\[ \frac{16}{9} \frac{(4\pi\alpha_s)^2}{Q^4} \frac{1}{\sqrt{2} f_\pi} \int [dx][dy]\left\{ \frac{2}{9} [11T_1 + 4T_2] \phi_{SS} - \frac{2}{\sqrt{3}} T_1 \phi_{AS} - \frac{2}{3} [T_1 + 2T_2] \phi_{AA} \right\} \quad (5.16) \]

\[ A(\gamma^* n \rightarrow n\pi^0) (Q^2) = \int [dx][dy] \phi_\rho^*(y) T_H^m(x, y, Q^2) \phi_\rho(x) = \]
\[ \frac{16}{9} \frac{(4\pi\alpha_s)^2}{Q^4} \frac{1}{f_\pi} \int [dx][dy]\left\{ \frac{13}{9} [T_1 - T_2] \phi_{SS} + \frac{1}{\sqrt{3}} T_1 \phi_{AS} + \frac{1}{3} [T_1 - T_2] \phi_{AA} \right\} \quad (5.17) \]

\[ A(\gamma^* n \rightarrow p\pi^-) (Q^2) = \int [dx][dy] \phi_\rho^*(y) T_H^m(x, y, Q^2) \phi_\rho(x) = \]
\[ \frac{16}{9} \frac{(4\pi\alpha_s)^2}{Q^4} \frac{1}{\sqrt{2} f_\pi} \int [dx][dy]\left\{ \frac{2}{9} [T_2 - T_1] \phi_{SS} + \frac{2}{\sqrt{3}} T_1 \phi_{AS} + \frac{2}{3} [T_2 - T_1] \phi_{AA} \right\} \quad (5.18) \]
5. Transition Form Factor Expansion

All considered types of form factors depend on combinations of $\phi_S$ and $\phi_A$. In order to express the transition form factors as functions of nucleon form factors, we have to constrain these distribution functions.

We already know from large $Q^2$ processes that $\phi_S$ dominates $\phi_A$. Therefore, we consider now the basic case that $\phi_S$ is arbitrary and $\phi_A$ can be neglected. In this case, one can express all transition form factors as functions of two nucleon form factors. It makes sense to apply the dominant nucleon form factors $G_M^p$ and $G_M^n$ for this expansion. These results are already obtained in [9].

$$A(\gamma^*p \rightarrow pp^0) = \frac{1}{f_\pi} \left( \frac{5}{6} G_M^p - \frac{4}{3} G_M^n \right)$$ (5.19)

$$A(\gamma^*p \rightarrow n\pi^+) = \frac{1}{\sqrt{2} f_\pi} \left( \frac{5}{3} G_M^p + \frac{4}{3} G_M^n \right)$$ (5.20)

$$A(\gamma^*n \rightarrow n\pi^0) = \frac{1}{f_\pi} \left( -13 \frac{6}{6} G_M^n \right)$$ (5.21)

$$A(\gamma^*n \rightarrow p\pi^-) = \frac{1}{\sqrt{2} f_\pi} \left( \frac{1}{2} G_M^n \right)$$ (5.22)

Moving away from the large $Q^2$ region, one cannot longer neglect $\phi_A$. Both components must be considered as arbitrary, but we can assume that $\phi_A$ is still small. That means, we can neglect the $\phi_{AA}$ parts for the form factor reduction. In order to express all transition form factors as functions of nucleon form factors, one has to use three nucleon form factors. We will add the form factor $G_A^n$ to the previous form factors $G_M^p$ and $G_M^n$. We avoid to use the form factor $G_A^n$, because it is suppressed in experiments. Taking into account reasonable antisymmetric contributions, we predict a more realistic description of the process and convincing agreement with experimental data consequentially.

$$A(\gamma^*p \rightarrow \pi^0) = \frac{1}{f_\pi} \left( \frac{25}{6} G_M^p + \frac{2}{3} G_M^n - 2 G_A^n \right)$$ (5.23)

$$A(\gamma^*p \rightarrow n\pi^+) = \frac{1}{\sqrt{2} f_\pi} \left( \frac{5}{6} G_M^p + \frac{5}{6} G_M^n + \frac{1}{2} G_A^n \right)$$ (5.24)

$$A(\gamma^*n \rightarrow n\pi^0) = \frac{1}{f_\pi} \left( \frac{10}{3} G_M^n - \frac{1}{6} G_M^n - 2 G_A^n \right)$$ (5.25)

$$A(\gamma^*n \rightarrow p\pi^-) = \frac{1}{\sqrt{2} f_\pi} \left( \frac{5}{6} G_M^p + \frac{5}{6} G_M^n - \frac{1}{2} G_A^n \right)$$ (5.26)

In the next chapter, we will compare the obtained results with experimental data. Therefore, we will apply the here derived transition form factor representations separately.
6. Comparison with Experimental Data

Our final task is to compare the obtained results for the nucleon structure functions with experimental data. We have the ability to apply experimental values for usual nucleon form factors in order to predict values for nucleon to pion nucleon transition form factors and nucleon structure functions. Data for the proton magnetic form factor are given in [22] and [23] and for the neutron magnetic form factor in [24] and [25]. Moreover, data for the isovector axial-vector form factor are given in [26]. Details about the experiments will be discussed in the next part of this work. One also needs values for multiple constants. Therefore, we can recommend [27].

When \( \phi_A \) can be neglected, we have to use experimental values in regions of \( Q^2 \) which are probably equal to or more than 10 GeV\(^2\). The proton magnetic form factor is given by
\[
Q^4 G_n^p(Q^2) = (1.0 \pm 0.1) \text{ GeV}^4
\]
and the neutron magnetic form factor is given by
\[
Q^4 G_n^n(Q^2) = -(0.5 \pm 0.1) \text{ GeV}^4
\]
in this area.

In the case that \( \phi_A \) is small, we have to use experimental values in regions of \( Q^2 \) which are probably less than or equal to 10 GeV\(^2\). For the proton magnetic form factor, one can apply
\[
Q^4 G_n^p(Q^2) = (1.1 \pm 0.1) \text{ GeV}^4
\]
and for the neutron magnetic form factor, one can apply
\[
Q^4 G_n^n(Q^2) = -(0.5 \pm 0.1) \text{ GeV}^4
\]
for the isovector axial-vector form factor, one can use
\[
Q^4 G_n^A(Q^2) = (1.5 \pm 0.1) \text{ GeV}^4
\]

Let us first apply the discussed connection between nucleon to pion nucleon transition form factors and usual nucleon form factors in order to obtain results for the transition form factors. These expressions depend on the pion decay constant given by \( f_\pi \approx 92.4 \text{ MeV} \).

We start with the scenario that \( \phi_A \) can be neglected. Inserting the required experimental values for the nucleon form factors, one gets the following list of results.

\[
Q^4 A(\gamma^* p \to p \pi^0)(Q^2) = (16.2 \pm 0.1) \text{ GeV}^3 \tag{6.1}
\]
\[
Q^4 A(\gamma^* p \to n \pi^+)(Q^2) = (7.7 \pm 0.1) \text{ GeV}^3 \tag{6.2}
\]
\[
Q^4 A(\gamma^* n \to n \pi^0)(Q^2) = (11.7 \pm 0.1) \text{ GeV}^3 \tag{6.3}
\]
\[
Q^4 A(\gamma^* n \to p \pi^-)(Q^2) = (-1.3 \pm 0.1) \text{ GeV}^3 \tag{6.4}
\]

We finish with the situation that \( \phi_A \) is small. After insertion of the corresponding experimental values for the nucleon form factors, we get another list of results.

\[
Q^4 A(\gamma^* p \to p \pi^0)(Q^2) = (13.5 \pm 0.1) \text{ GeV}^3 \tag{6.5}
\]
\[
Q^4 A(\gamma^* p \to n \pi^+)(Q^2) = (9.6 \pm 0.1) \text{ GeV}^3 \tag{6.6}
\]
\[
Q^4 A(\gamma^* n \to n \pi^0)(Q^2) = (8.1 \pm 0.1) \text{ GeV}^3 \tag{6.7}
\]
\[
Q^4 A(\gamma^* n \to p \pi^-)(Q^2) = (-1.9 \pm 0.1) \text{ GeV}^3 \tag{6.8}
\]

These values are just interesting from theoretical point of view. We remind that the corresponding transition states are undetected in experiments.

At next, we consider the ratio between a neutron and a proton structure function of the same kind at the production threshold of the pion. At this limit, the ratio is identical for all types of nucleon structure functions \( F \) and generated by the transition form factors.
6. Comparison with Experimental Data

At first, we consider the case that $\phi_A$ can be neglected again. When we insert the required results for the transition form factors, the following ratio will be obtained.

$$
\lim_{W \to W_{th}} \frac{F^n(W, Q^2)}{F^p(W, Q^2)} = \frac{\sum_{X = X^n} |A(\gamma^n \to X)|^2}{\sum_{X = X^p} |A(\gamma^p \to X)|^2} = 0.4 \pm 0.1 \tag{6.9}
$$

At last, we discuss the case that $\phi_A$ is small again. When we insert the corresponding results for the transition form factors, another ratio will be obtained.

$$
\lim_{W \to W_{th}} \frac{F^n(W, Q^2)}{F^p(W, Q^2)} = \frac{\sum_{X = X^n} |A(\gamma^n \to X)|^2}{\sum_{X = X^p} |A(\gamma^p \to X)|^2} = 0.3 \pm 0.1 \tag{6.10}
$$

We have got results which are independent of $Q^2$. This scaling behavior is already known from the parton model.

The obtained ratios are in good agreement with the results obtained in [17] and the perturbative QCD scaling limit expectation of $3/7$ for $x_B \to 0$, see [28].

Finally, we will integrate the nucleon structure functions in an area near the threshold in order to obtain results which can be compared with experimental data. The expressions depend on the axial coupling constant given by $g_A \approx 1.27$ and we have to apply values for the included masses $m_N \approx 0.94\, \text{GeV}$ and $m_A \approx 0.14\, \text{GeV}$.

We received experimental data for the integrated dominant proton structure function $F^p_2(W, Q^2)$ in the near threshold region. Starting from the threshold, the measurement was done for values of $W^2$ up to $2\, \text{GeV}^2$. These data have a spectrum of $6 < Q^2 < 30\, \text{GeV}^2$. Further information about this experiment can be taken from [29]. They integrated the quantity $Q^2 F^p_2(W, Q^2)$ in different areas near the threshold and realized that this quantity does not really depend on $Q^2$ for $Q^2 > 6\, \text{GeV}^2$, so it shows the best scaling. As expected, the best approximation was obtained for the lowest area of $W^2$, where the following integration was evaluated.

$$
\int_{th}^{1.4} dW^2 Q^6 F^p_2(W, Q^2) = (0.10 \pm 0.02) \, \text{GeV}^8 \tag{6.11}
$$

Concerning the integrated dominant neutron structure function $F^n_2(W, Q^2)$, we have no experimental data for comparisons at the moment. For several reasons, the required measurement can be done much easier on the proton than on the neutron.

Nevertheless, we decided to integrate all nucleon structure functions in the same area in order to make predictions for further experiments. One could use another integration region immediately.

Let us begin with the integrated proton structure functions based on the scenario that $\phi_A$ can be neglected. One gets the following list of results in this case.

$$
\int_{th}^{1.4} dW^2 Q^6 F^p_1(W, Q^2) = +(0.06 \pm 0.02) \, \text{GeV}^8 \tag{6.12}
$$

$$
\int_{th}^{1.4} dW^2 Q^6 F^p_2(W, Q^2) = +(0.11 \pm 0.02) \, \text{GeV}^8 \tag{6.13}
$$

$$
\int_{th}^{1.4} dW^2 Q^6 G^p_1(W, Q^2) = +(0.06 \pm 0.02) \, \text{GeV}^8 \tag{6.14}
$$

$$
\int_{th}^{1.4} dW^2 Q^6 G^p_2(W, Q^2) = -(0.05 \pm 0.02) \, \text{GeV}^8 \tag{6.15}
$$
Let us proceed with the integrated neutron structure functions based on the situation that $\phi_A$ can be neglected. One gets the following list of results in this case.

\[
\int_{th}^{1.4} dW^2 Q^6 F_1^n(W, Q^2) = +(0.02 \pm 0.02) \text{ GeV}^8
\]  
(6.16)

\[
\int_{th}^{1.4} dW^2 Q^6 F_2^n(W, Q^2) = +(0.05 \pm 0.02) \text{ GeV}^8
\]  
(6.17)

\[
\int_{th}^{1.4} dW^2 Q^6 G_1^n(W, Q^2) = +(0.02 \pm 0.02) \text{ GeV}^8
\]  
(6.18)

\[
\int_{th}^{1.4} dW^2 Q^6 G_2^n(W, Q^2) = -(0.02 \pm 0.02) \text{ GeV}^8
\]  
(6.19)

We obtained a result for the integrated proton structure function $F_2^p(W, Q^2)$ which is certainly in good agreement with the experimental value. Moreover, our results for this quantity and the integrated neutron structure function $F_2^n(W, Q^2)$ are the same as in [9].

Let us continue with the integrated proton structure functions based on the scenario that $\phi_A$ is small. We gain the following list of results in this case.

\[
\int_{th}^{1.4} dW^2 Q^6 F_1^p(W, Q^2) = +(0.05 \pm 0.02) \text{ GeV}^8
\]  
(6.20)

\[
\int_{th}^{1.4} dW^2 Q^6 F_2^p(W, Q^2) = +(0.10 \pm 0.02) \text{ GeV}^8
\]  
(6.21)

\[
\int_{th}^{1.4} dW^2 Q^6 G_1^p(W, Q^2) = +(0.05 \pm 0.02) \text{ GeV}^8
\]  
(6.22)

\[
\int_{th}^{1.4} dW^2 Q^6 G_2^p(W, Q^2) = -(0.04 \pm 0.02) \text{ GeV}^8
\]  
(6.23)

Let us end with the integrated neutron structure functions based on the scenario that $\phi_A$ is small. We gain the following list of results in this case.

\[
\int_{th}^{1.4} dW^2 Q^6 F_1^n(W, Q^2) = +(0.01 \pm 0.02) \text{ GeV}^8
\]  
(6.24)

\[
\int_{th}^{1.4} dW^2 Q^6 F_2^n(W, Q^2) = +(0.03 \pm 0.02) \text{ GeV}^8
\]  
(6.25)

\[
\int_{th}^{1.4} dW^2 Q^6 G_1^n(W, Q^2) = +(0.01 \pm 0.02) \text{ GeV}^8
\]  
(6.26)

\[
\int_{th}^{1.4} dW^2 Q^6 G_2^n(W, Q^2) = -(0.01 \pm 0.02) \text{ GeV}^8
\]  
(6.27)

We obtained a result for the integrated proton structure function $F_2^p(W, Q^2)$ which is in complete agreement with the experimental value. We predicted a more realistic description of the discussed process by taking into account a reasonable antisymmetric contribution and the perfect agreement with the experimental data will certainly confirm our statement.
7. Conclusion to Part I

In this part of our work, we calculated all nucleon structure functions in the region of large momentum transfer and small invariant masses. Therefore, we studied the scattering between an electron and a nucleon target due to exchange of a single virtual photon which carries large momentum. After the scattering process, we received an electron and a nucleon together with a produced pion with small momentum.

In order to derive the expressions for the nucleon structure functions, we calculated the cross section of the specified exclusive inelastic scattering process and also the cross section of inclusive inelastic electron nucleon scattering for comparison.

From theoretical point of view, the decomposition of the required nucleon to pion nucleon transition probability matrix elements was the important challenge. We were able to split these matrix elements into an exact threshold part plus a near threshold part. The exact threshold part was expanded in nucleon to pion nucleon transition form factors and the near threshold part was evaluated by using of soft pion theorems.

The remaining problem was the calculation of the required transition form factors. We applied the QCD factorization theorem to derive relations between nucleon to pion nucleon transition form factors and usual nucleon form factors under reasonable assumptions.

Applying this approach, we were able to express the formulas for the nucleon structure functions as functions of usual nucleon form factors only.

We used experimental values of usual nucleon form factors to derive representations for the nucleon to pion nucleon transition form factors and for the nucleon structure functions.

Finally, we compared our results with available experimental data or presented various predictions for further experiments. Using advanced assumptions for the form factor reduction, we obtained perfect agreement with the experimental data.

Nevertheless, further experimental data for the nucleon structure functions in the desired region are required to improve the knowledge about the nucleon structure. In the case of the proton target, substantial data for the structure functions are already available. In the case of the neutron target, usable data are still not existent. Of course, the corresponding measurements are much more problematic, because the neutron is an unstable particle, whereas the proton is a stable particle.

In order to present the essential achievement of these studies to the entire community, the main part of this work is published in [30].
Part II.

Leading Nucleon Form Factors at Large Momentum Transfer in Single Gauge Boson Exchange Approximation
8. Introduction to Part II

Our aim is to calculate all leading nucleon form factors at large momentum transfer and in single gauge boson exchange approximation. Therefore, we have to study various nucleon transition probability matrix elements including a single quark current. Moreover, one needs the representations by nucleon form factors. Expanding these matrix elements at large momentum transfer, one can reduce every decomposition to one dominant term depending on the leading nucleon form factor.

Studying leading nucleon form factors in the region of large momentum transfer, the formulation of the QCD factorization theorem in [10], [11], [12], and [13], [14], [15] was a great success. In [11] and [12], the expressions for the leading electromagnetic form factors were derived. Using the QCD factorization theorem, these expressions were improved in [18]. Moreover, the representation for the isovector axial-vector form factor was created in [19] with applications discussed in [20]. Similarly, the isoscalar axial-vector form factor was considered in [21]. Studying the behavior of vacuum to nucleon projection matrix elements, the leading electromagnetic form factors were calculated in [31] and [32] at the asymptotic limit. Hereby, an averaged value of $\alpha_s$ was used. This approach was also applied in [33] and improved in [34]. Furthermore, in [34], also non-asymptotic contributions of the leading twist nucleon distribution amplitude were taken into account to calculate the leading electromagnetic form factors, see also [35]. Another approach is to keep $\alpha_s$ inside the integral, evaluated in [36]. Despite the success of perturbative QCD, the formalism was criticized in [37]. Therefore, a modified formalism was discussed in [38]. Being more precise, the region of low momentum transfer was considered in the case of the pion, see [39]. Using the modified formalism in the case of the pion, the problems were cleared, see [40]. This technique was also applied for the nucleon in [41]. An improved calculation of the proton magnetic form factor, within the modified factorization scheme, was given in [42]. An analogous calculation for the neutron was done in [43]. A comprehensive work about the theory was published in [44]. A remaining problem has been pointed out in [45], where a different symmetry factor was used as in [36]. The same difference was also discussed in [46]. Meanwhile, nucleon distribution amplitudes of sub-leading twist were also extensively studied, see [47], with applications in [48].

In order to evaluate the required nucleon transition probability matrix elements at large momentum transfer, we will combine QCD perturbation theory with an expansion in nucleon distribution amplitudes. As already mentioned, we are interested in leading nucleon form factors at large momentum transfer and so we only have to consider the leading twist nucleon distribution amplitudes.

These distribution amplitudes can be expanded in polynomials. The leading polynomial can be expressed by a simple asymptotic structure related to the case of infinite momentum transfer. Moving away from this region, one has to apply non-leading polynomials as well. It can be shown that the second order polynomial is related to values of the momentum transfer which are still unreachable in experiments [47], but the third order polynomial can be used to compare our results with experimental data, see [44].

In this part, the different types of leading nucleon form factors will be studied indepen-
8. Introduction to Part II

dently. At the end of each form factor chapter, we want to compare the obtained results with experimental data. Therefore, we will discuss the realizable experiments and analyze the available data.

At first, we will evaluate nucleon transition probability matrix elements of two protons or two neutrons including the electromagnetic current. These structures appear in electromagnetic interactions, for instance, elastic scattering between an electron and a nucleon due to exchange of a single virtual photon. One should notice that these structures are also part of neutral weak interactions. We want to point out that we do not need to deal with the vector currents, because the corresponding form factors can be expressed by electromagnetic form factors. The required transition probability matrix elements are traditionally expressed in terms of two different form factor representations. One can use the Dirac form factor and the Pauli form factor or, equivalently, the magnetic form factor and the electric form factor.

At large momentum transfer, we get contributions for the Dirac form factor or the magnetic form factor only. Moreover, these form factors are identical in this case, because the Pauli form factor is suppressed by a power of the large momentum transfer. We mention that the form factor expansion does not tell us whether the electric form factor is power suppressed or not. It is convenient to express the expansion of these transition probability matrix elements in terms of the magnetic form factor and the Pauli form factor. We will concentrate our calculations on the proton magnetic form factor. The calculations for the neutron magnetic form factor are similar and we will only point out the differences during the procedure.

At next, we will proceed with the consideration of the nucleon transition probability matrix element of an initial proton and a final neutron including the isovector axial-vector current. We remark that we would get the same result by applying an initial neutron and a final proton, because the corresponding form factors do not depend on this choice. These structures appear in charged weak interactions as part of the exchange of a $W^+$ or $W^-$ boson. Moreover, it is possible to calculate the discussed matrix element with two protons or two neutrons. These calculations would also lead to the same form factor results, but these matrix elements appear in neutral weak interactions as part of the exchange of a $Z^0$ boson. Transition probability matrix elements including the isovector axial-vector current are expressed in terms of the isovector axial-vector form factor and the isovector pseudoscalar form factor. We notice that the isovector pseudoscalar form factor is suppressed by a power of the large momentum transfer. Consequently, we will calculate the isovector axial-vector form factor only.

At last, we will continue with the evaluation of the nucleon transition probability matrix element of two protons including the isoscalar axial-vector current. Applying two neutrons for this matrix element, one would get the same results for the appearing form factors. These structures only appear in non standard weak interaction theories, such as the exchange of an extra $Z^0$ boson. Transition probability matrix elements including the isoscalar axial-vector current are described in terms of the isoscalar axial-vector form factor and the isoscalar pseudoscalar form factor. We remark that the isoscalar pseudoscalar form factor is suppressed by a power of the large momentum transfer. Therefore, we will compute the isoscalar axial-vector form factor only.

Using the QCD factorization theorem for the matrix element evaluation, the dominant Dirac form factor of the proton was already calculated by multiple groups. The different results include a discrepancy which will be clarified in this work.
9. Magnetic Form Factor

In this chapter, we will calculate the nucleon magnetic form factors under the conditions discussed in the introduction. Therefore, one has to specify the technique which will be used in order to evaluate the required nucleon transition probability matrix elements.

9.1. General Construction

Let us start with (A.5) and (A.6) at large momentum transfer. At this limit, one gets the following representations.

\[
\langle p(p')|J'^m_\mu(0)|p(P)\rangle = G^p_M(Q^2)\tilde{N}(P')\gamma_\mu N(P) \tag{9.1}
\]

\[
\langle n(p')|J'^m_\mu(0)|n(P)\rangle = \tilde{G}^n_M(Q^2)\tilde{N}(P')\gamma_\mu N(P) \tag{9.2}
\]

In order to evaluate these matrix elements, one has to insert the expression of the electromagnetic current \( J'^m_\mu(0) = \sum_q e_q \bar{\psi}_q(0)\gamma_\mu \psi_q(0) \). Hereby, we have to sum over \( q = u, d \).

The large momentum transfer ensures that we can apply perturbation theory. Therefore, one has to insert the S-matrix including the interaction part of the QCD Lagrangian. Expanding the S-matrix up to the fourth order, one gets the first non-vanishing contributions. As mentioned in the appendix about Quantum Chromodynamics, we only have to apply the quark gluon interaction part. Let us explain this when we discuss the representations of the different interactions.

We mention that the introduced matrix elements cannot be calculated by applying perturbation theory only. Nevertheless, the perturbative expansion leads to an analyzable expression of the matrix elements.

We will show the result for the proton matrix element and concerning the neutron matrix element, it is only necessary to exchange \( p \rightarrow n \) for the nucleon states.

\[
\frac{(4\pi\alpha_s)^2}{24} \langle p(p')|\sum_q e_q \bar{\psi}_q(0)\gamma_\mu \psi_q(0)\rangle T \left[ \prod_{i=1}^4 \int d^4x_i \sum_{q_i} \bar{\psi}_{q_i}(x_i)\gamma_{\alpha_i}A^{\alpha_i}(x_i)\psi_{q_i}(x_i) \right]|p(P)\rangle \tag{9.3}
\]

This expansion can be described by Feynman diagrams and Wick contractions. Every diagram can be expressed by four vertices connected to one gluon and two quarks. Fortunately, it is not necessary to compute every diagram separately. Invariance between these diagrams ensure that the amount of diagrams which must be evaluated can significantly be reduced. Let us discuss these invariance in detail in the appendix about leading nucleon diagrams.

Adding the gluon gluon interaction part to the expansion, one gets additional diagrams. These diagrams can be expressed by three vertices connected to one gluon and two quarks together with one vertex connected to three gluons. Applying the third order in quark gluon interaction and the first order in gluon gluon interaction, one can generate these diagrams. It can be proved that the color factor concerning every diagram of this kind vanishes and so they do not contribute. That is the reason why we do not need the gluon gluon interaction part for our calculations.
9. Magnetic Form Factor

9.2. Sample Diagram Evaluation

All remaining diagrams have similar structures and according to this, we will calculate just one diagram for the proton magnetic form factor in detail. The entire diagram classification will be specified in the appendix about leading nucleon diagrams. The chosen diagram can be presented as follows. We have the incoming proton on the left and we have the outgoing proton on the right. The cross denotes the coupling to the electromagnetic current and the quark lines denote \( u, u, d \) from top to bottom. The designations at the vertices are the corresponding coordinates and the designations at the lines are the corresponding momenta. Concerning the neutron magnetic form factor, we have to exchange \( p \to n \) for the nucleon states and \( u \leftrightarrow d \) for the quark lines.

Let us begin with the determination of the color factor. Starting from the contracted representation of this diagram, one has to write out the color indices of all color dependent components. Afterwards, it is required to move these elements in the order which allows us to calculate the diagram. The indices \( (a, \ldots, i) \) at the squared brackets are the color indices and a summation over these indices is included. We present the contractions of all diagrams in the appendix about leading nucleon diagrams.

\[
\langle P | \bar{\psi}_u(x_1) | \overline{\psi}_u(0) \rangle \langle \psi_u(x_3) | \psi_u(x_2) \rangle \langle t^{\alpha_1} \rangle \left[ t^{\alpha_2} \right]_{de} A^{\alpha_1}_{a_1} \left( x_1 \right) A^{\alpha_2}_{a_2} \left( x_2 \right) \left[ t^{\alpha_3} \right]_{f} \left[ t^{\alpha_4} \right]_{h} A^{\alpha_3}_{a_3} \left( x_3 \right) A^{\alpha_4}_{a_4} \left( x_4 \right) \left( P \right) \right]
\]

\( \langle P | | \bar{\psi}_u(x_1) | \overline{\psi}_u(x_3) \rangle \langle \psi_u(x_2) | \psi_u(x_4) \rangle \langle t^{\alpha_1} \rangle \left[ t^{\alpha_2} \right]_{de} A^{\alpha_1}_{a_1} \left( x_1 \right) A^{\alpha_2}_{a_2} \left( x_3 \right) \left[ t^{\alpha_3} \right]_{f} \left[ t^{\alpha_4} \right]_{h} A^{\alpha_3}_{a_3} \left( x_4 \right) \left( P \right) \right]

Combining all terms and contracting the generators, one can derive the result of the color factor \( C_F \). Therefore, we need an expression for contracted generators. The required relation is given by \( [t^a]_{ij}[t^a]_{kl} = \frac{1}{2} \delta_{ik} \delta_{jl} - \frac{1}{9} \delta_{ij} \delta_{kl} \).

\[ C_F = \frac{1}{6} \epsilon_{bfg} \epsilon_{cde} \delta_{ab} \delta_{cd} \left[ t^{\alpha_1} \right]_{bc} \left[ t^{\alpha_2} \right]_{de} \left[ t^{\alpha_3} \right]_{fg} \left[ t^{\alpha_4} \right]_{hi} \delta_{a_1 a_2} \delta_{a_3 a_4} = \frac{2}{9} \] (9.4)

All specified diagrams can be described by the same color structure. According to this, it can be proved that the result of the color factor is identical for every required diagram.

Restarting from the contracted representation of this diagram, we must write out the Lorentz indices of all parts. Like in the previous case, we have to move these parts in the order which allows us to calculate the diagram. The indices \( (a, \ldots, j) \) at the squared brackets are now the Lorentz indices and a summation over these indices is included. Replacing positions
of quark fields, one gets an overall minus sign by respecting the corresponding commutation and anticommutation relations. Moreover, we get the electromagnetic charge of the quark which interacts with the electromagnetic current. One needs \( e_u = \frac{2}{3} \) and \( e_d = -\frac{1}{3} \). Finally, we obtain the following expression where we already included the color factor.

\[
- \frac{(4\pi\alpha_s)^2}{54} e_u \prod_{i=1}^{4} \int d^4 x_i \langle \gamma_{\mu} \delta \gamma_{\alpha_1} \delta \gamma_{\alpha_2} \delta \gamma_{\alpha_3} \rangle h[\gamma_{\alpha_4} | i^j]
\]

\[
[\bar{\psi}_u(x_1) \delta \bar{\psi}_u(0) e^{A_{\alpha_1}(x_1)} A_{\alpha_2}(x_2) A_{\alpha_3}(x_3) A_{\alpha_4}(x_4)]
\]

\[
(p(P') | [\bar{\psi}_u(x_1) [\bar{\psi}_u(x_3)]_k | \psi_d(x_4)]_i | 0 | 0 | \psi_u(0) | 0 | \psi_u(x_2) | 0 | \psi_d(x_4))]_j | p(P))
\]

In order to evaluate this expression, we need representations for the propagators and for the projection matrix elements. These components will be specified in the appendices about Quantum Chromodynamics and nucleon distribution amplitudes. Recombining the Lorentz structures, one gets the following result.

\[
\frac{(4\pi\alpha_s)^2}{864} e_u \prod_{i=1}^{4} \int d^4 x_i \int_{-1}^{2} \int_{-1}^{2} \int_{-1}^{2} \int_{-1}^{2} [du][dv] g^{\alpha_1 \alpha_2} g^{\alpha_3 \alpha_4} S
\]

\[
e^{-ix_1 \cdot (\Delta_1 - \Lambda_1 - \epsilon_1 p')} e^{-ix_2 \cdot (-\Delta_2 + \Lambda_1 + \epsilon_2 p)} e^{-ix_3 \cdot (\Delta_4 - \Lambda_2 - \epsilon_3 p')} e^{-ix_4 \cdot (\Delta_2 + \epsilon_3 p - \epsilon_3 p')}
\]

The integration over the coordinates produce delta functions which can be used to integrate over the momenta. They describe the momentum conservation at each vertex of the diagram.

Let us present these momentum conservation constraints for all diagrams in the appendix about leading nucleon diagrams.

\[
\Delta_1 = (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \\
\Lambda_1 = (v_2 + v_3)p' - (u_2 + u_3)p \\
\Delta_2 = (v_2 + v_3)p' - u_3 p \\
\Lambda_2 = u_3 p' - u_3 p
\]

The undeclared component \( S \) is the sum of all remaining structures connected with combinations of nucleon distribution amplitudes and nucleon spinors. We remind that we only work with leading twist distribution amplitudes and so one can omit the twist index at the distribution amplitudes. For further simplification, one can omit the dependence on the quark momentum fractions in these expressions. According to this, we use abbreviations, for example, \( AV \) instead of \( A_1(v_1, v_2, v_3)V_1(u_1, u_2, u_3) \). Moreover, we only have to deal with the large component of the spinor and therefore, one can use the standard notation \( \bar{N} \) and \( N \) for the nucleon spinors.

We will present the structures summarized in \( S \) for all diagrams in the appendix about leading form factor structures.

\[
S_1 = \bar{N}(P') \gamma_{a_4} N(P) \langle \gamma_{\mu} f_{\alpha a} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_1 \rangle (VV + AA)
\]

\[
S_2 = \bar{N}(P') \gamma_{a_4} \gamma_5 N(P) \langle \gamma_{\mu} f_{\alpha a} \gamma_{\alpha_3} \gamma_5 \gamma_{\alpha_1} \Delta_1 \rangle (-AV - VA)
\]

\[
S_3 = \bar{N}(P') \gamma^\lambda \gamma_{a_4} \gamma^\lambda N(P) \langle \gamma_{\mu} f_{\lambda a} \gamma_{\alpha_3} \lambda \gamma_{\alpha_1} \Delta_1 \rangle (-TT)
\]

Finally, we can present the result of the discussed diagram. This expression still depends on the integration over the quark momentum fractions.

The final composition of the results for all diagrams will be presented in the appendix about leading form factor results.

\[
\frac{(4\pi\alpha_s)^2}{216} e_u \int \frac{[du]}{u_3(u_2 + u_3)^2} \frac{[dv]}{v_3(v_2 + v_3)^2} [(V - A)^2 + 4T^2] \bar{N}(P') \gamma_{\mu} N(P)
\]
9. Magnetic Form Factor

9.3. Results of the Form Factors

In order to derive the final results for the nucleon magnetic form factors, we have to add the obtained results of the contributing diagrams. Furthermore, we must insert the nucleon distribution amplitudes. As discussed in the appendix about nucleon distribution amplitudes, we have to deal with one independent leading twist distribution amplitude only. The next step is to insert the most general polynomial for this distribution amplitude which is considered there. The other representations can be obtained by setting unnecessary coefficients to zero. Afterwards, we can evaluate the integration over the momentum fractions. This leads to the final results for the nucleon magnetic form factors.

9.3.1. Proton Magnetic Form Factor

Let us start with the final result for the proton magnetic form factor.

\[ G_M^p(Q^2) = \frac{(4\pi \alpha_s)^2 f_N^2}{Q^4} X_M^p(c_1, c_2, c_3, c_4, c_5, c_6) \]  \hspace{1cm} (9.6)

The hereby introduced function \( X_M^p \) depends on the polynomial coefficients of the applied distribution amplitude. Using specific numbers \( a_{ij} \), one gets the following representation.

\[ X_M^p(c_1, c_2, c_3, c_4, c_5, c_6) = \frac{25}{11664} \sum_{i=1}^{6} \sum_{j=1}^{i} a_{ij} c_i c_j \]  \hspace{1cm} (9.7)

These numbers are given by: \( a_{11} = 0, a_{21} = 4320, a_{31} = -1728, a_{41} = 6048, a_{51} = 432, a_{61} = -1296, a_{22} = 2160, a_{32} = 672, a_{42} = 4536, a_{52} = 1152, a_{62} = -240, a_{33} = -528, a_{43} = 1464, a_{53} = -168, a_{63} = -480, a_{44} = 2123, a_{54} = 1418, a_{64} = 72, a_{55} = 95, a_{65} = -180, a_{66} = -72. \)

9.3.2. Neutron Magnetic Form Factor

We finish with the final result for the neutron magnetic form factor.

\[ G_M^n(Q^2) = \frac{(4\pi \alpha_s)^2 f_N^2}{Q^4} X_M^n(c_1, c_2, c_3, c_4, c_5, c_6) \]  \hspace{1cm} (9.8)

The hereby defined function \( X_M^n \) also depends on the polynomial coefficients of the applied distribution amplitude. This function can be expressed by other numbers \( a_{ij} \).

\[ X_M^n(c_1, c_2, c_3, c_4, c_5, c_6) = \frac{25}{5832} \sum_{i=1}^{6} \sum_{j=1}^{i} a_{ij} c_i c_j \]  \hspace{1cm} (9.9)

These numbers are given by: \( a_{11} = 3888, a_{21} = 432, a_{31} = 3456, a_{41} = -972, a_{51} = 1836, a_{61} = 648, a_{22} = -336, a_{32} = 144, a_{42} = -960, a_{52} = -180, a_{62} = 120, a_{33} = 1008, a_{43} = -336, a_{53} = 1392, a_{63} = 240, a_{44} = -491, a_{54} = -395, a_{64} = 0, a_{55} = 523, a_{65} = 126, a_{66} = 24. \)
9.4. Comparison with Experimental Data

The final step is to compare the obtained results with available experimental data. Hereby, we can distinguish between the different order polynomials. The asymptotic contribution corresponds to asymptotic values of the momentum transfer and is only interesting from theoretical point of view. Applying the next contribution given by the conformal expansion, one has to deal with very large values of the momentum transfer which are unreachable in experiments, but one obtains a non-asymptotic behavior. When we use the last considered expression which is model dependent, we come in a region where we have a chance to compare the final results with experimental data. We use the models of Chernyak, Zhitnitsky (CZ) [31], Gari, Stefanis (GS) [33], King, Sachrajda (KS) [49], Chernyak, Ogloblin, Zhitnitsky (COZ) [35], and Stefanis, Bergmann (HET) [50]. In order to compare our results with those existing in the literature, we do not specify the values of the parameters $f_N$ and $\alpha_s$.

The determination of $\alpha_s$ has to be done with care, because it depends on the momentum transfer of the process, or more precisely, it depends on the gluon virtualities. Consequently, one gets two of them for every diagram independently. The main problem arises from the soft gluon region, where $\alpha_s$ becomes divergent. It is possible to apply different estimations to avoid this problem. In principle, one can introduce an averaged $\bar{\alpha}_s$ for all diagrams, see [31], [32]. This approach was also applied in [33] and improved in [34] for each diagram separately. Hereby, the value of $\bar{\alpha}_s \approx 0.3$ was used. Another approach is to keep $\alpha_s$ inside the integral, as done in [36]. This technique can change the slope of the form factors and improve the scaling behavior of them. Nevertheless, this cannot be done without introducing a new cutoff parameter. Taking into account soft gluon corrections [41], one may claim that the area of soft gluon virtualities is systematically underestimated. According to this, we prefer to use an averaged value of $\bar{\alpha}_s$ which is taken from an effective momentum transfer around 1 GeV$^2$. Consequently, we use $\bar{\alpha}_s = 0.45 \pm 0.05$. One should note that the obtained results contain large uncertainties due to the choice of $\alpha_s$.

9.4.1. Proton Magnetic Form Factor

Starting with the asymptotic case, one obtains $X_M^p = 0$ leading to $Q^4G_M^p(Q^2) = 0$ GeV$^4$. We gain a vanishing result at the asymptotic limit. When we apply the next contribution given by the conformal expansion, we get $X_M^p = 166$ rounded off the nearest full number leading to $Q^4G_M^p(Q^2) = (0.15 \pm 0.05)$ GeV$^4$. One obtains the first non-vanishing result which is small and positive. Finally, we consider the last discussed polynomial and insert the values for the coefficients from the different models into the function $X_M^p$. Through this procedure, one receives model dependent results for the proton magnetic form factor rounded off two decimal places in order $10^4$. The uncertainty must assumed to be large.

$$Q^4G_M^p(Q^2) = (4\pi\bar{\alpha}_s)^2 f_N^2 \cdot \begin{cases} 1.16 \cdot 10^3 \\ 1.16 \cdot 10^3 \\ 1.69 \cdot 10^3 \\ 1.34 \cdot 10^3 \\ 1.53 \cdot 10^3 \end{cases} = \begin{cases} (1.0 \pm 0.1) \text{ GeV}^4 \quad (\text{CZ}) \\ (1.0 \pm 0.1) \text{ GeV}^4 \quad (\text{GS}) \\ (1.5 \pm 0.1) \text{ GeV}^4 \quad (\text{KS}) \\ (1.2 \pm 0.1) \text{ GeV}^4 \quad (\text{COZ}) \\ (1.4 \pm 0.1) \text{ GeV}^4 \quad (\text{HET}) \end{cases} \quad (9.10)$$

This expansion of the applied distribution amplitude can be correlated to values of $Q^2$ in the area around 10 GeV$^2$. The dominant electromagnetic interaction ensures that this form factor can be measured in the required area. Moreover, the proton is a stable particle and so the measurement can be done on the proton directly. The mainly used experimental process
9. Magnetic Form Factor

applies an electron beam on a hydrogen target. This process allows the desired electron
proton scattering. We have experimental data concerning \( G_M^p(Q^2) \) for values of \( Q^2 \) which are
less than 10 GeV\(^2\) up to approximately 30 GeV\(^2\). That means, we can deal with experimental
values of a comprehensive region. The averaged experimental value for \( G_M^p(Q^2) \) is given
by \( Q^4 G_M^p(Q^2) = (1.0 \pm 0.1) \) GeV\(^4\). The model dependent results for the proton magnetic
form factor are in sufficient agreement with the experimental data. Details concerning the
experiments can mainly be taken from [22] and [23].

9.4.2. Neutron Magnetic Form Factor

At first, one has to consider the asymptotic case. In this situation, one obtains \( X_M^n = 50/3 \)
leading to \( Q^4 G_M^n(Q^2) = (15 \pm 5) \cdot 10^{-3} \) GeV\(^4\). We gain a non-vanishing but very small and
positive result at the asymptotic limit. Applying the next discussed contribution given by
the conformal expansion, one gets \( X_M^n = -74 \) rounded off the nearest full number leading to
\( Q^4 G_M^n(Q^2) = -(0.07 \pm 0.05) \) GeV\(^4\). We obtain a small and negative result and so the overall
sign of the result has been changed. At last, we use the final discussed polynomial and insert
the values for the coefficients from the different models into the function \( X_M^n \). Through this
procedure, one gains model dependent results for the neutron magnetic form factor rounded
off two decimal places in order \( 10^3 \). The uncertainty must assumed to be large.

\[
Q^4 G_M^n(Q^2) = (4\pi \bar{a}_s) f_N^2 \cdot \begin{cases}
-0.56 \cdot 10^3 \\
-0.11 \cdot 10^3 \\
-0.70 \cdot 10^3 \\
-0.63 \cdot 10^3 \\
-0.16 \cdot 10^3
\end{cases} = \begin{cases}
-(0.5 \pm 0.1) \text{ GeV}^4 \quad (CZ) \\
-(0.1 \pm 0.1) \text{ GeV}^4 \quad (GS) \\
-(0.6 \pm 0.1) \text{ GeV}^4 \quad (KS) \\
-(0.6 \pm 0.1) \text{ GeV}^4 \quad (COZ) \\
-(0.1 \pm 0.1) \text{ GeV}^4 \quad (HET)
\end{cases} \quad (9.11)
\]

This expansion of the used distribution amplitude can also be correlated to values of \( Q^2 \) in
the area around 10 GeV\(^2\). The dominant electromagnetic interaction ensures that this form
factor can be measured in the required area too. Unfortunately, the neutron is not a stable
particle and so the measurement cannot be done on the neutron directly. The mainly used
experimental process applies an electron beam on a deuterium target. This process cannot be
related to electron neutron scattering only, because electron proton scattering is also possible.
Moreover, this process can be considered as dominant, because the proton is charged while
the neutron is uncharged. Furthermore, real targets are mixed of hydrogen and deuterium.
We have experimental data concerning \( G_M^n(Q^2) \) for values of \( Q^2 \) which are less than 10 GeV\(^2\)
up to approximately 10 GeV\(^2\). That means, we just have got experimental values for the lower
area, but an extrapolation to the upper area is also possible. The extrapolated and averaged
experimental value for \( G_M^n(Q^2) \) is given by \( Q^4 G_M^n(Q^2) = -(0.5 \pm 0.1) \) GeV\(^4\). One can apply
the same error bar as before. The model dependent results for the neutron magnetic form
factor are in sufficient agreement with the extrapolated experimental data. Details concerning
the experiments can mainly be taken from [24] and [25].
10. Isovector Axial-Vector Form Factor

In this chapter, we will calculate the nucleon isovector axial-vector form factor under the conditions discussed in the introduction. Therefore, we will apply the same technique as used for the magnetic form factors.

10.1. General Construction

Let us start with (A.18) at large momentum transfer. One gets the following representation at this limit.

$$\langle N_f(P')|A_{\mu}^a(0)|N_i(P)\rangle = \frac{1}{2}G_A^a(Q^2)\bar{N}(P')\gamma_\mu\gamma_5\gamma_i^aN(P) \tag{10.1}$$

We will choose the transition process $p \rightarrow n$ for our calculations. Therefore, we set $i = 1$ meaning that the initial nucleon is a proton and $f = 2$ meaning that the final nucleon is a neutron. Moreover, we set $a = 1$ to create a transition. Inserting the expression of the isovector axial-vector current, we realize that just one part of this current can contribute.

$$\langle n(P')|\bar{\psi}_d(0)\gamma_\mu\gamma_5\psi_u(0)|p(P)\rangle = G_A^a(Q^2)\bar{N}(P')\gamma_\mu\gamma_5N(P) \tag{10.2}$$

Afterwards, one can create a perturbative expansion of this matrix element. According to this, one has to insert the S-matrix including the interaction part of the QCD Lagrangian. Expanding the S-matrix up to the fourth order, one gets the first non-vanishing contributions depending on the quark gluon interaction part only.

$$\frac{(4\pi\alpha_s)^2}{24}\langle n(P')|\bar{\psi}_d(0)\gamma_\mu\gamma_5\psi_u(0)|p(P)\rangle \prod_{i=1}^{4} \int d^4 x_i \sum_{q_i} \bar{\psi}_q(x_i)\gamma_\alpha A^{\alpha i}(x_i)\psi_q(x_i) \tag{10.3}$$

This expansion can be described by Feynman diagrams and Wick contractions. Every diagram can be presented by four vertices connected to two quarks and one gluon. It is not necessary to compute every diagram separately. Symmetry properties between these diagrams ensure that the amount of diagrams which must be evaluated can significantly be reduced.

When we add the gluon gluon interaction part to the expansion, we get additional diagrams. These diagrams can be presented by three vertices connected to two quarks and one gluon together with one vertex connected to three gluons. Applying the third order in quark gluon interaction and the first order in gluon gluon interaction, one can generate these diagrams. These graphs do not contribute, because their color factor is zero.

10.2. Sample Diagram Evaluation

Let us consider one diagram for the nucleon isovector axial-vector form factor in detail. We mention that all remaining diagrams have similar structures. The chosen diagram can be visualized as follows. We have the incoming proton on the left and we have the outgoing
neutron on the right. The cross denotes the coupling to the isovector axial-vector current and the quark lines denote\( u, u', d \) from top to bottom on the left and\( d, u, d' \) from top to bottom on the right. The designations at the vertices are the corresponding coordinates and the designations at the lines are the corresponding momenta.

We proceed with the calculation of the color factor. Starting from the contracted representation of this diagram, one has to write out the color indices of all color dependent components. Afterwards, these elements must be moved in the required order to calculate the diagram. The indices\( (a, \ldots, i) \) at the squared brackets are the color indices.

\[
[\psi_d(x_1)]_c [\bar{\psi}_d(0)]_a [\psi_u(x_3)]_d [\bar{\psi}_u(x_2)]_i [\tau^{a_1}]_{bc} [\tau^{a_2}]_{de} A_{a_1}^{\alpha_1}(x_1) A_{a_2}^{\alpha_2}(x_2) [\tau^{a_3}]_{fg} [\tau^{a_4}]_{hi} A_{a_3}^{\alpha_3}(x_3) A_{a_4}^{\alpha_4}(x_4) \\
\langle n(P') [\bar{\psi}_d(x_1)]_b [\bar{\psi}_d(x_4)]_a [\psi_u(x_2)]_f [0] [\psi_u(0)]_a [\bar{\psi}_u(x_2)]_e [\bar{\psi}_d(x_4)]_i | p(P) \rangle
\]

Combining all terms and contracting the generators, one can derive the result of the color factor \( C_F \) which is identical for every diagram.

\[
C_F = \frac{1}{6} \varepsilon_{b f e a c} \delta_{d g} [\tau^{a_1}]_{bc} [\tau^{a_2}]_{de} [\tau^{a_3}]_{fg} [\tau^{a_4}]_{hi} \delta^{a_1 a_2} \delta^{a_3 a_4} = -\frac{4}{9} \tag{10.4}
\]

At next, we have to evaluate the Lorentz structure and to include the color factor. Restarting from the contracted representation of this diagram, we must write out the Lorentz indices of all parts. As before, we have to move these parts in the order which allows us to calculate the diagram. The indices\( (a, \ldots, j) \) at the squared brackets are now the Lorentz indices.

\[
-\frac{(4\pi\bar{\alpha}_s)}{54} \int \prod_{i=1}^{4} \int d^4x_1 [\gamma_\mu \gamma_5]_{ab} [\gamma_{\alpha_1}]_{cd} [\gamma_{\alpha_2}]_{ef} [\gamma_{\alpha_3}]_{gh} [\gamma_{\alpha_4}]_{ij} \\
[\psi_d(x_1)]_i [\bar{\psi}_d(0)]_a [\psi_u(x_3)]_d [\bar{\psi}_u(x_2)]_e A_{a_1}^{\alpha_1}(x_1) A_{a_2}^{\alpha_2}(x_2) A_{a_3}^{\alpha_3}(x_3) A_{a_4}^{\alpha_4}(x_4) \\
\langle n(P') [\bar{\psi}_d(x_1)]_b [\bar{\psi}_d(x_4)]_a [\psi_u(x_2)]_f [0] [\psi_u(0)]_a [\bar{\psi}_u(x_2)]_e [\bar{\psi}_d(x_4)]_i | p(P) \rangle
\]

Inserting the required components, one gets the following result when we recombine the Lorentz indices.

\[
-\frac{(4\pi\bar{\alpha}_s)}{864} \int \prod_{i=1}^{4} \int d^4x_i \int d^4A_j \Delta_j + i0 \int \prod_{k=1}^{2} \int d^4\Lambda_k \Lambda_k + i0 \int [d\mu][d\nu] g^{\alpha_1 \alpha_2} g^{\alpha_3 \alpha_4} S \\
e^{-i\pi(\Delta_1 - \Delta_1 - v_1p')} e^{-i\pi(\Delta_2 + \Delta_2 + u_2p)} e^{-i\pi(\Delta_2 - \Delta_2 - v_3p')} e^{-i\pi(\Delta_2 + u_3p - v_3p')}
\]

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The integration over the coordinates produce delta functions which can be used to integrate over the momenta. They describe the momentum conservation at each vertex of the diagram.

\[ \Delta_1 = (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \]
\[ \Delta_2 = (v_2 + v_3)p' - u_3p \]
\[ A_1 = (v_2 + v_3)p' - (u_2 + u_3)p \]
\[ A_2 = v_2p' - u_3p \]

We must specify the structures summarized in \( S \). The main differences of the discussed form factors are described by this component. We notice that we only work with leading twist distribution amplitudes and so one can omit the twist index at the distribution amplitudes. For further simplification, one can omit the dependence on the quark momentum fractions in these expressions. Therefore, we use abbreviations, for example, \( AV \) instead of \( A_1(v_1, v_2, v_3)V_1(u_1, u_2, u_3) \). Moreover, we only have to work with the large component of the spinor and so one can use the basic notation \( \bar{N} \) and \( N \) for the nucleon spinors.

\[ S_1 = \bar{N}(P')\gamma_{\alpha_3}\Delta_2\gamma_{\alpha_2}\gamma_{\mu_2}\gamma_{\delta_5}\Delta_1\gamma_{\alpha_1}\gamma_{\delta_4}N(P)(V\bar{V} + AA + AV + VA) \]
\[ S_2 = \bar{N}(P')\gamma_{\nu_3}\gamma_{\alpha_3}\Delta_2\gamma_{\alpha_2}\gamma_{\mu_2}\gamma_{\delta_5}\Delta_1\gamma_{\alpha_1}\gamma_{\delta_4}N(P)(-TV + TA) \]
\[ S_3 = \bar{N}(P')\gamma_{\nu_3}\Delta_2\gamma_{\alpha_2}\gamma_{\mu_2}\gamma_{\delta_5}\Delta_1\gamma_{\alpha_1}\gamma_{\delta_4}\gamma_{\delta_3}N(P)(VT - AT) \]
\[ S_4 = \bar{N}(P')\gamma_{\nu_3}\Delta_2\gamma_{\alpha_2}\gamma_{\mu_2}\gamma_{\delta_5}\Delta_1\gamma_{\alpha_1}\gamma_{\delta_4}\gamma_{\delta_3}N(P)(-TT) \]

Finally, we will present the result of the diagram. This expression depends on the integration over the quark momentum fractions.

\[ \frac{(4\pi\hat{s}_0)^2}{216} \frac{1}{Q^4} \int \frac{[du]}{u_3(u_2 + u_3)^2} \frac{[dv]}{v_2(v_2 + v_3)^2} [2(T(V - A) + (V - A)T)] \bar{N}(P')\gamma_{\mu_2}\gamma_{\delta_5}N(P) \quad (10.5) \]

### 10.3. Result of the Form Factor

In order to derive the final result for the nucleon isovector axial-vector form factor, we have to add all corresponding diagram results. Moreover, one has to insert the expressions of the nucleon distribution amplitudes. We only have to deal with a single independent leading twist distribution amplitude. Let us insert the most general polynomial for this distribution amplitude which is discussed. The other representations can be obtained by setting unnecessary coefficients to zero. Afterwards, we can evaluate the integration over the momentum fractions. This leads to the final result for the nucleon isovector axial-vector form factor.

\[ G_A^\nu(Q^2) = \frac{(4\pi\hat{s}_0)^2}{Q^4} \frac{f_N^2}{X_\nu_A(c_1, c_2, c_3, c_4, c_5, c_6)} \quad (10.6) \]

The function \( X_\nu_A \) depends on the polynomial coefficients of the applied distribution amplitude. It can be written in the following form with numbers \( a_{ij} \).

\[ X_\nu_A(c_1, c_2, c_3, c_4, c_5, c_6) = \frac{25}{1944} \sum_{i=1}^{6} \sum_{j=1}^{i} a_{ij}c_ic_j \quad (10.7) \]

The introduced numbers are given by: \( a_{11} = 1296, a_{21} = 1872, a_{31} = 144, a_{41} = 1872, a_{51} = 216, a_{61} = -144, a_{22} = 640, a_{32} = 448, a_{42} = 1220, a_{52} = 432, a_{62} = -8, a_{33} = -176, a_{43} = 512, a_{53} = -252, a_{63} = -72, a_{44} = 549, a_{54} = 440, a_{64} = 42, a_{55} = -100, a_{65} = -30, a_{66} = -12. \]
10. Isovector Axial-Vector Form Factor

10.4. Comparison with Experimental Data

Finally, we have to compare the obtained result with available experimental data. Hereby, we can distinguish between the different order polynomials again. Let us also present the results without specifying $f_N$ and $\alpha_s$ like in the electromagnetic case. Furthermore, concerning $\alpha_s$, we will use an averaged value from an effective momentum transfer around 1 GeV$^2$. Consequently, we will apply $\bar{\alpha}_s = 0.45 \pm 0.05$.

We obtain $X_A^v = 50/3$ which leads to $Q^4G_A^v(Q^2) = (15 \pm 5) \cdot 10^{-3}$ GeV$^4$ at the asymptotic limit. This is a non-vanishing but very small and positive result. The next contribution is given by the conformal expansion. In this case, we gain $X_A^v = 247$ rounded off the nearest full number which leads to $Q^4G_A^v(Q^2) = (0.22 \pm 0.05)$ GeV$^4$. One gets a small and positive result. Let us now consider the model dependent polynomial. Inserting the values for the coefficients from the different models into the function $X_A^v$, one receives model dependent results for the nucleon isovector axial-vector form factor rounded off two decimal places in order $10^3$ but with large uncertainty:

$$Q^4G_A^v(Q^2) = (4\pi\bar{\alpha}_s)^2 f_N^2 \cdot \begin{pmatrix} 1.75 \cdot 10^3 \\ 1.31 \cdot 10^3 \\ 2.50 \cdot 10^3 \\ 2.04 \cdot 10^3 \\ 1.80 \cdot 10^3 \end{pmatrix} = \begin{pmatrix} (1.6 \pm 0.1) \text{ GeV}^4 & (CZ) \\ (1.2 \pm 0.1) \text{ GeV}^4 & (GS) \\ (2.2 \pm 0.1) \text{ GeV}^4 & (KS) \\ (1.8 \pm 0.1) \text{ GeV}^4 & (COZ) \\ (1.6 \pm 0.1) \text{ GeV}^4 & (HET) \end{pmatrix} .$$

(10.8)

This expansion of the required nucleon distribution amplitude can be correlated to values of $Q^2$ in the region around 10 GeV$^2$. This form factor appears in standard weak interactions. Consequently, the desired processes are generally suppressed by the dominant electromagnetic interaction. Nevertheless, it is possible to choose processes for the measurement which can be separated from the electromagnetic interaction. In order to avoid the electromagnetic interaction, one can consider neutrino neutron scattering, because the neutrino as well as the neutron are uncharged. The problems are the facts that the neutron is an unstable particle and the slightly interacting neutrinos must be produced through decays before. Therefore, it is complicated to measure this form factor in the needed area. The mainly used experimental process applies an neutrino beam on a deuterium target. This process cannot be related to neutrino neutron scattering only, because neutrino proton scattering is also possible. Moreover, real targets are mixed of hydrogen and deuterium. The measured transition process is considered as quasielastic. We just have got experimental data for values of $Q^2$ being less than 10 GeV$^2$. Predicting the behavior in the required region, one can extract a value for $G_A^v(Q^2)$ which is given by $Q^4G_A^v(Q^2) = (1.5 \pm 0.2)$ GeV$^4$. Details concerning the experiments can mainly be taken from [26].
11. Isoscalar Axial-Vector Form Factor

In this chapter, we will calculate the nucleon isoscalar axial-vector form factor under the conditions discussed in the introduction. Therefore, we will apply the same technique as used for the magnetic form factors.

11.1. General Construction

We will start with (A.17) at large momentum transfer. One gets the following expression at this limit.

\[ \langle N_f(P')|A^0(0)|N_i(P)\rangle = \frac{1}{2}G^A_\lambda(Q^2)\bar{N}(P')\gamma_\mu\gamma_5\delta_{fi}N(P) \quad (11.1) \]

Let us choose the process \( p \rightarrow p \) for our calculations. Therefore, we set \( i = 1 \) and \( f = 1 \) meaning that the initial and the final nucleon are both protons. Additionally, we insert the expression of the isoscalar axial-vector current.

\[ \langle p(P')|\sum_q \bar{\psi}_q(0)\gamma_\mu\gamma_5\psi_q(0)|p(P)\rangle = G^A_\lambda(Q^2)\bar{N}(P')\gamma_\mu\gamma_5N(P) \quad (11.2) \]

Afterwards, one can introduce a perturbative expansion of this matrix element. According to this, one has to insert the S-matrix including the interaction part of the QCD Lagrangian. Expanding the S-matrix up to the fourth order, one gets the first non-vanishing contributions depending on the quark gluon interaction part only.

\[ \frac{(4\pi\alpha_s)^2}{24}\langle p(P')|\sum_q \bar{\psi}_q(0)\gamma_\mu\gamma_5\psi_q(0)|p(P)\rangle \prod_{i=1}^4 \int d^4x_i \sum_q \bar{\psi}_q(x_i)\gamma_\alpha A^{\alpha_i}(x_i)\psi_q(x_i) |p(P')\rangle \quad (11.3) \]

This expansion can be described by Feynman diagrams and Wick contractions. Every diagram can be presented by four vertices connected to one gluon and two quarks. It is not necessary to compute every diagram separately. Symmetry properties between these diagrams ensure that the amount of diagrams which must be evaluated can significantly be reduced.

When we add the gluon gluon interaction part to the expansion, we get additional diagrams. These diagrams can be presented by three vertices connected to one gluon and two quarks together with one vertex connected to three gluons. Applying the third order in quark gluon interaction and the first order in gluon gluon interaction, one can generate these diagrams. These graphs do not contribute, because their color factor is zero.

11.2. Sample Diagram Evaluation

Let us consider one diagram for the nucleon isoscalar axial-vector form factor in detail. We mention that all remaining diagrams have similar structures. The chosen diagram can be visualized as follows. We have the incoming proton on the left and we have the outgoing
11. Isoscalar Axial-Vector Form Factor

proton on the right. The cross denotes the coupling to the isoscalar axial-vector current and the quark lines denote \( u, u, d \) from top to bottom. The designations at the vertices are the corresponding coordinates and the designations at the lines are the corresponding momenta.

We continue with the calculation of the color factor. Starting from the contracted representation of this diagram, one has to write out the color indices of all color dependent components. Afterwards, these elements must be moved in the required order to calculate the diagram. The indices \( (a, \ldots, i) \) at the squared brackets are the color indices.

\[
\begin{align*}
&\langle P' | [\bar{\psi}_u(x_1)]_c [\bar{\psi}_u(0)]_a [\psi_u(x_3)]_b [\bar{\psi}_d(x_2)]_f [\bar{\psi}_d(0)]_i | P \rangle \\
&\langle P' | [\bar{\psi}_u(x_1)]_b [\bar{\psi}_u(x_3)]_j [\bar{\psi}_d(x_4)]_i [\psi_u(x_2)]_d [\psi_u(x_2)]_e | P \rangle
\end{align*}
\]

Combining all terms and contracting the generators, we derive the result of the color factor \( C_F \). This result is the same for every diagram.

\[
C_F = \frac{1}{6} \varepsilon_{f j h} \varepsilon_{a c d} \delta_{c d} g_{a i} [t^a]_{bc} [t^b]_{de} [t^c]_{fg} [t^d]_{hi} \delta^a \delta^b \delta^c = \frac{4}{9} \tag{11.4}
\]

At next, one has to evaluate the Lorentz structure including the color factor. Restarting from the contracted representation of this diagram, we must write out the Lorentz indices of all parts. As before, we have to move these parts in the order which allows us to calculate the diagram. The indices \( (a, \ldots, j) \) at the squared brackets are now the Lorentz indices.

\[
\begin{align*}
&- \frac{(4 \pi \alpha_s)^2}{54} \prod_{i=1}^{4} \int \frac{d^4 x_i}{(2\pi)^4} \int \frac{d^4 \Delta_j}{\Delta_j^2 + i0} \int \frac{d^4 \Delta_k}{\Delta_k^2 + i0} \int [d u_i] [d v_i] g^{a_1 a_2} g^{a_3 a_4} \mathcal{S} \\
&e^{-ix_1 \cdot (\Delta_1 - \Lambda_1 - v_1 P')} e^{-ix_2 \cdot (-\Delta_2 + \Lambda_1 + u_2 P)} e^{-ix_3 \cdot (\Delta_2 - \Lambda_2 - v_2 P')} e^{-ix_4 \cdot (\Lambda_2 + u_3 P - v_3 P')}
\end{align*}
\]
11.3. Result of the Form Factor

The integration over the coordinates produce delta functions which can be used to integrate over the momenta. They describe the momentum conservation at each vertex of the diagram.

\[
\begin{align*}
\Delta_1 &= (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \\
\Delta_2 &= (v_2 + v_3)p' - u_3p \\
\Lambda_1 &= (v_2 + v_3)p' - (u_2 + u_3)p \\
\Lambda_2 &= v_3p' - u_3p
\end{align*}
\]

We have to specify the structures summarized in \( S \). The main differences of the discussed form factors are described by this component. We notice that we only work with leading twist distribution amplitudes and so one can omit the twist index at the distribution amplitudes. For further simplification, one can omit the dependence on the quark momentum fractions in these expressions. Therefore, we use abbreviations, for example, \( AV \) instead of \( A_1(v_1, v_2, v_3)V_1(u_1, u_2, u_3) \). Moreover, we only have to work with the large component of the spinor and so one can use the basic notation \( \bar{N} \) and \( N \) for the nucleon spinors.

\[
S_1 = \bar{N}(P')\gamma_{\alpha_4}N(P)\text{Tr}[\gamma_{\mu}\gamma_5\gamma_{\alpha_2}\bar{\Delta}_2\gamma_{\alpha_3}\gamma_{\alpha_1}\Delta_1](VV + AA)
\]

\[
S_2 = \bar{N}(P')\gamma_{\alpha_4}\gamma_5N(P)\text{Tr}[\gamma_{\mu}\gamma_5\gamma_{\alpha_2}\bar{\Delta}_2\gamma_{\alpha_3}\gamma_5\gamma_{\alpha_1}\Delta_1](AV - VA)
\]

\[
S_3 = \bar{N}(P')\gamma_{\alpha_4}\gamma_5\gamma^N(P)\text{Tr}[\gamma_{\mu}\gamma_5\gamma_{\alpha_2}\bar{\Delta}_2\gamma_{\alpha_3}\gamma_{\lambda'}\gamma_{\alpha_1}\Delta_1](-TT)
\]

Finally, we will present the result of the diagram. This expression depends on the integration over the quark momentum fractions.

\[
\frac{(4\pi\bar{\alpha}_s)^2}{216} \frac{1}{Q^4} \int \frac{[du]}{u_3(u_2 + u_3)^2 v_3(v_2 + v_3)^2} \left[ (V - A)^2 + 4T^2 \right] \bar{N}(P')\gamma_{\mu}\gamma_5N(P)
\]

11.3. Result of the Form Factor

In order to derive the final result for the nucleon isoscalar axial-vector form factor, one has to add all corresponding diagram results. Moreover, we have to insert the expressions of the nucleon distribution amplitudes. We only have to deal with a single independent leading twist distribution amplitude. Let us insert the most general polynomial for this distribution amplitude which is discussed. The other representations can be obtained by setting unnecessary coefficients to zero. Afterwards, we can evaluate the integration over the momentum fractions. This leads to the final result for the nucleon isoscalar axial-vector form factor.

\[
G_A^s(Q^2) = \frac{(4\pi\bar{\alpha}_s)^2 f_N^2}{Q^4} X_A^s(c_1, c_2, c_3, c_4, c_5, c_6)
\]

The function \( X_A^s \) depends on the polynomial coefficients of the applied distribution amplitude. It can be written in the following form with numbers \( a_{ij} \).

\[
X_A^s(c_1, c_2, c_3, c_4, c_5, c_6) = -\frac{25}{3888} \sum_{i=1}^{6} \sum_{j=1}^{i} a_{ij} c_i c_j
\]

The introduced numbers are given by: \( a_{11} = 7776, a_{21} = 3456, a_{31} = 3456, a_{41} = -216, a_{51} = -216, a_{61} = 1728, a_{22} = -304, a_{32} = 1472, a_{42} = -1500, a_{52} = 312, a_{62} = 480, a_{33} = -304, a_{43} = 312, a_{53} = -1500, a_{63} = 480, a_{44} = -907, a_{54} = -124, a_{64} = 144, a_{55} = -907, a_{65} = 144, a_{66} = 72.\)
11. Isoscalar Axial-Vector Form Factor

11.4. Comparison with Experimental Data

Finally, we have to compare the obtained result with available experimental data. Hereby, one can distinguish between the different order polynomials again. Let us also present the results without specifying $f_N$ and $\alpha_s$ like in the electromagnetic case. Furthermore, concerning $\alpha_s$, we will use an averaged value from an effective momentum transfer around 1 GeV$^2$. Consequently, we will apply $\bar{\alpha}_s = 0.45 \pm 0.05$.

We obtain $X^A_\alpha = -50$ which leads to $Q^4G^A_\alpha(Q^2) = -(45 \pm 5) \cdot 10^{-3}$ GeV$^4$ at the asymptotic limit. This is a non-vanishing but very small and negative result. The next contribution is given by the conformal expansion. In this case, we gain $X^A_\alpha = 256$ rounded off the nearest full number which leads to $Q^4G^A_\alpha(Q^2) = (0.23 \pm 0.05)$ GeV$^4$. One gets a small and positive result and so the sign of the result has been changed. Let us now consider the model dependent polynomial. Inserting the values for the coefficients from the different models into the function $X^A_\alpha$, one receives model dependent results for the nucleon isoscalar axial-vector form factor rounded off two decimal places in order $10^3$ but with large uncertainty.

$$
Q^4G^A_\alpha(Q^2) = (4\pi\bar{\alpha}_s)^2 f_N^2 \cdot 
\begin{pmatrix}
1.66 \cdot 10^3 \\
3.10 \cdot 10^3 \\
2.61 \cdot 10^3 \\
1.89 \cdot 10^3 \\
3.71 \cdot 10^3
\end{pmatrix} = 
\begin{pmatrix}
(1.5 \pm 0.1) \text{ GeV}^4 \\
(2.3 \pm 0.1) \text{ GeV}^4 \\
(1.7 \pm 0.1) \text{ GeV}^4 \\
(3.3 \pm 0.1) \text{ GeV}^4
\end{pmatrix} \text{ (CZ)} \\
\begin{pmatrix}
(2.8 \pm 0.1) \text{ GeV}^4 \\
(2.3 \pm 0.1) \text{ GeV}^4 \\
(1.7 \pm 0.1) \text{ GeV}^4 \\
(3.3 \pm 0.1) \text{ GeV}^4
\end{pmatrix} \text{ (GS)} \\
\begin{pmatrix}
(2.8 \pm 0.1) \text{ GeV}^4 \\
(2.3 \pm 0.1) \text{ GeV}^4 \\
(1.7 \pm 0.1) \text{ GeV}^4 \\
(3.3 \pm 0.1) \text{ GeV}^4
\end{pmatrix} \text{ (KS)} \\
\begin{pmatrix}
(2.8 \pm 0.1) \text{ GeV}^4 \\
(2.3 \pm 0.1) \text{ GeV}^4 \\
(1.7 \pm 0.1) \text{ GeV}^4 \\
(3.3 \pm 0.1) \text{ GeV}^4
\end{pmatrix} \text{ (COZ)} \\
\begin{pmatrix}
(2.8 \pm 0.1) \text{ GeV}^4 \\
(2.3 \pm 0.1) \text{ GeV}^4 \\
(1.7 \pm 0.1) \text{ GeV}^4 \\
(3.3 \pm 0.1) \text{ GeV}^4
\end{pmatrix} \text{ (HET)}
$$

11.5. Comparison with Experiment

This expansion of the required nucleon distribution amplitude can be correlated to values of $Q^2$ in the region around 10 GeV$^2$. This form factor appears in non standard weak interaction theories only. Therefore, the needed special processes are suppressed by the dominant electromagnetic interaction. Moreover, these processes are also suppressed by standard weak interactions. Consequently, it is not possible to construct an experiment in order to measure this form factor independently. Under these conditions, it is really complicated to measure this form factor at usable values of the momentum transfer. We just have got experimental data for values of $Q^2$ which are significantly less than 10 GeV$^2$. Under these circumstances, it makes no sense to compare the obtained results. According to this, we can only consider our results as a prediction for further experiments. A comparison with experimental data is impossible at the moment.
12. Discussion of the Discrepancy

Finally, we will compare our results for the proton magnetic form factor with previous results obtained in calculations based on the QCD factorization theorem. Using this technique, one can split the required nucleon transition probability matrix element into one hard part and two soft parts. Afterwards, these parts can be calculated separately. For further information about this theory, it is recommended to have a look on [10], [11], [12], and [13], [14], [15]. Hereby, we focus on the known discrepancy concerning the symmetry factor.

The starting point of our considerations is the calculation of the desired form factor performed in [36]. After some time, the discussed form factor was recalculated in [45]. They obtained a result which is precisely a factor of 2 smaller than the previous result. They mentioned that they do not understand the origin of this discrepancy. A newer calculation of the proton magnetic form factor was done in [46]. Their result is in agreement with [45]. They realized that the symmetry factor in [36] is precisely a factor of $\sqrt{2}$ larger than in [45]. Nevertheless, they mentioned that they do not know which symmetry factor is correct. However, our result of the proton magnetic form factor is the same as in [46]. At this point, we should try to find the origin of the discrepancy between the older and the newer results. Looking in [12], we can see that the normalization of the soft parts is not fixed, but a required normalization connected to the number of colors is discussed. We realized that we need a normalization connected to the number of possible color states. In the case of the pion, one gets no difference, because the normalization by the number of colors as well as the normalization by the number of possible color states would lead to the factor $1/\sqrt{3}$. This is not true in the case of the nucleon. The normalization by the number of colors would obviously lead to the factor $1/\sqrt{3}$ too, but the normalization by the number of possible color states would require the factor $1/\sqrt{6}$ creating a difference of $\sqrt{2}$. Because two soft parts are required, one has to square the normalization factor. We claim that this is the source for the factor of 2 difference in previous calculations. Indeed, when we compare the normalization of [36] and [45], we find the normalization factor $1/\sqrt{3}$ in [36] and $1/\sqrt{6}$ in [45] as expected.
13. Conclusion to Part II

We calculated all leading nucleon form factors at large momentum transfer and in single gauge boson exchange approximation. We started with the evaluation of the nucleon magnetic form factors. Hereby, we concentrated our considerations on the proton form factor. Concerning the neutron form factor, we just specified the differences during the calculations. Afterwards, we studied the isovector axial-vector form factor. For the computation of this form factor, we used the proton to neutron transition. At last, we discussed the isoscalar axial-vector form factor. Hereby, we considered two proton states for the form factor evaluation.

In order to evaluate the required nucleon transition probability matrix elements, we combined QCD perturbation theory with an expansion in nucleon distribution amplitudes. In this part of our work, we had to use leading twist nucleon distribution amplitudes only.

It was our aim to compare the obtained results with experimental data. Therefore, it was necessary to study the behavior of the single independent leading twist nucleon distribution amplitude.

This distribution amplitude could be expanded in polynomials. The first order polynomial could be expressed by a simple asymptotic structure related to the case of infinite momentum transfer. Taking into account the second order polynomial, we realized that this construction is related to values of the momentum transfer which are still unreachable in experiments. Therefore, it was necessary to use the third order polynomial. In this case, we were able to describe the region of the momentum transfer $Q^2$ around 10 GeV$^2$. In this area, experimental data is available. The disadvantage of this polynomial is that the coefficients are non-perturbative parameters which are not fixed. According to this, we used non-perturbative models for the coefficients.

Finally, we were able to compare with experimental data. Hereby, we discussed the realizable experiments and analyzed the available data. The model dependent results of all calculated nucleon form factors are essentially in good agreement with the available experimental data. Nevertheless, further experimental data at large momentum transfer are required in order to gain a deeper understanding of the nucleon form factors. This would obviously lead to an improved knowledge concerning the structure of the nucleons.

Finally, we discussed the known discrepancy concerning the symmetry factor in previous works. We argued that taking into account the color normalization appropriately, the discrepancy can be resolved.

In order to present the important results of these studies to the entire community, the main part of this work is published in [51].
Part III.

Evaluation of the Nucleon Helicity Flip Form Factor at Large Momentum Transfer using One and Two Virtual Photons
14. Introduction to Part III

Elastic electron nucleon scattering mediated by the electromagnetic interaction is the most important process to receive information about the nucleon structure within Quantum Chromodynamics. Applying the basic one photon exchange approximation, the required nucleon transition probability matrix elements are traditionally expressed by the Dirac form factor and the Pauli form factor or, equivalently, the magnetic form factor and the electric form factor. For convenience, we use the representation by the magnetic form factor and the Pauli form factor, specified in (A.5) and (A.6). Let us concentrate our considerations on the proton form factors like in the previous part. The required exchanges to obtain the contributions for the neutron form factors were already discussed there.

At large momentum transfer $Q^2$, one just gets the contribution for the magnetic form factor with the power behavior of $Q^{-4}$. This form factor was measured in a comprehensive region and calculated with different techniques, basically with the QCD factorization theorem. Among other form factors, the magnetic form factor was extensively studied in the previous part of our work. Hereby, we realized that the results for the magnetic form factor and the Dirac form factor are identical at this limit.

Moving to intermediate values of the momentum transfer, one also gets contributions for the Pauli form factor with the power behavior of $Q^{-6}$. The different power behavior arises from the helicity flip of the nucleon and so this form factor is also known as helicity flip form factor. In this part of our work, we will evaluate the nucleon helicity flip form factor. Concerning this form factor, experimental data are also available. Moreover, one has discovered a different behavior depending on the type of the experiment.

The basic information were taken from unpolarized cross sections. Using the Rosenbluth separation technique [52], several experiments were performed. Hereby, early experiments did not show significant double photon corrections, see [53], [54], [55], [56], [57], [58], [59], [60], or radiative corrections, see [61], [62]. Further experiments were executed in [63], [64], [65], and with taking into account radiative corrections, see [66], [67]. Moreover, the available data were fitted in [68]. The consequences of radiative corrections were considered in [69].

The discussed technique is useful at low $Q^2$, but at larger $Q^2$, the contribution of the helicity flip form factor is suppressed by the momentum transfer. However, the electric form factor seems to have the same power behavior as the magnetic form factor. In order to measure the desired form factor at larger values of $Q^2$, one has to study polarized cross sections. During the last years, the experimental requirements have been created and so various experiments have been performed. Hereby, one needs a polarized electron beam. From the experimental perspective, the polarization transfer method, discussed in [70], seems to be in favor. Hereby, one has to measure the polarization of the recoil proton [71], [72], [73], [74], [75], [76], [77], [78]. The alternative is to use polarized proton targets [79], [80]. Concerning these data, the electric form factor seems to be power suppressed compared to the magnetic form factor. The different measurements were compared in [81], [82], [83].

From the theoretical perspective, the calculation of the desired form factor in the large $Q^2$ region is problematic. Using the QCD factorization theorem and the basic one photon exchange, the Pauli form factor was studied in [84]. In this work, divergent integrals were
obtained and therefore a cutoff parameter related to an effective size of the nucleon was introduced. This form factor was also considered in [85]. Hereby, different models were considered and a cutoff parameter related to an effective mass of the nucleon was discussed with different logarithmic power behavior.

In order to understand the different behavior in the discussed experiments, it has been suggested that the two photon exchange contribution can cause this situation. Therefore, an advanced form factor parametrization was developed in [86]. The modified magnetic form factor was calculated in [87]. The obtained corrections cannot describe the experiments without an input from the helicity flip form factor. Therefore, it is necessary to study the required form factor in the one and two photon exchange approximation.

Let us apply the technique specified in the previous part to evaluate the form factor. We have to combine QCD perturbation theory with an expansion in nucleon distribution amplitudes again. Hereby, we have to use the combination of leading and sub-leading twist nucleon distribution amplitudes. Moreover, we have to specify the frame. We will show this in the appendix about nucleon distribution amplitudes.

We will start with the one photon exchange and we will finish with the two photon exchange. Therefore, we will choose one sample diagram for the computation of the helicity flip form factor. In the one photon exchange approximation, one can separate the leptonic and the hadronic part. Consequently, it is sufficient to evaluate the corresponding nucleon transition probability matrix element. In the two photon exchange approximation, the leptonic and the hadronic part are not separated. According to this, we have to calculate the scattering amplitude of the process and to derive the dependence on another nucleon transition probability matrix element. In order to generalize this approach, we will start with the evaluation of the scattering amplitude for the one and two photon exchange approximation.

Using our technique, we will obtain a divergent result for the form factor. Nevertheless, the structure of the divergency can be extracted. Concerning the modified helicity flip form factor, we will obtain the dependence on one additional variable. Calculating the experimental cross section of the process and using the momentum transfer and the scattering angle as variables, one gets a different general behavior in the one and two photon exchange approximation. In the first case, the form factor depends on the momentum transfer only. This was already known and so the Rosenbluth separation technique could be applied. In the second case, the form factor depends on the momentum transfer and apart from that, it depends on the scattering angle additionally. That means, the Rosenbluth separation technique cannot be used in this case. Moreover, the obtained power behavior of the helicity flip form factor can describe the experimental data based on polarized cross sections qualitatively. According to this, we can explain the different behavior in the experiments using unpolarized or polarized cross sections. Finally, we will discuss the required modifications to avoid the divergency.
15. One Photon Exchange Approximation

Let us start with the presentation of the desired diagram. In the upper part, we see the incoming electron on the left and the outgoing electron on the right. In the lower part, we have the incoming proton on the left and the outgoing proton on the right. The required quark lines denote $u, u, d$ from top to bottom. The designations at the vertices are the corresponding coordinates and the designations at the lines are the corresponding momenta.

Our aim is to derive the expression for the scattering amplitude. Applying QED Feynman rules, one can evaluate the leptonic part of the diagram directly.

$$\mathcal{M} = -i(4\pi\alpha_{em}) \prod_{i=1}^{2} \int \frac{d^4 y_i}{(2\pi)^4} \int \frac{d^4 q}{q^2 + i0} \bar{u}(l')\gamma^\mu u(l) \langle p(P') | J^{em}_\mu(y_1) | p(P) \rangle e^{iq(y_2-y_1)} e^{-iq(l-l')}
$$

We can solve the integration over $y_2$ immediately. One gets the known momentum conservation delta function describing $q = l - l'$. Moreover, we can replace $\langle p(P') | J^{em}_\mu(y_1) | p(P) \rangle$ with $\langle p(P') | J^{em}_\mu(0) | p(P) \rangle e^{-i(q_{P'} - q_P)\cdot(l' - l)}$. Afterwards, we can solve the integration over $y_1$. This leads to the known momentum conservation delta function describing $q = P' - P$. Using these delta functions, the integration over $q$ can be evaluated.

$$\mathcal{M} = -\frac{i(4\pi\alpha_{em})}{q^2} \bar{u}(l')\gamma^\mu u(l) \langle p(P') | J^{em}_\mu(0) | p(P) \rangle \quad (15.1)
$$

Applying (A.5), we introduce the expansion in nucleon electromagnetic form factors. Let us use the standard notation $Q^2 = -q^2$. For convenience, we define the averaged nucleon momentum $2P = p + p'$. At large momentum transfer, one gets the reduction $2P = p + p'$.
15. One Photon Exchange Approximation

\[ \mathcal{M} = \frac{i(4\pi\alpha_{em})}{Q^2} \bar{u}(l') \gamma^\mu u(l) \tilde{N}(P') \left[ G_M^p(Q^2)\gamma_\mu - F_2^p(Q^2)\frac{\not{P}_\mu}{m_N} \right] N(P) \] (15.2)

The advantage of this expression is the separation of the leptonic and the hadronic part. Consequently, we only need to consider the matrix element \( \langle p(P');J^em(0)|p(P) \rangle \). We already used this behavior for the calculation of the magnetic form factor, but now we want to compute the Pauli form factor which is also known as helicity flip form factor.

Applying the S-matrix expansion including the interaction part of the QCD Lagrangian, one gets the following leading expression for the desired matrix element.

\[ \frac{(4\pi\alpha_s)^2}{24} \langle p(P')| \sum_q e_q \bar{\psi}_q(0)\gamma_\mu \psi_q(0) T \left[ \prod_{i=1}^4 d^4x_i \sum_{q_i} \bar{\psi}_{q_i}(x_i)\gamma_{\alpha_i} A^{a_i}(x_i)\psi_{q_i}(x_i) \right] |p(P) \rangle \]

This expansion can be described by 42 Feynman diagrams and Wick contractions. We can extract the representation of the diagram which we want to study.

Let us begin with the determination of the color factor. Therefore, one has to examine the color structure of the diagram, denoting the color indices with \((a, \ldots, i)\).

\[ \langle p(P')| \sum_q e_q \bar{\psi}_q(0) T \left[ \prod_{i=1}^4 d^4x_i \sum_{q_i} \bar{\psi}_{q_i}(x_i)\gamma_{\alpha_i} A^{a_i}(x_i)\psi_{q_i}(x_i) \right] |p(P) \rangle \]

Combining all terms and contracting the generators, one gets the color factor.

\[ C_F = \frac{1}{6} \varepsilon_{bfg} \varepsilon_{ace} \delta_{cd} \delta_{[\alpha_1]} \varepsilon_{[\alpha_2]} \varepsilon_{[\alpha_3]} \varepsilon_{[\alpha_4]} = \frac{4}{9} \] (15.3)

We continue with the evaluation of the Lorentz structure of the diagram, designating the Lorentz indices with \((a, \ldots, j)\). Including \( C_F \), we obtain the following expression.

\[ -\frac{(4\pi\alpha_s)^2}{54} e_u \sum_{i=1}^4 \int d^4x_i \langle \gamma_\mu | \bar{\psi}(0) \gamma_\alpha | c_1 | \bar{\psi}(0) \gamma_\alpha | c_2 \rangle \varepsilon_{[\alpha_1]} \varepsilon_{[\alpha_2]} \varepsilon_{[\alpha_3]} \varepsilon_{[\alpha_4]} \]

\[ \langle p(P')| \sum_q e_q \bar{\psi}_q(0) T \left[ \prod_{i=1}^4 d^4x_i \sum_{q_i} \bar{\psi}_{q_i}(x_i)\gamma_{\alpha_i} A^{a_i}(x_i)\psi_{q_i}(x_i) \right] |p(P) \rangle \]

In order to evaluate this expression, we have to apply the representations for the propagators and for the projection matrix elements.

\[ e^{-ix_1 \cdot (\Delta_1 - \Lambda_1 - v_1 p')} e^{-ix_2 \cdot (-\Delta_2 + \Lambda_1 + u_2 p')} e^{-ix_3 \cdot (\Delta_2 - \Lambda_2 - v_2 p')} e^{-ix_4 \cdot (\Delta_2 + u_3 p - v_3 p')} \]

Computing the integrations, one gets the required momentum conservation constraints.

\[ \Delta_1 = (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \]
\[ \Lambda_1 = (v_2 + v_3)p' - (u_2 + u_3)p \]

\[ \Delta_2 = (v_2 + v_3)p' - u_3 p \]
\[ \Lambda_2 = v_3 p' - u_3 p \]

The component \( S \) is the sum of all required structures connected with combinations of nucleon distribution amplitudes and nucleon spinors. Let us omit the dependence on the quark momentum fractions. Moreover, we use the standard notation for the nucleon spinors.
We get the following structures for initial twist-4 and final twist-3.

\[
S_1 = (m_N/Q^2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_3} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(V_1 V_2 + A_1 A_2 + V_1 V_3 - A_1 A_3)
\]

\[
S_2 = (m_N/Q^2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(A_1 V_2 + V_1 A_2 - A_1 V_3 + V_1 A_3)
\]

\[
S_3 = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(-V_1 V_3 + A_1 A_3)
\]

\[
S_4 = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} i \sigma_{\lambda \mu} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(T_1 S_1)
\]

\[
S_5 = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} i \sigma_{\lambda \mu} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(T_1 P_1)
\]

\[
S_7 = (2m_N/Q^2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} i \sigma_{\mu \nu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} i \sigma_{\lambda \mu} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(T_1 T_2 - T_1 T_3 + T_1 T_7)
\]

\[
S_8 = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} i \sigma_{\lambda \mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} i \sigma_{\lambda \mu} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(T_1 T_2 + 2T_1 T_7)
\]

\[
S_9 = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} i \sigma_{\lambda \mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} i \sigma_{\lambda \mu} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(T_1 T_7)
\]

We get the following structures for initial twist-3 and final twist-4.

\[
S_{10} = (m_N/Q^2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(V_2 V_1 + A_2 A_1 + V_3 V_1 - A_3 A_1)
\]

\[
S_{11} = (m_N/Q^2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(V_2 A_1 + A_2 V_1 + V_3 A_1 - A_3 V_1)
\]

\[
S_{12} = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(-V_3 V_1 + A_3 A_1)
\]

\[
S_{13} = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(-V_3 A_1 + A_3 V_1)
\]

\[
S_{14} = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(-S_1 T_1)
\]

\[
S_{15} = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(P_1 T_1)
\]

\[
S_{16} = (2m_N/Q^2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} i \sigma_{\lambda \mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} i \sigma_{\lambda \mu} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(T_2 T_1 - T_2 T_3 + T_2 T_7)
\]

\[
S_{17} = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} i \sigma_{\lambda \mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} i \sigma_{\lambda \mu} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(T_2 T_1 + 2T_2 T_7)
\]

\[
S_{18} = (m_N/2) \bar{N}(P') \gamma_{\alpha_4} \gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\alpha_1} N(P) \text{Tr}[\gamma_{\mu} i \sigma_{\lambda \mu} \gamma_{\alpha_2} \Delta_{2} \gamma_{\alpha_3} i \sigma_{\lambda \mu} \gamma_{\alpha_1} \Delta_{1}]
\]
\[
(T_7 T_1)
\]

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15. One Photon Exchange Approximation

Computing every structure, one always gets the dependence on $N(P')P_NN(P)$ as predicted. One can simplify the expression by exchanging $u \leftrightarrow v$ for contributions of initial twist-3 and final twist-4. Consequently, we obtain a representation depending on initial twist-4 and final twist-3 only. For instance, we transform $A_2(v_1, v_2, v_3)V_1(u_1, u_2, u_3) \to V_1(v_1, v_2, v_3)A_2(u_1, u_2, u_3)$.

Let us now present the result of the discussed diagram depending on the integration over the quark momentum fractions. When we compare with the separated hadronic part of (15.2), we can extract the contribution to the desired form factor. In order to get the complete result, one also needs the contributions of the other diagrams designated by $C$.

$$F_2^p(Q^2) = -\frac{(4\pi\alpha_s)^2 e_u m_N^2}{108} \frac{1}{Q^6} \int \frac{[du]}{u_3(u_2 + u_3)^2} \frac{[dv]}{v_2^2(v_2 + v_3)^2} D + C$$

(15.4)

The component $D$ is the sum of the remaining twist combinations of distribution amplitudes connected with multiple quark momentum fractions.

$$D_1 = |V_1V_2 + A_1A_2|(2(u_2 + u_3)(v_2 + v_3))$$
$$D_2 = |V_1V_3 - A_1A_3|(v_3 - (u_2 + u_3)(v_2 + v_3))$$
$$D_3 = |V_1A_3 - A_1V_3|(v_3 + (u_2 + u_3)(v_2 + v_3))$$
$$D_4 = |T_1S_1 - T_1P_1|(2v_3 - 2(u_2 + u_3)v_3)$$
$$D_5 = |T_1T_3 + T_1T_7|(2v_3 - 2(u_2 + u_3)v_3)$$

Finally, we must insert the nucleon distribution amplitudes. As discussed in the appendix about nucleon distribution amplitudes, we have to deal with one independent twist-3 distribution amplitude and with three independent twist-4 distribution amplitudes. One can insert the most general polynomials for these distribution amplitudes which are considered there. The other representations can be obtained by setting unnecessary coefficients to zero. In both cases, we can consider the first order polynomial given by the asymptotic form and the second order polynomial described by the conformal expansion. Afterwards, we can evaluate the integration over the momentum fractions.

Unfortunately, the corresponding integration is divergent. This divergency arises from endpoint singularities. That means, the integrals get divergent when a quark has no momentum or the full momentum of the nucleon. In order to analyze the structure of the divergency, one can introduce a cutoff parameter $\Omega$. Therefore, one has to respect that in the case of infinite momentum transfer the integration must go from zero to one for every quark momentum fraction. According to this, we always integrate from $\Omega/Q^2$ to $1 - \Omega/Q^2$, keeping in mind that the introduced parameter has the same dimension as the momentum transfer.

Computing the modified integration, we can extract the structure of the divergency. One obtains the same general behavior for the second order polynomial as for the first order polynomial. Moreover, this behavior is identical for all other required diagrams as well. Consequently, we can generally express the power behavior of the helicity flip form factor depending on the cutoff parameter.

$$F_2^p(Q^2) \propto Q^{-6} \ln^2(Q^2/\Omega)$$

(15.5)

We derived the expected power behavior of $Q^{-6}$ and we obtained a double logarithmic divergency in the case of $\Omega \to 0$. This behavior is in agreement with [84].
16. Two Photon Exchange Approximation

Let us begin with the discussion about an important behavior of this situation. In the one photon case, the inversion of the lepton direction delivers the same contribution to the scattering amplitude. This statement is not true in the two photon case, because the inversion of the lepton direction produces another diagram. Therefore, one has to distinguish between the box diagram and the cross diagram. The corresponding contributions to the scattering amplitude must be calculated separately.

We start with the presentation of the box diagram. In the upper part, we see the incoming electron on the left and the outgoing electron on the right. In the lower part, we have the incoming proton on the left and the outgoing proton on the right. The required quark lines denote $u, u, d$ from top to bottom. The designations at the vertices are the corresponding coordinates and the designations at the lines are the corresponding momenta.

At next, we have to derive the expression for the scattering amplitude of the box diagram. Applying QED Feynman rules, one can evaluate the leptonic part of the diagram directly. Moreover, one can neglect the electron mass in the propagator:

$$\mathcal{M}_B = -\frac{i(4\pi\alpha_{em})^2}{24} \prod_{i=1}^{4} \frac{d^4 y_i}{(2\pi)^4} \frac{d^4 q_j}{q_j^2 + i0} \int \frac{d^4 \Gamma}{\Gamma^2 + i0} \bar{u}(l') \gamma_{\mu_2} P \gamma_{\mu_1} u(l)$$

$$\langle p(P')|J_{\mu_2}^c(y_2)J_{\mu_1}^c(y_1)|p(P)\rangle e^{iq_1 \cdot (y_3 - y_1)} e^{iq_2 \cdot (y_4 - y_2)} e^{-i\Gamma \cdot (y_4 - y_3)} e^{-iy_3 \cdot l} e^{iy_4 \cdot l'}$$

The integration over $y_3$ leads to the momentum conservation delta function describing $\Gamma = l - q_1$ and the integration over $y_4$ leads to the momentum conservation delta function.
16. Two Photon Exchange Approximation

which describes $\Gamma = l' + q_2$. Combining these delta functions, the integration over $\Gamma$ can be solved. One gets the representation $2\Gamma = (l + l') + (q_2 - q_1)$.

\[
\mathcal{M}_B = -\frac{i(4\pi\alpha_{em})^2}{24} \prod_{i=1}^2 \int \frac{d^4y_i}{(2\pi)^4} \prod_{j=1}^2 \int \frac{d^4q_j}{q_j^2 + i0} \frac{1}{\Gamma^2} \bar{u}(l')\gamma^\mu_1 P\gamma^\mu_2 u(l)
\]

\[\langle p(P')|J_{\mu_2}^em(y_2)J_{\mu_1}^em(y_1)|p(P)\rangle e^{-iq_1\cdot y_1} e^{iq_2\cdot (y_3 - y_2)} e^{-i\Gamma\cdot (y_4 - y_3)} e^{-iy_3\cdot l'} e^{iy_4\cdot l'}
\]

We finish with the presentation of the cross diagram. In the upper part, we see the incoming electron on the right and the outgoing electron on the left. In the lower part, we have the incoming proton on the left and the outgoing proton on the right. The designated quark lines denote $u$, $d$, $s$ from top to bottom. The designations at the vertices are the corresponding coordinates and the designations at the lines are the corresponding momenta.

\[\mathcal{M}_C = -\frac{i(4\pi\alpha_{em})^2}{24} \prod_{i=1}^2 \int \frac{d^4y_i}{(2\pi)^4} \prod_{j=1}^2 \int \frac{d^4q_j}{q_j^2 + i0} \frac{1}{\Gamma^2} \bar{u}(l')\gamma^\mu_1 P\gamma^\mu_2 u(l)
\]

\[\langle p(P')|J_{\mu_2}^em(y_2)J_{\mu_1}^em(y_1)|p(P)\rangle e^{-iq_1\cdot y_1} e^{iq_2\cdot (y_3 - y_2)} e^{-i\Gamma\cdot (y_4 - y_3)} e^{-iy_3\cdot l'} e^{iy_4\cdot l'}
\]

The integration over $y_3$ leads to the momentum conservation delta function describing $\Gamma = l - q_2$ and the integration over $y_4$ leads to the momentum conservation delta function which describes $\Gamma = l' + q_1$. Combining these delta functions, the integration over $\Gamma$ can be evaluated. One gets the representation $2\Gamma = (l + l') + (q_1 - q_2)$.

\[
\mathcal{M}_C = -\frac{i(4\pi\alpha_{em})^2}{24} \prod_{i=1}^2 \int \frac{d^4y_i}{(2\pi)^4} \prod_{j=1}^2 \int \frac{d^4q_j}{q_j^2 + i0} \frac{1}{\Gamma^2} \bar{u}(l')\gamma^\mu_1 P\gamma^\mu_2 u(l)
\]

\[\langle p(P')|J_{\mu_2}^em(y_2)J_{\mu_1}^em(y_1)|p(P)\rangle e^{-iq_1\cdot y_1} e^{iq_2\cdot (y_3 - y_2)} e^{-i\Gamma\cdot (y_4 - y_3)} e^{-iy_3\cdot l'} e^{iy_4\cdot l'}
\]

At next, we have to derive the expression for the scattering amplitude of the cross diagram. Applying QED Feynman rules, one can evaluate the leptonic part of the diagram directly. Furthermore, one can neglect the electron mass in the propagator.
The overall result for the scattering amplitude is given by \( \mathcal{M} = \mathcal{M}_B + \mathcal{M}_C \). Let us now introduce the expansion in nucleon electromagnetic form factors. Unfortunately, the leptonic and the hadronic part are not separated in this case. Nevertheless, one can show the existence of a separated representation for \( \mathcal{M} \). Therefore, one has to modify (A.5). Whereas the basic expression just depends one the nucleon momenta, the modified expression also depends on the lepton momenta. The derivation can be taken from [86]. Therefore, we can use \( Q^2 = -q^2 \) and assume \( q = l-l' \) together with \( q = p' - p \). Furthermore, we need the formula \( 2\tilde{P} = P + P' \) and the reduction \( 2\tilde{P} = p + p' \) again. Moreover, we define the averaged lepton momentum \( 2\tilde{L} = l + l' \).

\[
\mathcal{M} = \frac{i(4\pi \alpha_{em})}{Q^2} \bar{u}(l') \gamma^\mu u(l) \tilde{N}(P') \left[ \tilde{G}_M^p \gamma^\mu - \tilde{F}_2^p \frac{\tilde{P}_\mu}{m_N} + \tilde{F}_3^p \frac{\tilde{E}_\mu}{m_N^2} \right] N(P) \tag{16.3}
\]

All form factors depend on \( Q^2 \) as usual, but they also depend on one additional variable. Therefore, we choose the dimensionless quantity \( \omega \) defined by \( \omega = 4(\tilde{P} \cdot \tilde{L})/Q^2 \). At large \( Q^2 \), one gets the boundary condition \( \omega \geq 1 \). In principle, one can generally expand every form factor as \( \tilde{F} = F + \delta F \), where \( F \) is the single photon exchange contribution and \( \delta F \) is the multi photon exchange contribution. We do not use this decomposition because we consider the one and two photon exchange separately. The form factor \( \tilde{F}_3^p(\omega, Q^2) \) does not appear in single photon exchange approximation. The form factor \( \tilde{G}_M^p(\omega, Q^2) \) designates the two photon contribution to the already known magnetic form factor. The form factor \( \tilde{F}_2^p(\omega, Q^2) \) describes the two photon contribution to the helicity flip form factor and we want to compute this component. The other components are considered in [87].

Let us now study the matrix element \( \langle p(P') | J_{em}^{\mu_2}(y_2) J_{em}^{\mu_1}(y_1) | p(P) \rangle \). The evaluation of this matrix element must be combined with the other terms in (16.1) and (16.2) to derive a result for the form factor. Applying the S-matrix Lagrangian one gets the following leading expression for this matrix element.

\[
-\frac{(4\pi \alpha_s)}{2} \langle p(P') \prod_{j=2}^2 \sum_{q_j} e_{q_j} \bar{u}_{q_j}(y_j) \gamma_\mu_j u_{q_j}(y_j) T \left[ \prod_{i=1}^2 \int d^4 x_i \sum_{q_i} \bar{\psi}_{q_i}(x_i) \gamma_\alpha_i A_\alpha_i(x_i) \psi_{q_i}(x_i) \right] p(P) \rangle
\]

This expansion can be described by 12 Feynman diagrams and Wick contractions. We can extract the representation of the diagram which we want to study.

Let us begin with the determination of the color factor. Therefore, one has to examine the color structure of the diagram. We denote the color indices with \( (a, \ldots, f) \).

\[
\langle p(P') | [\bar{u}_{a}(y_1)]_b [\bar{u}_{a}(y_2)]_c [\bar{u}_{a}(x_1)]_{cd} \tilde{F}_F [A_{\alpha_1}^a(x_1) A_{\alpha_2}^a(x_2)]
\]

Combining all terms and contracting the generators, one can derive the result of the color factor for every diagram.

\[
C_F = \frac{1}{6} \varepsilon_{abc} \varepsilon_{dab} [t^{a_1} \omega d] [t^{a_2} \omega d] \delta^{a_1 a_2} - \frac{2}{3}
\tag{16.4}
\]

We continue with the evaluation of the Lorentz structure of the diagram. Therefore, we designate the Lorentz indices with \( (a, \ldots, h) \). Hereby, we have to distinguish between the box contribution and the cross contribution. Nevertheless, we have to include \( C_F \) in both representations.
16. Two Photon Exchange Approximation

At first, we present the expression for (16.1). Hereby, we have to keep in mind the already derived constraint $\Gamma = (l + l') + (q_2 - q_1)$.

$$\mathcal{M}_B = \frac{i(4\pi\alpha_s)(4\pi\alpha_{em})^2e_u^2}{72} \sum_{i=1}^{2} \int d^4x_i \sum_{j=1}^{2} \int d^4y_j \sum_{k=1}^{2} \int d^4q_k \frac{1}{q^2_k + i\Gamma^2} \tilde{u}(l')\gamma^\mu\gamma^\nu\bar{P}_\nu\gamma^\mu u(l)$$

$$[\gamma_{\mu_2} ab [\gamma_{\mu_1} cd [\gamma_{\alpha_1} e f [\gamma_{\alpha_2} gh [\tilde{\psi}_u(x_1)]f[\tilde{\psi}_u(y_2)]a A^\alpha_1(x_1)A^\alpha_2(x_2) e^{-i\gamma_1\cdot q_1} e^{-i\gamma_2\cdot q_2}$$

$$\langle p(P')|[\tilde{\psi}_u(y_1)]c[\tilde{\psi}_u(x_1)]e[\tilde{\psi}_d(x_2)]d|0\rangle\langle0|\tilde{\psi}_u(y_1)]d[\tilde{\psi}_u(y_2)]b[\psi_d(x_2)]h|p(P')\rangle$$

At last, we present the expression for (16.2). Therefore, we have to keep in mind the already derived constraint $\Gamma = (l + l') + (q_1 - q_2)$.

$$\mathcal{M}_C = \frac{i(4\pi\alpha_s)(4\pi\alpha_{em})^2e_u^2}{72} \sum_{i=1}^{2} \int d^4x_i \sum_{j=1}^{2} \int d^4y_j \sum_{k=1}^{2} \int d^4q_k \frac{1}{q^2_k + i\Gamma^2} \tilde{u}(l')\gamma^\mu\gamma^\nu\bar{P}^\nu\gamma^\mu u(l)$$

$$[\gamma_{\mu_2} ab [\gamma_{\mu_1} cd [\gamma_{\alpha_1} e f [\gamma_{\alpha_2} gh [\tilde{\psi}_u(x_1)]f[\tilde{\psi}_u(y_2)]a A^\alpha_1(x_1)A^\alpha_2(x_2) e^{-i\gamma_1\cdot q_1} e^{-i\gamma_2\cdot q_2}$$

$$\langle p(P')|[\tilde{\psi}_u(y_1)]c[\tilde{\psi}_u(x_1)]e[\tilde{\psi}_d(x_2)]d|0\rangle\langle0|\tilde{\psi}_u(y_1)]d[\tilde{\psi}_u(y_2)]b[\psi_d(x_2)]h|p(P')\rangle$$

In order to evaluate these expressions, we have to apply the representations for the propagators and the projection matrix elements.

We present the expression for (16.1) at first again. Obviously, we have to keep in mind the already derived condition $\Gamma = (l + l') + (q_2 - q_1)$.

$$\mathcal{M}_B = -\frac{i(4\pi\alpha_s)(4\pi\alpha_{em})^2e_u^2}{1152} \sum_{i=1}^{2} \int d^4x_i \sum_{j=1}^{2} \int d^4y_j \sum_{k=1}^{2} \int d^4q_k \int \frac{d^4\Delta}{\Delta^2 + i0} \int \frac{d^4\Lambda}{\Lambda^2 + i0}$$

$$\frac{1}{\Gamma^2} u(l')\gamma^\mu\gamma^\nu\bar{P}_\nu\gamma^\mu u(l) \int [du][dv]g^{\gamma_1\gamma_2 S} e^{-i\gamma_1\cdot (q_1 + u_1p - v_1p)} e^{-i\gamma_2\cdot (q_2 - \Delta + u_2p)} e^{-ix_1\cdot (\Delta - \Lambda - v_2p')} e^{-ix_2\cdot (\Lambda + u_3p - v_3p')}$$

We present the expression for (16.2) at last again. Obviously, we have to keep in mind the already derived condition $\Gamma = (l + l') + (q_1 - q_2)$.

$$\mathcal{M}_C = -\frac{i(4\pi\alpha_s)(4\pi\alpha_{em})^2e_u^2}{1152} \sum_{i=1}^{2} \int d^4x_i \sum_{j=1}^{2} \int d^4y_j \sum_{k=1}^{2} \int d^4q_k \int \frac{d^4\Delta}{\Delta^2 + i0} \int \frac{d^4\Lambda}{\Lambda^2 + i0}$$

$$\frac{1}{\Gamma^2} u(l')\gamma^\mu\gamma^\nu\bar{P}_\nu\gamma^\mu u(l) \int [du][dv]g^{\gamma_1\gamma_2 S} e^{-i\gamma_1\cdot (q_1 + u_1p - v_1p)} e^{-i\gamma_2\cdot (q_2 - \Delta + u_2p)} e^{-ix_1\cdot (\Delta - \Lambda - v_2p')} e^{-ix_2\cdot (\Lambda + u_3p - v_3p')}$$

The appearing integrations are identical in both cases. After computation of these integrations, one gets the required momentum conservation constraints. We notice that the photon momenta do not depend on $\omega$ consequentially:

$$q_1 = v_1p' - u_1p$$
$$q_2 = (v_2 + v_3)p' - (u_2 + u_3)p$$

$$\Delta = (v_2 + v_3)p' - u_3p$$
$$\Lambda = v_3p' - u_3p$$

The component $S$ is the sum of all required structures connected with combinations of nucleon distribution functions and nucleon spinors. Let us omit the dependence on the quark momentum fractions. Moreover, we use the standard notation for the nucleon spinors.
We get the following structures for initial twist-4 and final twist-3.

\[ S_1 = (m_N/Q^2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(V_1V_2 + A_1A_2 + V_1V_3 - A_1A_3) \]

\[ S_2 = (m_N/Q^2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(A_1V_2 + V_1A_2 - A_1V_3 + V_1A_3) \]

\[ S_3 = (m_N/2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(-V_1V_3 + A_1A_3) \]

\[ S_4 = (m_N/2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(-A_1V_3 + V_1A_3) \]

\[ S_5 = (m_N)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(T_1S_1) \]

\[ S_6 = (m_N)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(T_1P_1) \]

\[ S_7 = (2m_N/Q^2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(T_1T_2 - T_1T_3 + T_1T_2) \]

\[ S_8 = (m_N/Q^2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(T_1T_2 + 2T_1T_3) \]

\[ S_9 = (m_N/2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(T_1T_3) \]

We get the following structures for initial twist-3 and final twist-4.

\[ S_{10} = (m_N/Q^2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(V_2V_1 + A_2A_1 + V_3V_1 - A_3A_1) \]

\[ S_{11} = (m_N/Q^2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(V_2A_1 + A_2V_1 + V_3A_1 - A_3V_1) \]

\[ S_{12} = (m_N/2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(-V_3V_1 + A_3A_1) \]

\[ S_{13} = (m_N/2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(-V_3A_1 + A_3V_1) \]

\[ S_{14} = (m_N)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(-S_1T_1) \]

\[ S_{15} = (m_N)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(P_1T_1) \]

\[ S_{16} = (2m_N/Q^2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(T_2T_1 - T_2T_3 + T_2T_1) \]

\[ S_{17} = (m_N/Q^2)\tilde{N}(P')\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(T_2T_1 + 2T_2T_3) \]

\[ S_{18} = (m_N/2)\tilde{N}(P')i\sigma^{\kappa\kappa'}\gamma_{\alpha_2}\gamma_{\gamma_5}\gamma_{\alpha_1}N(P) \text{ Tr}[\gamma_{\mu_1}\gamma_{\mu_2}\Delta_{\gamma_{\alpha_1}}\gamma_{\alpha_1}] \\
(T_7T_1) \]

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16. Two Photon Exchange Approximation

In order to evaluate these structures, we have to consider the kinematics. One can express the scalar products of the momenta as functions of \( Q^2 \) and \( \omega \). Consequently, we obtain the formulas 4\((l \cdot p) = Q^2(\omega + 1) \) and 4\((l \cdot p') = Q^2(\omega - 1) \) and also 4\((l' \cdot p) = Q^2(\omega + 1) \) and 4\((l' \cdot p') = Q^2(\omega + 1) \).

Computing every structure, we get the dependence on multiple combinations of lepton and nucleon spinors. Nevertheless, it is possible to express all these combinations as functions of the desired component \( \bar{u}(l')\gamma^\mu u(l)\bar{N}(P')\bar{P}_\mu N(P) \) only. Therefore, we have to expand several matrices using the parametrization \( 8M = 2\text{Tr}(M) + 2\text{Tr}(\gamma_5 M)\gamma_5 + 2\text{Tr}(\gamma_\mu M)\gamma^\mu - 2\text{Tr}(\gamma_5 \gamma_5 M)\gamma^\mu \gamma_5 - \text{Tr}(i\sigma_{\mu\nu} M)i\sigma^{\mu\nu} \), which is valid for an arbitrary matrix \( M \) in this work. Furthermore, we have to use that the combination \( \bar{u}(l')\gamma^\mu\gamma_5 u(l)\bar{N}(P')\bar{P}_\mu \gamma_5 N(P) \) does not contribute.

We start with the expressions with two free Dirac matrices between the nucleon spinors. The first relation is \( \bar{u}(l')\gamma^\mu\gamma_5 u(l)\bar{N}(P')\gamma_\mu\gamma_5 N(P) = -2\bar{u}(l')\gamma^\mu u(l)\bar{N}(P')\bar{P}_\mu N(P) \).

The second relation is \( \bar{u}(l')\gamma^\mu\gamma_5 u(l)\bar{N}(P')\gamma_\mu\gamma_5 N(P) = -2\bar{u}(l')\gamma^\mu u(l)\bar{N}(P')\bar{P}_\mu N(P) \).

The third relation is \( \bar{u}(l')\gamma^\mu\gamma_5 u(l)\bar{N}(P')\gamma_\mu\gamma_5 N(P) = -2\bar{u}(l')\gamma^\mu u(l)\bar{N}(P')\bar{P}_\mu N(P) \).

The fourth relation is \( \bar{u}(l')\gamma^\mu\gamma_5 u(l)\bar{N}(P')\gamma_\mu\gamma_5 N(P) = -2\bar{u}(l')\gamma^\mu u(l)\bar{N}(P')\bar{P}_\mu N(P) \).

We finish with the expressions with one free Dirac matrix between the nucleon spinors. The first relation is given by \( \bar{u}(l')\gamma^\mu u(l)\bar{N}(P')\gamma_\mu N(P) = -\bar{u}(l')\gamma^\mu u(l)\bar{N}(P')\bar{P}_\mu N(P) \).

The second relation is given by \( \bar{u}(l')\gamma^\mu u(l)\bar{N}(P')\gamma_\mu N(P) = +\bar{u}(l')\gamma^\mu u(l)\bar{N}(P')\bar{P}_\mu N(P) \).

Applying these formulas, one can compute the box and the cross structures. Afterwards, they can be combined to get the overall result for the scattering amplitude \( \mathcal{M} = \mathcal{M}_B + \mathcal{M}_C \). Using \( u_1 + u_2 + u_3 = 1 \) and \( v_1 + v_2 + v_3 = 1 \), one gets convenient representations for all components. We notice that in the obtained result for \( \mathcal{M} \), the leptonic and the hadronic part are separated now.

One can simplify the received expression by exchanging \( u \leftrightarrow v \) for contributions of initial twist-3 and final twist-4. Consequently, we obtain a representation depending on initial twist-4 and final twist-3 only. For instance, we transform \( \mathcal{A}_2(u_1, u_2, u_3) \rightarrow V_1(v_1, v_2, v_3) A_2(u_1, u_2, u_3) \).

Let us now present the result of the discussed diagram connection depending on the integration over the quark momentum fractions. When we compare with (16.3), we can extract the contribution to the desired form factor. In order to get the complete result, one also needs the contributions of the other diagrams designated by \( \mathcal{C} \).

\[
\tilde{F}_2^m(\omega, Q^2) = \left(4\pi\alpha_s(4\pi\alpha_em)^2\frac{m_N^2}{72}\int\frac{[du]}{u_1u_3(u_2 + u_3)v_1v_3^2(v_2 + v_3)^2}\frac{[dv]}{(u_1(v_2 + v_3) + (u_2 + u_3)v_1)^2 - (u_1 - v_1)^2(\omega^2)}\right)\mathcal{D} + \mathcal{C}
\]  

(16.5)

We realize that the dependence on \( Q^2 \) and \( \omega \) is factorized in the obtained result. According to this, one can study the dependence on \( \omega \) independently.

The component \( \mathcal{D} \) is the sum of the remaining twist combinations of distribution amplitudes connected with multiple quark momentum fractions.

\[
\mathcal{D}_1 = |V_1V_2 + A_1A_2|(2u_1(u_2 + u_3)(v_2 + v_3) + 2v_1(v_2 + v_3)^2)
\]
\[
\mathcal{D}_2 = |V_1V_3 - A_1A_3|(4u_1v_3(v_2 + v_3) - u_1(u_2 + u_3)(v_2 + v_3) - v_1(v_2 + v_3)^2)
\]
\[
\mathcal{D}_3 = |V_2A_3 - A_1V_3|(4u_1v_3(v_2 + v_3) + u_1(u_2 + u_3)(v_2 + v_3) + v_1(v_2 + v_3)^2)
\]
\[
\mathcal{D}_4 = |T_1S_1 - T_1P_1|(8u_1v_3(v_2 + v_3) - 2u_1(u_2 + u_3)v_3 - 2v_1v_3(v_2 + v_3))
\]
\[
\mathcal{D}_5 = |T_1T_3 + T_1T_7|(-8u_1v_3(v_2 + v_3) - 2u_1(u_2 + u_3)v_3 - 2v_1v_3(v_2 + v_3))
\]
Finally, we must insert the nucleon distribution amplitudes, like in the previous case. We have to deal with one independent twist-3 distribution amplitude and with three independent twist-4 distribution amplitudes again. One can insert the most general polynomials for these distribution amplitudes which are discussed in this work. The other representations can be obtained by setting unnecessary coefficients to zero. In both cases, we can consider the first order polynomial given by the asymptotic form and the second order polynomial described by the conformal expansion. Afterwards, we can evaluate the integration over the momentum fractions.

Unfortunately, the corresponding integration is divergent. This divergency arises from endpoint singularities. That means, the integrals get divergent when a quark has no momentum or the full momentum of the nucleon. This behavior is similar to the one photon exchange, but now we get another singularity. This divergency just appears at the limit $\omega = 1$ and it is also an endpoint singularity. We already realized that the dependence on $Q^2$ and $\omega$ is factorized in the obtained expression. Therefore, one can fix the value of the dimensionless quantity $\omega$ and study the dependence on $Q^2$ independently. Hereby, we have to note that the additional variable has a physical meaning. The existence of this variable is a fundamental consequence of the two photon exchange, and it influences the experiments.

In order to analyze the structure of the divergency, one can introduce a cutoff parameter $\Omega$ as applied in the previous case. Therefore, one has to respect that in the case of infinite momentum transfer the integration must go from zero to one for every quark momentum fraction. According to this, we always integrate from $\Omega / Q^2$ to $1 - \Omega / Q^2$, keeping in mind that the introduced parameter has the same dimension as the momentum transfer. We mention that the cutoff parameter of the one photon exchange can differ from the cutoff parameter of the two photon exchange.

Computing the modified integration, we can extract the structure of the divergency. One obtains the same general behavior for the second order polynomial as for the first order polynomial. Moreover, this behavior is identical for all other required diagrams as well. Consequently, we can generally express the power behavior of the helicity flip form factor depending on the cutoff parameter.

$$\tilde{F}^p_\lambda(Q^2) \propto Q^{-6} \ln^2(Q^2/\Omega)$$  \hspace{1cm} (16.6)

We derived the expected power behavior of $Q^{-6}$ and we obtained a double logarithmic divergency in the case of $\Omega \to 0$. This behavior is in agreement with [84].

We want to emphasize that the result has the same power behavior for the helicity flip form factor in the one and two photon exchange approximation. According to this, one can study these contributions simultaneously. Hereby, we have to mention that the two photon contribution depends on the additional variable $\omega$. This variable can be related to the scattering angle of the experimental cross section. Consequently, we get the general behavior discussed in the introduction.
17. Conclusion to Part III

In the final part of our work, we evaluated the nucleon helicity flip form factor in the region of large momentum transfers. Hereby, we studied the exchange of one and two virtual photons separately.

Therefore, we started with the calculation of the scattering amplitude in both cases. Afterwards, we had to evaluate the appearing nucleon transition probability matrix elements. In order to evaluate these matrix elements, we combined QCD perturbation theory with an expansion in nucleon distribution amplitudes. Hereby, we had to use the combination of leading and sub-leading twist nucleon distribution amplitudes. Moreover, it was necessary to specify the frame.

Using this technique, we obtained a divergent result for the considered form factor in both cases. Nevertheless, the structure of the divergence could be extracted. We derived the same power behavior for the helicity flip form factor in the one and two photon exchange approximation.

Concerning the modified helicity flip form factor of the two photon exchange, we obtained the dependence on one additional variable. In order to understand the meaning of this variable, we considered the connection to the experimental cross section. Using the momentum transfer and the scattering angle as variables, we obtained that the helicity flip form factor of the two photon exchange depends on the scattering angle additionally. Consequently, the Rosenbluth separation technique cannot be used in this case. Furthermore, we realized that the obtained power behavior of the helicity flip form factor can describe the experimental data based on polarized cross sections qualitatively. These conclusions can explain the different behavior in the experiments using unpolarized or polarized cross sections.

Finally, we have to discuss the required modifications to avoid the divergency. The appearing double logarithmic singularities indicate the existence of not included soft contributions. This is a consequence of the factorization approach where possible contributions from remaining soft spectator quarks are considered as power suppressed. Meanwhile, there are evidences that those contributions cannot be neglected. Using a soft effective theory, the behavior of soft contributions is discussed in [88]. In this work, it has been pointed out that the discussed soft contributions have the same power behavior as the factorized contributions and so they must be taken into account. Unfortunately, the required techniques to get all possible soft contributions are still in development.

The studies about various nucleon form factors in multi photon exchange approximation including factorizable and non-factorizable contributions are an interesting topic which requires further investigations. Meanwhile, also comprehensive reviews were written, see [89] and [90] to get an overview about the obtained achievement.

In order to present the important conclusions of these studies to the entire community, the main part of this work is published in [91].
18. Summary of the Dissertation

In the first part, we studied all nucleon structure functions in the region of large momentum transfer and small invariant masses. Therefore, we considered the scattering between an electron and a nucleon target due to exchange of a single virtual photon which carries large momentum. After the scattering process, we received an electron and a nucleon together with a produced pion with small momentum. In order to derive the expressions for the nucleon structure functions, we calculated the cross section of this exclusive inelastic scattering process and also the cross section of inclusive inelastic electron nucleon scattering for comparison. The required nucleon to pion nucleon transition probability matrix elements were expressed by nucleon to pion nucleon transition form factors and usual nucleon form factors. We used the QCD factorization theorem to derive relations between nucleon to pion nucleon transition form factors and usual nucleon form factors under reasonable assumptions. Applying this approach, experimental values of the nucleon form factors were used to get results for the transition form factors and the structure functions. Finally, we compared our results with available experimental data or presented various predictions for further experiments. Using advanced assumptions for the form factor reduction, we obtained perfect agreement with the experimental data.

In the second part, we calculated all leading nucleon form factors at large momentum transfer and in single gauge boson exchange approximation. In order to evaluate the required nucleon transition probability matrix elements, we combined QCD perturbation theory with an expansion in nucleon distribution amplitudes. Hereby, we had to use leading twist nucleon distribution amplitudes only. Furthermore, we applied third order polynomials and corresponding models for these distribution amplitudes to compare the obtained results with experimental data. The obtained results of all calculated nucleon form factors are essentially in good agreement with the available experimental data. Finally, we discussed the known discrepancy concerning the symmetry factor in previous works. We argued that taking into account the color normalization appropriately, the discrepancy can be resolved.

In the third part, we evaluated the nucleon helicity flip form factor in the region of large momentum transfers. Hereby, we studied the exchange of one and two virtual photons separately. Therefore, we calculated the required scattering amplitudes and the appearing nucleon transition probability matrix elements by using the combination of QCD perturbation theory and an expansion in nucleon distribution amplitudes. We had to apply leading and sub-leading twist nucleon distribution amplitudes. Using this technique, we obtained a divergent result for the considered form factor in both cases, but with the same power behavior. Concerning the helicity flip form factor of the two photon exchange, we obtained the dependence on one additional variable which can be related to the scattering angle of the experimental cross section. This behavior causes problems for the interpretation of unpolarized cross sections. Furthermore, we realized that the obtained power behavior of the helicity flip form factor can describe the experimental data based on polarized cross sections qualitatively. These conclusions can explain the different behavior in the experiments using unpolarized or polarized cross sections. Finally, we discussed the required modifications to avoid the divergency.
A. Nucleon Form Factors

We will present definitions of quark currents and nucleon form factors. We always work with an initial nucleon $N_i$ with momentum $P$ and spin $S$ and a final nucleon $N_f$ with momentum $P'$ and spin $S'$. One has to deal with the spinor $N(P,S)$ for the initial nucleon and the spinor $N(P',S')$ for the final nucleon. The nucleon mass is denoted by $m_N$. Values of the constants can be taken from [27]. Let us omit the spin dependence, because the form factors depend on the momentum transfer $Q^2 = -q^2 = -(P' - P)^2$ only.

A.1. Electromagnetic Form Factors

These form factors occur in the parametrization of identical nucleon matrix elements including the electromagnetic current representable as $J_{\mu}^{em}(0) = \tilde{\psi}(0)\gamma_\mu (\frac{1}{2} + \frac{4}{\tau})\psi(0)$.

We start with the definition of the Dirac form factor $F_1$ and the Pauli form factor $F_2$.

\[
\langle p(P')|J_{\mu}^{em}(0)|p(P)\rangle = \bar{N}(P') \left[ F_1^p(Q^2)\gamma_\mu + F_2^p(Q^2)\frac{i\sigma_{\mu\nu}(P' - P)^\nu}{2m_N} \right] N(P) \quad (A.1)
\]

\[
\langle n(P')|J_{\mu}^{em}(0)|n(P)\rangle = \bar{N}(P') \left[ F_1^n(Q^2)\gamma_\mu + F_2^n(Q^2)\frac{i\sigma_{\mu\nu}(P' - P)^\nu}{2m_N} \right] N(P) \quad (A.2)
\]

At zero momentum transfer, one gets $F_1^p(0) = 1$ and $F_1^n(0) = 0$ and also $F_2^p(0) = \kappa_p$ and $F_2^n(0) = \kappa_n$. One obtains $\kappa_p \approx 1.79$ as the anomalous magnetic moment of the proton and $\kappa_n \approx -1.91$ as the anomalous magnetic moment of the neutron.

Let us define the magnetic form factor $G_M$ and the electric form factor $G_E$ at next.

\[
\langle p(P')|J_{\mu}^{em}(0)|p(P)\rangle = \bar{N}(P') \left[ G_M^p(Q^2)\gamma_\mu + \frac{G_E^p(Q^2) - G_M^p(Q^2)(P' + P)^\mu}{1 + Q^2/4m_N^2} \right] N(P) \quad (A.3)
\]

\[
\langle n(P')|J_{\mu}^{em}(0)|n(P)\rangle = \bar{N}(P') \left[ G_M^n(Q^2)\gamma_\mu + \frac{G_E^n(Q^2) - G_M^n(Q^2)(P' + P)^\mu}{1 + Q^2/4m_N^2} \right] N(P) \quad (A.4)
\]

The electric form factor is given by $G_E(Q^2) = F_1(Q^2) - (Q^2/4m_N^2)F_2(Q^2)$ leading to $G_E^p(0) = 1$ and $G_E^n(0) = 0$. These allow us to interpret the electric form factor at zero momentum transfer as the electric charge of the nucleon. The magnetic form factor is given by $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$ leading to $G_M^p(0) = \mu_p = 1 + \nu_p$ and $G_M^n(0) = \mu_n = \kappa_n$. The magnetic form factor at zero momentum transfer delivers the magnetic moment of the nucleon. One obtains $\mu_p \approx 2.79$ as the magnetic moment of the proton and $\mu_n \approx -1.91$ as the magnetic moment of the neutron. We also get $F_2(Q^2) = (G_M(Q^2) - G_E(Q^2))/(1 + (Q^2/4m_N^2))$ and $F_1(Q^2) = (G_E(Q^2) + (Q^2/4m_N^2)G_M(Q^2))/(1 + (Q^2/4m_N^2))$ as the inverse formulas.

Combining these relations, one gets a convenient dependence on form factors.

\[
\langle p(P')|J_{\mu}^{em}(0)|p(P)\rangle = \bar{N}(P') \left[ G_M^p(Q^2)\gamma_\mu - F_2^p(Q^2)(P' + P)^\mu \right] N(P) \quad (A.5)
\]

\[
\langle n(P')|J_{\mu}^{em}(0)|n(P)\rangle = \bar{N}(P') \left[ G_M^n(Q^2)\gamma_\mu - F_2^n(Q^2)(P' + P)^\mu \right] N(P) \quad (A.6)
\]
A.2. Vector Form Factors

Let us begin with the definition of the isoscalar vector current $V_{\mu}^0(0) = \frac{1}{2} \bar{\psi}(0) \gamma_{\mu} \gamma_5 \psi(0)$ which is used to produce the isoscalar vector form factors.

$$\langle N_f(P') | W_{\mu}(0) | N_i(P) \rangle = \tilde{N}(P') \left[ F_1^v(Q^2) \gamma_{\mu} + F_2^v(Q^2) \frac{i \sigma_{\mu\nu} (P' - P)^\nu}{2m_N} \right] \frac{\delta_{f_i}}{2} N(P)$$  \hspace{1cm} (A.7)

We proceed with the definition of the isovector vector current $V_{\mu}^a(0) = \frac{1}{2} \bar{\psi}(0) \gamma_{\mu} \tau^a \psi(0)$ which produces the isovector vector form factors.

$$\langle N_f(P') | V_{\mu}^a(0) | N_i(P) \rangle = \tilde{N}(P') \left[ F_1^a(Q^2) \gamma_{\mu} + F_2^a(Q^2) \frac{i \sigma_{\mu\nu} (P' - P)^\nu}{2m_N} \right] \frac{\tau_{f_i}^a}{2} N(P)$$  \hspace{1cm} (A.8)

The electromagnetic current can be expressed by these currents. We get the important connection $3 J_{\mu e m}^r(0) = V_{\mu}^0(0) + 3 V_{\mu}^3(0)$. Matching this relation between identical nucleon states, one gets connections between electromagnetic form factors and vector form factors.

At first, we present the formulas for the electromagnetic form factors.

$$F_1^v(Q^2) = \frac{1}{6} F_1^v(Q^2) + \frac{1}{2} F_1^a(Q^2)$$  \hspace{1cm} (A.9)

$$F_2^v(Q^2) = \frac{1}{6} F_2^v(Q^2) + \frac{1}{2} F_2^a(Q^2)$$  \hspace{1cm} (A.10)

$$F_1^a(Q^2) = \frac{1}{6} F_1^a(Q^2) - \frac{1}{2} F_1^v(Q^2)$$  \hspace{1cm} (A.11)

$$F_2^a(Q^2) = \frac{1}{6} F_2^a(Q^2) - \frac{1}{2} F_2^v(Q^2)$$  \hspace{1cm} (A.12)

At last, we provide the formulas for the vector form factors.

$$F_1^v(Q^2) = 3 F_1^v(Q^2) + F_1^a(Q^2)$$  \hspace{1cm} (A.13)

$$F_2^v(Q^2) = 3 F_2^v(Q^2) + F_2^a(Q^2)$$  \hspace{1cm} (A.14)

$$F_1^a(Q^2) = F_1^a(Q^2) - F_1^v(Q^2)$$  \hspace{1cm} (A.15)

$$F_2^a(Q^2) = F_2^a(Q^2) - F_2^v(Q^2)$$  \hspace{1cm} (A.16)

A.3. Axial-Vector Form Factors

Let us start with the definition of the isoscalar axial-vector current $A_{\mu}^0(0) = \frac{1}{2} \bar{\psi}(0) \gamma_{\mu} \gamma_5 \tau_3 \psi(0)$ producing the isoscalar axial-vector form factor and the isoscalar pseudoscalar form factor.

$$\langle N_f(P') | A_{\mu}^0(0) | N_i(P) \rangle = \tilde{N}(P') \left[ G_A^v(Q^2) \gamma_{\mu} + G_P^v(Q^2) \frac{(P' - P)^\mu}{2m_N} \right] \gamma_5 \frac{\delta_{f_i}}{2} N(P)$$  \hspace{1cm} (A.17)

We finish with the definition of the isovector axial-vector current $A_{\mu}^a(0) = \frac{1}{2} \bar{\psi}(0) \gamma_{\mu} \gamma_5 \tau^a \psi(0)$ producing the isovector axial-vector form factor and the isovector pseudoscalar form factor.

$$\langle N_f(P') | A_{\mu}^a(0) | N_i(P) \rangle = \tilde{N}(P') \left[ G_A^a(Q^2) \gamma_{\mu} + G_P^a(Q^2) \frac{(P' - P)^\mu}{2m_N} \right] \gamma_5 \frac{\tau_{f_i}^a}{2} N(P)$$  \hspace{1cm} (A.18)

The pseudoscalar form factor can be related to the axial-vector form factor at zero momentum transfer due to $G_P^v(0) = 4 \frac{m_N^2}{m_N^2} G_A^v(0)$. This connection can be obtained by applying current conservation at the limit $(P - P')^2 \to m_N^2$ which is close to zero.

The axial-vector form factor at zero momentum transfer is known as the axial coupling constant given by $G_A^v(0) = g_A \approx 1.27$. 

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B. Quantum Chromodynamics

Let us introduce some required basic elements of quantum field theory. We will mainly consider aspects of Quantum Chromodynamics.

QCD describes the strong interaction between quark fields $\psi_q$ and gluon fields $A_\mu^a$ and can be expressed by the QCD Lagrangian. We hide the gluon index at the field strength tensor and its summation in this Lagrangian and apply the notation $A_\mu(x) = t^a A_\mu^a(x)$.

$$\mathcal{L}_{\text{QCD}}(x) = \sum_q \bar{\psi}_q(x) \left\{ i\gamma^\mu \left[ \partial_\mu - igA_\mu(x) \right] - m_q \right\} \psi_q(x) - \frac{1}{4} F^{\mu\nu}(x) F_{\mu\nu}(x) \quad (B.1)$$

The quark flavor index $q$ designates the six quark types and $m_q$ is the corresponding quark mass. Moreover, the gluon index $a$ describes the eight gluons.

Let us introduce the field strength tensor $F^a_{\mu\nu}$ at next. The generators $t^a$ and the structure constants $f^{abc}$ are connected by the relation $[t^a, t^b] = i f^{abc} t^c$.

$$F^a_{\mu\nu}(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g f^{abc} A_\mu^b(x) A_\nu^c(x) \quad (B.2)$$

This Lagrangian can be written as sum of a free part and an interaction part. The important interaction part $\mathcal{L}_I(x)$ can be split in two components. One component generates the quark-gluon interaction denoted by $\mathcal{L}_{Iqg}(x)$ and the other component generates the gluon-gluon interaction denoted by $\mathcal{L}_{gg}(x)$. We only need the quark-gluon interaction because all required contributions generated by the gluon-gluon interaction vanish.

$$\mathcal{L}_{Iqg}(x) = g \sum_q \bar{\psi}_q(x) \gamma^\mu A_\mu(x) \psi_q(x) = g \sum_q \bar{\psi}_q(x) \gamma^\mu t^a A_\mu^a(x) \psi_q(x) \quad (B.3)$$

The interaction part of this Lagrangian is used in connection with the S-matrix given by $S = \text{T exp}(i \int d^4x \mathcal{L}_I(x))$. The strong coupling constant $g^2 = 4\pi\alpha_s$ depends on the momentum transfer of the considered process.

We introduce the lepton and quark propagator noting that we need the massless limit.

$$\bar{\psi}_q(x) \psi_q(y) = \int \frac{d^4 \Delta}{(2\pi)^4} \frac{i(\Delta + m)}{\Delta^2 - m^2 + i0} e^{-i\Delta \cdot (x-y)} \quad (B.4)$$

We also need the photon and gluon propagator frequently.

$$A_{\mu}(x) A_{\nu}(y) = \int \frac{d^4 \Lambda}{(2\pi)^4} \frac{-ig_{\mu\nu}}{\Lambda^2 + i0} e^{i\Lambda \cdot (x-y)} \quad (B.5)$$

Let us proceed with properties of fermion spinors $u$ and antifermion spinors $\bar{v}$. We denote the momentum with $p$ and the spin with $s$ and the mass with $m$. Some relations require to distinguish between spin $\uparrow$ leading to the upper sign and spin $\downarrow$ leading to the lower sign. One needs $\bar{u}(p, s') u(p, s) = -\bar{v}(p, s') v(p, s) = 2m \delta_{ss'}$ and $\bar{u}(p, s') v(p, s) = \bar{v}(p, s') u(p, s) = 0$.

We obtain $(p' - m) u(p, s) = \bar{u}(p, s)(p' - m) = (p' + m) v(p, s) = \bar{v}(p, s)(p' + m) = 0$ when we apply the momentum as projector.
B. Quantum Chromodynamics

We gain \((\gamma_5 \gamma = 1) u(p, s) = \bar{u}(p, s)(\gamma_5 \gamma \pm 1) = (\gamma_5 \gamma = 1) v(p, s) = \bar{v}(p, s)(\gamma_5 \gamma \pm 1) = 0\) when we use the spin as projector.

One can introduce an expression for the momentum under the basic condition \(p^2 = m^2\).

\[
p_{\mu} = \frac{1}{2} \bar{u}(p, s) \gamma_{\mu} u(p, s) = \frac{1}{2} \bar{v}(p, s) \gamma_{\mu} v(p, s) \tag{B.6}
\]

Under the constraint \(s^2 = -1\) and \(s \cdot p = 0\), we can introduce an expression for the spin by a covariant spin vector similar to the covariant momentum vector.

\[
s_{\mu} = \pm \frac{1}{2m} \bar{u}(p, s) \gamma_\mu \gamma_5 u(p, s) = \mp \frac{1}{2m} \bar{v}(p, s) \gamma_\mu \gamma_5 v(p, s) \tag{B.7}
\]

One gets the completeness relations by summing over the spin.

\[
\sum_{s=\uparrow, \downarrow} u(p, s) \bar{u}(p, s) = \gamma^\prime + m \tag{B.8}
\]

\[
\sum_{s=\uparrow, \downarrow} v(p, s) \bar{v}(p, s) = \gamma^\prime - m \tag{B.9}
\]

When we do not sum over the spin, we have to deal with the covariant spin vector.

\[
u(p, s) \bar{u}(p, s) = (\gamma^\prime + m) \frac{1 + \gamma_5 \gamma}{2} \tag{B.10}
\]

\[
v(p, s) \bar{v}(p, s) = (\gamma^\prime - m) \frac{1 + \gamma_5 \gamma}{2} \tag{B.11}
\]

Finally, we will summarize some basic properties of Dirac matrices which are used frequently. At first, we specify the transformation between upper and lower indices \(\gamma_\mu = g_{\mu \nu} \gamma^\nu\). Moreover, we present the anticommutation relation \(\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu \nu}\). At next, we notice the basic contraction \(\gamma^\mu \gamma_\mu = 4\). Furthermore, we specify the formula concerning the adjoint expression \((\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0\). The results for the components are given by \((\gamma^0)^2 = 1\) and \((\gamma^i)^2 = -1\) and also \((\gamma^0)^\dagger = \gamma^0\) and \((\gamma^i)^\dagger = -\gamma^i\).

Additionally, we introduce a combination of Dirac matrices defined by \(\sigma^{\mu \nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\). The corresponding adjoint expression is given by \((\sigma^{\mu \nu})^\dagger = \gamma^0 \sigma^{\mu \nu} \gamma^0\).

Let us now provide the definition of \(\gamma^5\). Therefore, we have to notice the basic properties \(\{\gamma^5, \gamma^\mu\} = 0\) and \((\gamma^5)^2 = 1\) and \((\gamma^5)^\dagger = \gamma^5\).

\[
\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = -\frac{i}{4!} \epsilon_{\alpha \beta \gamma \delta} \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta = -\frac{i}{4!} \epsilon_{\alpha \beta \gamma \delta} \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \gamma_5 \tag{B.12}
\]

In the definition of \(\gamma^5\) already appeared the total antisymmetric tensor \(\epsilon_{\alpha \beta \gamma \delta}\). Let us introduce its trace representation \(-4i \epsilon_{\alpha \beta \gamma \delta} = \text{Tr}[\gamma^5 \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta]\). One has to deal with the convention \(\epsilon_{0123} = -\epsilon_{0123} = 1\).

We also need the Schouten identity expressible by the following formula.

\[
p^\alpha \epsilon_{\beta \gamma \delta} + p^\beta \epsilon_{\gamma \delta \alpha} + p^\gamma \epsilon_{\delta \alpha \beta} + p^\delta \epsilon_{\alpha \beta \gamma} + p^\epsilon \epsilon_{\alpha \beta \gamma \delta} = 0 \tag{B.13}
\]

Finally, we consider some properties of the charge conjugation matrix. This matrix respects the basic identities \(C^{-1} = C\) and \(C^T = -C\).

The charge conjugation matrix satisfies the general transformation for Dirac matrices \((\gamma^5)^T = -C \gamma^5 C^{-1}\). Consequently, one gets \((\sigma^{\mu \nu})^T = -C \sigma^{\mu \nu} C^{-1}\) and \((\gamma^5)^T = C \gamma^5 C^{-1}\). All other relations can be derived by using these identities.
C. Nucleon Distribution Amplitudes

We have to consider definitions and properties of nucleon distribution amplitudes. Let us start with the required nucleon projection matrix elements.

\[
\begin{align*}
0[\psi_u(x_1)]_a [\psi_u(x_2)]_b [\psi_d(x_3)]_\gamma |p(P)\rangle & \quad \text{and} \quad \langle p(P')|[\bar{\psi}_u(x_1)]_a [\bar{\psi}_u(x_2)]_b [\bar{\psi}_d(x_3)]_\gamma |0\rangle \\
0[\bar{\psi}_d(x_1)]_a [\bar{\psi}_d(x_2)]_b [\bar{\psi}_u(x_3)]_\gamma |n(P)\rangle & \quad \text{and} \quad \langle n(P')|[\bar{\psi}_u(x_1)]_a [\bar{\psi}_u(x_2)]_b [\bar{\psi}_d(x_3)]_\gamma |0\rangle
\end{align*}
\]  

(C.1)

These matrix elements can be expressed by an expansion in nucleon distribution amplitudes. This expansion is identical for both vacuum to nucleon projection matrix elements and the expansion of the corresponding nucleon to vacuum projection matrix elements can be derived consequently. This part of our work is based on [47] and so we must multiply the introduced matrix elements in (C.1) by a factor of 4.

In order to evaluate this construction, we have to specify the kinematics. Working at large momentum transfer \(Q^2 = -q^2 = -(p' - P)^2\), one can neglect \(m_N^2\) compared to \(Q^2\). Under this assumption, we obtain \(P \cdot P' = Q^2/2\). In massless case, one can work with \(P^2 = P'^2 = 0\), but in massive case, one has to apply \(P^2 = P'^2 = m_N^2\). It is convenient to use the light cone frame for our calculations. We prefer to use light cone vectors which can be related to the nucleon momenta directly. One can define two light like vectors \(p\) and \(p'\) under the constraint \(p^2 = p'^2 = 0\) and \(p \cdot p' = Q^2/2\). For massless nucleons, we can express the nucleon momenta on the light cone due to \(P_\mu = p_\mu\) and \(P'_\mu = p'_{\mu}\), but for massive nucleons, one cannot work with such direct expressions. It is recommended to deal with \(P_\mu = p_\mu + (m_N^2/Q^2)p'_\mu\) and \(P'_\mu = p'_{\mu} + (m_N^2/Q^2)p_{\mu}\) in this case.

We proceed with the derivation of the equation of motion relations in this frame. Let us focus on the initial nucleon spinor \(N(P)\) because the relations for the final nucleon spinor \(\bar{N}(P')\) can be derived in the same way. We write \(N(P)\) as sum of a large component \(N^+(P)\) given by \(Q^2N^+(P) = \rho\eta'N(P)\) and a small component \(N^-(P)\) given by \(Q^2N^-(P) = \rho'\eta N(P)\). Applying the general equation of motion relation, one gets \(\rho N(P) = m_N N^+(P)\) and \(\rho' N(P) = (Q^2/m_N) N^-(P)\). Finally, we obtain a complete set of equation of motion relations given by \(\rho N^+(P) = 0\) and \(\rho' N^+(P) = (Q^2/m_N) N^-(P)\) and also \(\rho' N^-(P) = m_N N^+(P)\) and \(\rho' N^-(P) = 0\). The final result of our calculations should only depend on \(N^+(P)\) and the specified frame gives us the possibility to eliminate \(N^-(P)\) by using of \(N^-(P) = (m_N/Q^2)\rho N^+(P)\) and by applying \(\rho'\) on the opposite spinor.

For convenience, we will use a contracted notation for \(\sigma_{\mu\nu}\), for example, \(\sigma_{p'\nu} = \sigma^{\mu\nu} p_\mu p'_\nu\). Moreover, we introduce a Dirac matrix transverse to the light cone vectors expressible by the formula \(\gamma_\perp = \gamma_\mu - (2/Q^2)(p' p_\mu + p_{p'\mu})\).

The basic representation of every initial nucleon distribution amplitude \(F\) and final nucleon distribution amplitude \(F^*\) is given by an integration over the quark momentum fractions. Therefore, one can apply the conventional notation \(|du| = du_1 du_2 du_3 \delta(u_1 + u_2 + u_3 - 1)\). One can show the important connection \(F^*(u_1, u_2, u_3) = F(u_1, u_2, u_3)\).

\[
F = \int |du| e^{-i(x_1 u_1 + x_2 u_2 + x_3 u_3)} F(u_1, u_2, u_3) \\
F^* = \int |du| e^{i(x_1 p_1' + x_2 p_2' + x_3 p_3')} F^*(u_1, u_2, u_3)
\]  

(C.2)  

(C.3)
C. Nucleon Distribution Amplitudes

C.1. Twist-3 Distribution Amplitudes

Let us consider the expansion of the left hand side in (C.1) in twist-3 distribution amplitudes at first. For convenience, one can omit the dependence on the momentum $P$ concerning the expressions of the nucleon spinors:

$$V_1(pC)_{\alpha\beta}(\gamma_5 N^+)_{\gamma} + A_1(p\gamma_5 C)_{\alpha\beta}(N^+)_{\gamma} + T_1(i\sigma_{\perp}pC)_{\alpha\beta}(\gamma^+\gamma_5 N^+)_{\gamma} \quad (C.4)$$

At next, we derive the expansion of the right hand side in (C.1) in twist-3 distribution amplitudes. One can also omit the dependence on the momentum $P'$ for the expressions of the nucleon spinors:

$$V_1^*(\bar{N}^+\gamma_5 \gamma(C\not{p})_{\alpha\beta} - A_1^*(\bar{N}^+\gamma_5 \gamma(C\not{p})_{\alpha\beta} = T_1^*(\bar{N}^+\gamma_5 \gamma^+)_{\gamma}(C\sigma_{\perp}p')_{\alpha\beta} \quad (C.5)$$

We have got three distribution amplitudes in twist-3, but they are not independent. That gives us the opportunity to define a single independent distribution amplitude called $\Phi_3$. One has to deal with the definition $\Phi_3(u_1, u_2, u_3) = (V_1 - A_1)(u_1, u_2, u_3)$. Basic symmetry deliver the relation $(V_1 + A_1)(u_1, u_2, u_3) = (V_1 - A_1)(u_2, u_1, u_3)$ and the tensor structure can be described by the expression $2T_1(u_1, u_2, u_3) = (V_1 - A_1)(u_1, u_3, u_2) + (V_1 - A_1)(u_2, u_3, u_1)$.

With these connections between distribution amplitudes and the definition of $\Phi_3$, one can express all three distribution amplitudes as functions of $\Phi_3$ only.

$$2V_1(u_1, u_2, u_3) = \Phi_3(u_1, u_2, u_3) + \Phi_3(u_2, u_1, u_3) \quad (C.6)$$

$$2A_1(u_1, u_2, u_3) = \Phi_3(u_2, u_1, u_3) - \Phi_3(u_1, u_2, u_3) \quad (C.7)$$

$$2T_1(u_1, u_2, u_3) = \Phi_3(u_1, u_3, u_2) + \Phi_3(u_2, u_3, u_1) \quad (C.8)$$

At next, we want to discuss the polynomial expansion of the specified independent distribution amplitude $\Phi_3$. The required coefficients are non-perturbative parameters which are scale dependent and have to be modeled or taken from the lattice. We use the standard hadronic scale of 1 GeV.

The first order expression of this distribution amplitude is given by the well known asymptotic form $\Phi_3(u_1, u_2, u_3) = 120u_1u_2u_3f_N$. Hereby, we have to deal with the constant $f_N = 5.3 \cdot 10^{-3}$ GeV$^2$ taken from [47]. Meanwhile, this parameter is also studied on the lattice, see [92].

When we consider a non-asymptotic form, we get more complicated structures. Keeping in mind $u_1 + u_2 + u_3 = 1$, one can expand $\Phi_3$ in polynomials which depend on $u_1$ and $u_3$ only. We choose this representation by tradition.

We proceed with $\Phi_3(u_1, u_2, u_3) = 120u_1u_2u_3f_N[c_1 + c_2u_1 + c_3u_3]$ as the second order polynomial representation. The here introduced coefficients are given by the conformal expansion, see [47]. We obtain $c_1 = -1.91$ and $c_2 = 7.96$ and also $c_3 = 0.75$ rounded off two decimal places and with acceptable uncertainty.

Moreover, we have to consider the third order polynomial. We get the expression $\Phi_3(u_1, u_2, u_3) = 120u_1u_2u_3f_N[c_1 + c_2u_1 + c_3u_3 + c_4u_1^2 + c_5u_3^2 + c_6u_1u_3]$. The coefficients must be taken from models. There are five models discussed in the literature. The coefficients are rounded off two decimal places again and the corresponding error bars must assumed to be large. Information about the models can be taken from [44]. We use the models of Chernyak, Zhitnitsky (CZ) [31], Gari, Stefanis (GS) [33], King, Sachrajda (KS) [49], Chernyak, Ogloblin, Zhitnitsky (COZ) [35], and Stefanis, Bergmann (HET) [50]. The abbreviation HET means heterotic conception.
### C.2. Twist-4 Distribution Amplitudes

At first, we consider the expansion of the left hand side in (C.1) in twist-4 distribution amplitudes. We obtain the following expression similar to the previous case.

\[
\begin{align*}
V_2(pC)_{\alpha\beta}(\gamma_5N^-)_{\gamma} + A_2(p\gamma_5C)_{\alpha\beta}(N^-)_{\gamma} + T_2(i\sigma_{1\perp\gamma}C)_{\alpha\beta}(\gamma^+\gamma_5N^-)_{\gamma} + \\
S_1(m_N)(C)_{\alpha\beta}(\gamma_5N^+)_{\gamma} + P_1(m_N)(\gamma_5C)_{\alpha\beta}(N^+)_{\gamma} + \\
V_3(m_N/2)(\gamma_\perp\gamma_5C)_{\alpha\beta}(\gamma^+\gamma_5N^+)_{\gamma} + A_3(m_N/2)(\gamma_\perp\gamma_5C)_{\alpha\beta}(\gamma^+\gamma_5N^+)_{\gamma} + \\
T_3(2m_N/Q^2)(i\sigma_{\perp\gamma}C)_{\alpha\beta}(\gamma_5N^+)_{\gamma} - T_7(m_N/2)(i\sigma_{\perp\gamma}C)_{\alpha\beta}(\gamma^+\gamma_5N^+)_{\gamma}
\end{align*}
\]

(C.9)

At last, we derive the expansion of the right hand side in (C.1) in twist-4 distribution amplitudes. We gain the corresponding representation similar to the previous case.

\[
\begin{align*}
V_2^*(N^-\gamma_5)(C\gamma)p'_{\alpha\beta} - A_2^*(N^-\gamma_5)(C\gamma)p'_{\alpha\beta} - T_2^*(N^-\gamma_5\gamma^+)_{\gamma}(C\gamma)p'_{\alpha\beta} - \\
S_1^*(m_N)(\bar{N}^+\gamma_5\gamma)(C)_{\alpha\beta} - P_1^*(m_N)(\bar{N}^+\gamma)(\gamma_5)_{\alpha\beta} + \\
V_3^*(m_N/2)(\bar{N}^+\gamma_5\gamma^+)_{\gamma}(C\gamma_\perp)_{\alpha\beta} - A_3^*(m_N/2)(\bar{N}^+\gamma^+)_{\gamma}(C\gamma_\perp)_{\alpha\beta} - \\
T_3^*(2m_N/Q^2)(\bar{N}^+\gamma_5\gamma)(C\gamma_\perp\gamma)p'_{\alpha\beta} - T_7^*(m_N/2)(\bar{N}^+\gamma_5\gamma)p'_{\alpha\beta} - T_7^*(m_N/2)(\bar{N}^+\gamma_5\gamma)p'_{\alpha\beta} -
\end{align*}
\]

(C.10)

One obtains nine distribution amplitudes in twist-4. Using relations between them, the amount of independent distribution amplitudes can be reduced to three. They are called \( \Phi_4 \) and \( \Psi_4 \) and also \( \Xi_4 \). Let us define these distribution amplitudes at next. We get \( \Phi_4(u_1, u_2, u_3) = (V_2 - A_2)(u_1, u_2, u_3) \) and \( \Psi_4(u_1, u_2, u_3) = (V_3 - A_3)(u_1, u_2, u_3) \) and also \( \Xi_4(u_1, u_2, u_3) = (T_3 - T_7 + S_1 + P_1)(u_1, u_2, u_3) \). Basic symmetry considerations deliver the relations \( (V_2 + A_2)(u_1, u_2, u_3) = (V_3 + A_3)(u_1, u_2, u_3) \) together with \( (V_3 + A_3)(u_1, u_2, u_3) = (V_3 - A_3)(u_1, u_2, u_3) \). Moreover, one obtains a tensor expression given by \( 2T_2(u_1, u_2, u_3) = (T_3 - T_7 + S_1 + P_1)(u_3, u_2, u_1) + (T_3 - T_7 + S_1 + P_1)(u_3, u_2, u_1) \). Furthermore, one gets the representation \( (T_3 + T_7 + S_1 + P_1)(u_1, u_2, u_3) = (V_3 - A_3)(u_3, u_1, u_2) + (V_3 - A_3)(u_2, u_3, u_1) \). Applying these connections between distribution amplitudes and the definitions of the specified independent distribution amplitudes, one can express all nine distribution amplitudes as functions of \( \Phi_4, \Psi_4 \) and \( \Xi_4 \).

\[
\begin{align*}
2V_2(u_1, u_2, u_3) &= \Phi_4(u_1, u_2, u_3) + \Phi_4(u_2, u_1, u_3) \\
2A_2(u_1, u_2, u_3) &= \Phi_4(u_2, u_1, u_3) - \Phi_4(u_1, u_2, u_3) \\
2V_3(u_1, u_2, u_3) &= \Psi_4(u_1, u_2, u_3) + \Psi_4(u_2, u_1, u_3) \\
2A_3(u_1, u_2, u_3) &= \Psi_4(u_2, u_1, u_3) - \Psi_4(u_1, u_2, u_3) \\
2T_2(u_1, u_2, u_3) &= \Xi_4(u_3, u_1, u_2) + \Xi_4(u_3, u_2, u_1)
\end{align*}
\]

(C.11) (C.12) (C.13) (C.14) (C.15)
C. Nucleon Distribution Amplitudes

\[
4T_3(u_1, u_2, u_3) = \Phi_4(u_2, u_3, u_1) + \Phi_4(u_1, u_3, u_2)
+ \Psi_4(u_3, u_1, u_2) + \Psi_4(u_3, u_2, u_1)
+ \Xi_4(u_1, u_2, u_3) + \Xi_4(u_2, u_1, u_3) \tag{C.16}
\]

\[
4T_7(u_1, u_2, u_3) = \Phi_4(u_2, u_3, u_1) + \Phi_4(u_1, u_3, u_2)
+ \Psi_4(u_3, u_1, u_2) + \Psi_4(u_3, u_2, u_1)
- \Xi_4(u_1, u_2, u_3) - \Xi_4(u_2, u_1, u_3) \tag{C.17}
\]

\[
4S_1(u_1, u_2, u_3) = \Phi_4(u_2, u_3, u_1) - \Phi_4(u_1, u_3, u_2)
+ \Psi_4(u_3, u_1, u_2) - \Psi_4(u_3, u_2, u_1)
+ \Xi_4(u_1, u_2, u_3) - \Xi_4(u_2, u_1, u_3) \tag{C.18}
\]

\[
4P_1(u_1, u_2, u_3) = \Phi_4(u_1, u_3, u_2) - \Phi_4(u_2, u_3, u_1)
+ \Psi_4(u_3, u_2, u_1) - \Psi_4(u_3, u_1, u_2)
+ \Xi_4(u_1, u_2, u_3) - \Xi_4(u_2, u_1, u_3) \tag{C.19}
\]

At next, we want to discuss the polynomial expansions of the three introduced independent distribution amplitudes. The required coefficients are non-perturbative parameters which are scale dependent and have to be modeled or taken from the lattice. We use the standard hadronic scale of 1 GeV like in the previous case.

The first order polynomial expressions of these distribution amplitudes are given by asymptotic forms. One gets \(\Phi_4(u_1, u_2, u_3) = 12u_1u_2(f_N + \lambda_1)\) and \(\Psi_4(u_1, u_2, u_3) = 12u_1u_3(f_N - \lambda_1)\) and also \(\Xi_4(u_1, u_2, u_3) = 4u_2u_3\lambda_2\). We have to deal with the values \(\lambda_1 = -27 \cdot 10^{-3}\) GeV\(^2\) and \(\lambda_2 = 51 \cdot 10^{-3}\) GeV\(^2\) taken from [47]. Meanwhile, these parameters are also studied on the lattice, see [92].

When we consider a non-asymptotic form, we get more complicated structures again. Keeping in mind \(u_1 + u_2 + u_3 = 1\), one can expand every distribution amplitude in polynomials which depend on \(u_1\) and \(u_3\) only.

Let us start with \(\Phi_4(u_1, u_3, u_3) = 12u_1u_2(f_N + \lambda_1)[c_1 + c_2u_1 + c_3u_3]\) as the second order polynomial representation for \(\Phi_4\). The here introduced coefficients are given by the conformal expansion, see [47]. We obtain \(c_1 = 2.02\) and \(c_2 = -5.96\) and also \(c_3 = 6.83\) rounded off two decimal places and with acceptable uncertainty.

We proceed with \(\Psi_4(u_1, u_2, u_3) = 12u_1u_3(f_N - \lambda_1)[c_1 + c_2u_1 + c_3u_3]\) as the second order polynomial representation concerning \(\Psi_4\). The here required coefficients are given by the conformal expansion, see [47]. One gets \(c_1 = -1.46\) and \(c_2 = -0.73\) and also \(c_3 = 6.88\) rounded off two decimal places and with acceptable uncertainty.

Let us finish with \(\Xi_4(u_1, u_2, u_3) = 4u_2u_3\lambda_2[c_1 + c_2u_1 + c_3u_3]\) as the second order polynomial representation for \(\Xi_4\). The hereby introduced coefficients are also given by the conformal expansion, see [47]. We gain \(c_1 = 4.94\) and \(c_2 = -6.58\) and also \(c_3 = -6.56\) rounded off two decimal places and with acceptable uncertainty too.
D. Leading Form Factor Diagrams

In this appendix, we will study the expansions specified in (9.3), (10.3), (11.3). These expansions can be presented in form of Feynman diagrams and Wick contractions. Furthermore, we will present the momentum conservation constraints for the diagrams.

The general decomposition in diagrams can be classified by the current coupling to a chosen quark. One gets 14 diagrams for every quark, but they are not independent. Calculating the diagrams described by the current coupling to the identical quarks in the initial nucleon, we get the same results. Moreover, a general interchange of the incoming and outgoing nucleon can be expressed by the interchange of the corresponding quark momentum fractions of these nucleons. We notice that the transition expression of the isovector axial-vector form factor does not allow the current coupling to the quark which occurs once in the initial nucleon.

Fixing the positions of the quark lines in the manner that the single quark in the initial nucleon is at the top or bottom position, one has to deal with the groups of seven diagrams which will be considered here.

Concerning the electromagnetic case, we will especially consider the proton matrix element. The expressions for the neutron matrix element can be obtained by an exchange of $p \rightarrow n$ for the nucleons together with an interchange of $u \leftrightarrow d$ for every quark. In the case of the axial-vector matrix elements, we will consider the discussed processes.

We have to deal with the following list of momentum conservation relations for the seven diagrams when the single quark in the initial nucleon is at the bottom position. The expressions concerning the seven diagrams when this quark is at the top position can be obtained by the interchange of $u_1 \leftrightarrow u_3$ and $v_1 \leftrightarrow v_3$. This list can be applied for all form factors except for the isovector axial-vector form factor. The calculation of this form factor requires the interchange of $v_2 \leftrightarrow v_3$.

\begin{align*}
\Delta_1 &= (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \\
\Lambda_1 &= (v_2 + v_3)p' - (u_2 + u_3)p \\
\Delta_2 &= (v_2 + v_3)p' - u_3p \\
\Lambda_2 &= v_3p' - u_3p \\
\Delta_1 &= (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \\
\Lambda_1 &= (v_2 + v_3)p' - (u_2 + u_3)p \\
\Delta_2 &= (u_2 + u_3)p - v_3p' \\
\Lambda_2 &= v_3p' - u_3p \\
\Delta_1 &= (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \\
\Lambda_1 &= (v_2 + v_3)p' - (u_2 + u_3)p \\
\Delta_2 &= (v_2 + v_3)p' - u_2p \\
\Lambda_2 &= u_2p - v_2p' \\
\Delta_1 &= (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \\
\Lambda_1 &= (v_2 + v_3)p' - (u_2 + u_3)p \\
\Delta_2 &= (u_2 + u_3)p - v_2p' \\
\Lambda_2 &= u_2p - v_2p' \\
\Delta_1 &= (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \\
\Lambda_1 &= (v_2 + v_3)p' - (u_2 + u_3)p \\
\Delta_2 &= (v_1 + v_3)p' - u_3p \\
\Lambda_2 &= v_3p' - u_3p \\
\Delta_1 &= (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \\
\Lambda_1 &= (v_2 + v_3)p' - (u_2 + u_3)p \\
\Delta_2 &= (v_1 + v_3)p' - u_2p \\
\Lambda_2 &= v_2p' - u_2p \\
\Delta_1 &= (v_1 + v_2 + v_3)p' - (u_2 + u_3)p \\
\Lambda_1 &= (v_2 + v_3)p' - (u_2 + u_3)p \\
\Delta_2 &= (v_1 + v_3)p' - u_3p \\
\Lambda_2 &= v_3p' - u_3p \\
\Delta_1 &= (u_1 + u_2)p - v_2p' \\
\Lambda_1 &= v_2p' - u_2p \\
\Delta_2 &= (v_1 + v_3)p' - u_3p \\
\Lambda_2 &= v_3p' - u_3p
\end{align*}
We start our presentation with the seven diagrams expressed by the electromagnetic current coupling to an up quark.

The cross denotes the coupling to the electromagnetic current and the quark lines are in order $u$, $u$, $d$ from top to bottom. These Feynman diagrams are related to the following Wick contractions.

\[
\langle p(P') | \bar{\psi}_u(0) \gamma_\mu \psi_u(0) \bar{\psi}_u(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_u(x_1) \bar{\psi}_u(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \bar{\psi}_u(x_3) \psi_u(x_3) \bar{\psi}_d(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{\psi}_d(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \bar{\psi}_u(0) \gamma_\mu \psi_u(0) \bar{\psi}_u(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_u(x_1) \bar{\psi}_u(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_d(x_2) \bar{\psi}_u(x_3) \psi_u(x_3) \bar{\psi}_d(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{\psi}_d(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \bar{\psi}_u(0) \gamma_\mu \psi_u(0) \bar{\psi}_u(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_u(x_1) \bar{\psi}_d(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_d(x_2) \bar{\psi}_u(x_3) \psi_u(x_3) \bar{\psi}_d(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{\psi}_d(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \bar{\psi}_u(0) \gamma_\mu \psi_u(0) \bar{\psi}_u(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_u(x_1) \bar{\psi}_d(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_d(x_2) \bar{\psi}_u(x_3) \psi_u(x_3) \bar{\psi}_d(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{\psi}_d(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \bar{\psi}_u(0) \gamma_\mu \psi_u(0) \bar{\psi}_u(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_u(x_1) \bar{\psi}_d(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_d(x_2) \bar{\psi}_u(x_3) \psi_u(x_3) \bar{\psi}_d(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{\psi}_d(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \bar{\psi}_u(0) \gamma_\mu \psi_u(0) \bar{\psi}_u(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_u(x_1) \bar{\psi}_d(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_d(x_2) \bar{\psi}_u(x_3) \psi_u(x_3) \bar{\psi}_d(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{\psi}_d(x_4) | p(P) \rangle
\]
We continue our presentation with the seven diagrams expressed by the electromagnetic current coupling to the down quark.

The cross denotes the coupling to the electromagnetic current and the quark lines are in order $d, u, u$ from top to bottom. These Feynman diagrams are related to the following Wick contractions.

\[
\langle p(P') | \tilde{\psi}_d(0) \gamma_{\mu} \psi_d(0) \tilde{\psi}_d(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_d(x_1) \tilde{\psi}_u(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \tilde{\psi}_u(x_3) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \psi_u(x_3) \tilde{\psi}_u(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \psi_u(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \tilde{\psi}_d(0) \gamma_{\mu} \psi_d(0) \tilde{\psi}_d(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_d(x_1) \tilde{\psi}_u(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \tilde{\psi}_u(x_3) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \psi_u(x_3) \tilde{\psi}_u(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \psi_u(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \tilde{\psi}_d(0) \gamma_{\mu} \psi_d(0) \tilde{\psi}_d(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_d(x_1) \tilde{\psi}_u(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \tilde{\psi}_u(x_3) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \psi_u(x_3) \tilde{\psi}_u(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \psi_u(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \tilde{\psi}_d(0) \gamma_{\mu} \psi_d(0) \tilde{\psi}_d(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_d(x_1) \tilde{\psi}_u(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \tilde{\psi}_u(x_3) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \psi_u(x_3) \tilde{\psi}_u(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \psi_u(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \tilde{\psi}_d(0) \gamma_{\mu} \psi_d(0) \tilde{\psi}_d(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_d(x_1) \tilde{\psi}_u(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \tilde{\psi}_u(x_3) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \psi_u(x_3) \tilde{\psi}_u(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \psi_u(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \tilde{\psi}_d(0) \gamma_{\mu} \psi_d(0) \tilde{\psi}_d(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_d(x_1) \tilde{\psi}_u(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \tilde{\psi}_u(x_3) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \psi_u(x_3) \tilde{\psi}_u(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \psi_u(x_4) | p(P) \rangle
\]

\[
\langle p(P') | \tilde{\psi}_d(0) \gamma_{\mu} \psi_d(0) \tilde{\psi}_d(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_d(x_1) \tilde{\psi}_u(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \tilde{\psi}_u(x_3) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \psi_u(x_3) \tilde{\psi}_u(x_4) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \psi_u(x_4) | p(P) \rangle
\]
Let us now present the seven diagrams described by the isovector axial-vector current coupling to an up quark.

The cross denotes the coupling to the isovector axial-vector current and the quark lines are in order \( u, u, d \) from top to bottom on the left and in order \( d, u, d \) from top to bottom on the right.

These Feynman diagrams are connected to the following Wick contractions:

\[
\langle n(P') | \bar{\psi}_d(0) \gamma_\mu \gamma_5 \psi_u(0) | \bar{\psi}_u(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_d(x_1) \bar{\psi}_d(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \psi_u(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{\psi}_d(x_4) | p(P) \rangle
\]

\[
\langle n(P') | \bar{\psi}_d(0) \gamma_\mu \gamma_5 \psi_u(0) | \bar{\psi}_u(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \psi_d(x_1) \bar{\psi}_d(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \psi_u(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \psi_u(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{\psi}_d(x_4) | p(P) \rangle
\]
At next, we present the seven diagrams given by the isoscalar axial-vector current coupling to an up quark.

\[
\langle p(P')| \bar{u}(0)\gamma_\mu \gamma_5 \bar{u}(0)\gamma_\alpha \psi_\alpha(x_1)A^{\alpha_1}(x_1)\psi_\alpha(x_1)\bar{u}_\alpha(x_2)A^{\alpha_2}(x_2)\psi_\alpha(x_2)\bar{u}_\alpha(x_3)A^{\alpha_3}(x_3)\psi_\alpha(x_3)\bar{u}_\alpha(x_4)A^{\alpha_4}(x_4)\psi_\alpha(x_4)|p(P)\rangle
\]

\[
\langle p(P')| \bar{u}(0)\gamma_\mu \gamma_5 \bar{u}(0)\gamma_\alpha \psi_\alpha(x_1)A^{\alpha_1}(x_1)\psi_\alpha(x_1)\bar{d}(x_2)A^{\alpha_2}(x_2)\psi_\alpha(x_2)\bar{d}(x_3)A^{\alpha_3}(x_3)\psi_\alpha(x_3)\bar{d}(x_4)A^{\alpha_4}(x_4)\psi_\alpha(x_4)|p(P)\rangle
\]

\[
\langle p(P')| \bar{u}(0)\gamma_\mu \gamma_5 \bar{u}(0)\gamma_\alpha \psi_\alpha(x_1)A^{\alpha_1}(x_1)\psi_\alpha(x_1)\bar{d}(x_2)A^{\alpha_2}(x_2)\psi_\alpha(x_2)\bar{d}(x_3)A^{\alpha_3}(x_3)\psi_\alpha(x_3)\bar{d}(x_4)A^{\alpha_4}(x_4)\psi_\alpha(x_4)|p(P)\rangle
\]

\[
\langle p(P')| \bar{u}(0)\gamma_\mu \gamma_5 \bar{u}(0)\gamma_\alpha \psi_\alpha(x_1)A^{\alpha_1}(x_1)\psi_\alpha(x_1)\bar{u}_\alpha(x_2)A^{\alpha_2}(x_2)\psi_\alpha(x_2)\bar{u}_\alpha(x_3)A^{\alpha_3}(x_3)\psi_\alpha(x_3)\bar{u}_\alpha(x_4)A^{\alpha_4}(x_4)\psi_\alpha(x_4)|p(P)\rangle
\]

\[
\langle p(P')| \bar{u}(0)\gamma_\mu \gamma_5 \bar{u}(0)\gamma_\alpha \psi_\alpha(x_1)A^{\alpha_1}(x_1)\psi_\alpha(x_1)\bar{u}_\alpha(x_2)A^{\alpha_2}(x_2)\psi_\alpha(x_2)\bar{u}_\alpha(x_3)A^{\alpha_3}(x_3)\psi_\alpha(x_3)\bar{u}_\alpha(x_4)A^{\alpha_4}(x_4)\psi_\alpha(x_4)|p(P)\rangle
\]

\[
\langle p(P')| \bar{u}(0)\gamma_\mu \gamma_5 \bar{u}(0)\gamma_\alpha \psi_\alpha(x_1)A^{\alpha_1}(x_1)\psi_\alpha(x_1)\bar{u}_\alpha(x_2)A^{\alpha_2}(x_2)\psi_\alpha(x_2)\bar{u}_\alpha(x_3)A^{\alpha_3}(x_3)\psi_\alpha(x_3)\bar{u}_\alpha(x_4)A^{\alpha_4}(x_4)\psi_\alpha(x_4)|p(P)\rangle
\]

The cross denotes the coupling to the isoscalar axial-vector current and the quark lines are in order u, u, d from top to bottom. These Feynman diagrams are connected to the following Wick contractions.
At last, we present the seven diagrams given by the isoscalar axial-vector current coupling to the down quark.

The cross denotes the coupling to the isoscalar axial-vector current and the quark lines are in order $d$, $u$, $u$ from top to bottom. These Feynman diagrams are connected to the following Wick contractions:

$$\langle P'| \bar{d}(0) \gamma_\mu \gamma_5 \gamma_\mu \bar{d}(0) \bar{d}(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \bar{d}(x_1) \bar{u}(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \bar{u}(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \bar{u}(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{u}(x_4) | P \rangle$$

$$\langle P'| \bar{d}(0) \gamma_\mu \gamma_5 \gamma_\mu \bar{d}(0) \bar{d}(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \bar{d}(x_1) \bar{u}(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \bar{u}(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \bar{u}(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{u}(x_4) | P \rangle$$

$$\langle P'| \bar{d}(0) \gamma_\mu \gamma_5 \gamma_\mu \bar{d}(0) \bar{d}(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \bar{d}(x_1) \bar{u}(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \bar{u}(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \bar{u}(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{u}(x_4) | p(P) \rangle$$

$$\langle P'| \bar{d}(0) \gamma_\mu \gamma_5 \gamma_\mu \bar{d}(0) \bar{d}(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \bar{d}(x_1) \bar{u}(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \bar{u}(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \bar{u}(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{u}(x_4) | p(P) \rangle$$

$$\langle P'| \bar{d}(0) \gamma_\mu \gamma_5 \gamma_\mu \bar{d}(0) \bar{d}(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \bar{d}(x_1) \bar{u}(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \bar{u}(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \bar{u}(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{u}(x_4) | p(P) \rangle$$

$$\langle P'| \bar{d}(0) \gamma_\mu \gamma_5 \gamma_\mu \bar{d}(0) \bar{d}(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \bar{d}(x_1) \bar{u}(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \bar{u}(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \bar{u}(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{u}(x_4) | p(P) \rangle$$

$$\langle P'| \bar{d}(0) \gamma_\mu \gamma_5 \gamma_\mu \bar{d}(0) \bar{d}(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \bar{d}(x_1) \bar{u}(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \bar{u}(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \bar{u}(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{u}(x_4) | p(P) \rangle$$

$$\langle P'| \bar{d}(0) \gamma_\mu \gamma_5 \gamma_\mu \bar{d}(0) \bar{d}(x_1) \gamma_{\alpha_1} A^{\alpha_1}(x_1) \bar{d}(x_1) \bar{u}(x_2) \gamma_{\alpha_2} A^{\alpha_2}(x_2) \bar{u}(x_2) \gamma_{\alpha_3} A^{\alpha_3}(x_3) \bar{u}(x_3) \gamma_{\alpha_4} A^{\alpha_4}(x_4) \bar{u}(x_4) | p(P) \rangle$$
E. Leading Form Factor Structures

In this appendix, we will present the appearing structures of the expansions introduced in (9.3), (10.3), (11.3). These structures are summarized in the component $S$ specified in the corresponding chapters. We will deal with the same diagrams as considered in the appendix about leading nucleon diagrams.

We start our presentation with the structures concerning the seven diagrams expressed by the electromagnetic current coupling to an up quark.

\[ S_1 = \bar{N}(P')\gamma_{\alpha_4}N(P)\mathrm{Tr}[\gamma_\mu\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4](V + AA) \]
\[ S_2 = \bar{N}(P')\gamma_{\alpha_4}\gamma_5N(P)\mathrm{Tr}[\gamma_\mu\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4](AV - VA) \]
\[ S_3 = \bar{N}(P')\gamma^\nu\gamma_{\alpha_4}\gamma^\lambda N(P)\mathrm{Tr}[\gamma_\mu\sigma_{\lambda\gamma_2}\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4](-TT) \]

\[ S_1 = \bar{N}(P')\gamma_{\alpha_4}\gamma_2\gamma_{\alpha_3}N(P)\mathrm{Tr}[\gamma_\mu\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4]V + AA \]
\[ S_2 = \bar{N}(P')\gamma_{\alpha_4}\gamma_5\gamma_2\gamma_{\alpha_2}N(P)\mathrm{Tr}[\gamma_\mu\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4](-AV - VA) \]
\[ S_3 = \bar{N}(P')\gamma^\nu\gamma_{\alpha_4}\gamma^\lambda N(P)\mathrm{Tr}[\gamma_\mu\sigma_{\lambda\gamma_2}\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_2}\gamma_1\gamma_4](-TT) \]

\[ S_1 = \bar{N}(P')\gamma_{\alpha_2}\gamma_2\gamma_{\alpha_1}N(P)\mathrm{Tr}[\gamma_\mu\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4]V + AA \]
\[ S_2 = \bar{N}(P')\gamma_{\alpha_2}\gamma_5\gamma_2\gamma_{\alpha_3}N(P)\mathrm{Tr}[\gamma_\mu\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4](-AV - VA) \]
\[ S_3 = \bar{N}(P')\gamma^\nu\gamma_{\alpha_2}\gamma^\lambda N(P)\mathrm{Tr}[\gamma_\mu\sigma_{\lambda\gamma_2}\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4](-TT) \]

\[ S_1 = \bar{N}(P')\gamma_{\alpha_2}\gamma_2\gamma_{\alpha_3}N(P)\mathrm{Tr}[\gamma_\mu\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4]V + AA \]
\[ S_2 = \bar{N}(P')\gamma_{\alpha_2}\gamma_5\gamma_2\gamma_{\alpha_1}N(P)\mathrm{Tr}[\gamma_\mu\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4](-AV - VA) \]
\[ S_3 = \bar{N}(P')\gamma^\nu\gamma_{\alpha_2}\gamma^\lambda N(P)\mathrm{Tr}[\gamma_\mu\sigma_{\lambda\gamma_2}\gamma_{\alpha_3}\gamma_2\gamma_{\alpha_1}\gamma_4](-TT) \]

We continue our presentation with the structures concerning the seven diagrams expressed by the electromagnetic current coupling to the down quark.
E. Leading Form Factor Structures

\[ S_1 = \bar{N} (P') \gamma_0 \Delta \gamma_\mu N(P) \text{Tr}[\bar{\psi} \gamma_\alpha \Delta 2 \gamma_\alpha \gamma_\mu \gamma_0 N(P)(V V + AA) \]
\[ S_2 = \bar{N} (P') \gamma_1 \bar{\psi} \gamma_0 \gamma_\mu N(P) \text{Tr}[\bar{\psi} \gamma_\alpha \Delta 2 \gamma_\alpha \gamma_\mu \gamma_0 N(P)(-AV - VA) \]
\[ S_3 = \bar{N} (P') \gamma_2 \gamma_1 \Delta \gamma_\mu N(P) \text{Tr}[\bar{\psi} \gamma_\alpha \gamma_\alpha \Delta 2 \gamma_\alpha \gamma_\mu \gamma_0 N(P)(-TT) \]
\[ S_4 = \bar{N} (P') \gamma_3 \gamma_0 \gamma_\mu N(P) \text{Tr}[\bar{\psi} \gamma_\alpha \gamma_\alpha \Delta 2 \gamma_\alpha \gamma_\mu \gamma_0 N(P)(V V + AA) \]
\[ S_5 = \bar{N} (P') \gamma_4 \gamma_1 \gamma_\mu N(P) \text{Tr}[\bar{\psi} \gamma_\alpha \gamma_\alpha \Delta 2 \gamma_\alpha \gamma_\mu \gamma_0 N(P)(-AV - VA) \]
\[ S_6 = \bar{N} (P') \gamma_5 \gamma_2 \gamma_\mu N(P) \text{Tr}[\bar{\psi} \gamma_\alpha \gamma_\alpha \Delta 2 \gamma_\alpha \gamma_\mu \gamma_0 N(P)(-TT) \]

Let us now present the structures concerning the seven diagrams described by the isovector axial-vector current coupling to an up quark.

\[ S_1 = \bar{N} (P') \gamma_0 \Delta 2 \gamma_\alpha \gamma_\mu \gamma_5 \Delta 1 \gamma_\alpha \bar{\psi} \gamma_\alpha N(P)(V V + AA + AV + VA) \]
\[ S_2 = \bar{N} (P') \gamma_1 \bar{\psi} \gamma_0 \gamma_\alpha \gamma_\mu \gamma_5 \Delta 1 \gamma_\alpha \bar{\psi} \gamma_\alpha N(P)(-TV + TA) \]
\[ S_3 = \bar{N} (P') \gamma_2 \gamma_1 \gamma_\mu \gamma_5 \Delta 1 \gamma_\alpha \bar{\psi} \gamma_\alpha N(P)(VT - AT) \]
\[ S_4 = \bar{N} (P') \gamma_3 \gamma_0 \gamma_\mu \gamma_5 \Delta 1 \gamma_\alpha \gamma_\alpha \gamma_\mu \gamma_0 N(P)(-TT) \]
\[ S_5 = \bar{N} (P') \gamma_4 \gamma_1 \gamma_\mu \gamma_5 \Delta 1 \gamma_\alpha \gamma_\alpha \gamma_\mu \gamma_0 N(P)(V V + AA + AV + VA) \]
\[ S_6 = \bar{N} (P') \gamma_5 \gamma_2 \gamma_\mu \gamma_5 \Delta 1 \gamma_\alpha \gamma_\alpha \gamma_\mu \gamma_0 N(P)(-TV + TA) \]
\[ S_7 = \bar{N} (P') \gamma_6 \gamma_3 \gamma_\mu \gamma_5 \Delta 1 \gamma_\alpha \gamma_\alpha \gamma_\mu \gamma_0 N(P)(VT - AT) \]
\[ S_8 = \bar{N} (P') \gamma_7 \gamma_4 \gamma_\mu \gamma_5 \Delta 1 \gamma_\alpha \gamma_\alpha \gamma_\mu \gamma_0 N(P)(-TT) \]
\[ S_1 = \bar{N}(P')\gamma_{\alpha_3}\gamma_{\mu\gamma_5}\Delta_1 \gamma_{\alpha_1} \gamma_{\alpha_2} \Delta_2 N(P)(VV + AA + AV + VA) \]
\[ S_2 = \bar{N}(P')\gamma_{\alpha_3}\gamma_{\mu\gamma_5}\Delta_1 \gamma_{\alpha_1} \gamma_{\alpha_2} \Delta_2 N(P)(TV + TA) \]
\[ S_3 = \bar{N}(P')\gamma_{\alpha_3} \gamma_{\alpha_4} \Delta_{\gamma_5} \gamma_{\alpha_1} \gamma_{\alpha_2} \gamma_{\alpha_2} N(P)(VT - AT) \]
\[ S_4 = \bar{N}(P')\gamma_{\alpha_3} \gamma_{\alpha_1} \gamma_{\alpha_2} \Delta_{\gamma_5} \gamma_{\alpha_1} \gamma_{\alpha_4} \Delta_{\gamma_2} \gamma_{\alpha_3} \gamma_{\alpha_3} N(P)(TT) \]

At next, we present the structures for the seven diagrams given by the isoscalar axial-vector current coupling to an up quark:

\[ S_1 = \bar{N}(P')\gamma_{\alpha_4} N(P) \mc{Tr}[\gamma_{\mu\gamma_5}\gamma_{\alpha_3} \gamma_{\mu} \gamma_{\alpha_2} \gamma_{\alpha_2} N(P)(VV + AA) \]
\[ S_2 = \bar{N}(P')\gamma_{\alpha_4} N(P) \mc{Tr}[\gamma_{\mu\gamma_5}\gamma_{\alpha_3} \gamma_{\mu} \gamma_{\alpha_2} \gamma_{\alpha_2} N(P)(TV + TA) \]
\[ S_3 = \bar{N}(P')\gamma_{\alpha_4} N(P) \mc{Tr}[\gamma_{\mu\gamma_5}\gamma_{\alpha_3} \gamma_{\mu} \gamma_{\alpha_2} \gamma_{\alpha_2} N(P)(VT - AT) \]
\[ S_4 = \bar{N}(P')\gamma_{\alpha_4} N(P) \mc{Tr}[\gamma_{\mu\gamma_5}\gamma_{\alpha_3} \gamma_{\mu} \gamma_{\alpha_2} \gamma_{\alpha_2} N(P)(TT) \]

\[ S_5 = \bar{N}(P')\gamma_{\alpha_4} N(P) \mc{Tr}[\gamma_{\mu}\gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\mu} \gamma_{\alpha_2} \gamma_{\alpha_2} N(P)(VV + AA) \]
\[ S_6 = \bar{N}(P')\gamma_{\alpha_4} N(P) \mc{Tr}[\gamma_{\mu}\gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\mu} \gamma_{\alpha_2} \gamma_{\alpha_2} N(P)(TV + TA) \]
\[ S_7 = \bar{N}(P')\gamma_{\alpha_4} N(P) \mc{Tr}[\gamma_{\mu}\gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\mu} \gamma_{\alpha_2} \gamma_{\alpha_2} N(P)(VT - AT) \]
\[ S_8 = \bar{N}(P')\gamma_{\alpha_4} N(P) \mc{Tr}[\gamma_{\mu}\gamma_{\alpha_5} \gamma_{\alpha_3} \gamma_{\mu} \gamma_{\alpha_2} \gamma_{\alpha_2} N(P)(TT) \]

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\[ S_1 = \bar{N}(P)^\gamma_{\alpha_2} \Delta_{\gamma_{\alpha_1}} N(P) \text{Tr}[\gamma_\mu \gamma_5 p_{\alpha_3} p_{\alpha_4}] (VV + AA) \]

\[ S_2 = \bar{N}(P)^\gamma_{\alpha_2} \Delta_2 \gamma_{\alpha_1} \gamma_5 N(P) \text{Tr}[\gamma_\mu \gamma_5 p_{\alpha_3} p_{\alpha_4}] (-AV - VA) \]

\[ S_3 = \bar{N}(P)^\gamma_{\alpha_2} \Delta_{\gamma_{\alpha_1}} N(P) \text{Tr}[\gamma_\mu \gamma_5 i\sigma_{\lambda\rho} \gamma_{\alpha_3} \gamma_{\lambda\rho} \gamma_{\alpha_4}] (-TT) \]

\[ S_4 = \bar{N}(P)^\gamma_{\alpha_2} N(P) \text{Tr}[\gamma_\mu \gamma_5 p_{\alpha_3} p_{\alpha_4}] (VV + AA) \]

\[ S_5 = \bar{N}(P)^\gamma_{\alpha_2} \Delta_2 \gamma_{\alpha_1} \gamma_5 N(P) \text{Tr}[\gamma_\mu \gamma_5 p_{\alpha_3} p_{\alpha_4}] (-AV - VA) \]

\[ S_6 = \bar{N}(P)^\gamma_{\alpha_2} \Delta_{\gamma_{\alpha_1}} N(P) \text{Tr}[\gamma_\mu \gamma_5 i\sigma_{\lambda\rho} \gamma_{\alpha_3} \gamma_{\lambda\rho} \gamma_{\alpha_4}] (-TT) \]

\[ S_7 = \bar{N}(P)^\gamma_{\alpha_2} N(P) \text{Tr}[\gamma_\mu \gamma_5 \Delta_1 \gamma_{\alpha_1} p_{\alpha_3} p_{\alpha_4}] (VV + AA) \]

\[ S_8 = \bar{N}(P)^\gamma_{\alpha_2} \Delta_1 \gamma_{\alpha_1} \gamma_5 N(P) \text{Tr}[\gamma_\mu \gamma_5 p_{\alpha_3} p_{\alpha_4}] (-AV - VA) \]

\[ S_9 = \bar{N}(P)^\gamma_{\alpha_2} \Delta_{\gamma_{\alpha_1}} N(P) \text{Tr}[\gamma_\mu \gamma_5 i\sigma_{\lambda\rho} \gamma_{\alpha_3} \gamma_{\lambda\rho} \gamma_{\alpha_4}] (TT) \]

At last, we present the structures for the seven diagrams given by the isoscalar axial-vector current coupling to the down quark.

\[ S_1 = \bar{N}(P)^\gamma_{\alpha_1} \Delta_1 \gamma_{\mu_5} N(P) \text{Tr}[\gamma_\mu_i \gamma_5 p_{\alpha_3} p_{\alpha_4}] (VV + AA) \]

\[ S_2 = \bar{N}(P)^\gamma_{\alpha_1} \Delta_2 \gamma_{\mu_5} N(P) \text{Tr}[\gamma_\mu_i \gamma_5 p_{\alpha_3} p_{\alpha_4}] (-AV - VA) \]

\[ S_3 = \bar{N}(P)^\gamma_{\alpha_1} \Delta_{\mu_5} N(P) \text{Tr}[i\sigma_{\lambda\rho} \gamma_{\alpha_3} \gamma_{\lambda\rho} \gamma_{\alpha_4}] (-TT) \]

\[ S_4 = \bar{N}(P)^\gamma_{\alpha_1} \Delta_1 \gamma_{\mu_5} N(P) \text{Tr}[\gamma_\mu_i \gamma_5 p_{\alpha_3} p_{\alpha_4}] (VV + AA) \]

\[ S_5 = \bar{N}(P)^\gamma_{\alpha_1} \Delta_2 \gamma_{\mu_5} N(P) \text{Tr}[\gamma_\mu_i \gamma_5 p_{\alpha_3} p_{\alpha_4}] (-AV - VA) \]

\[ S_6 = \bar{N}(P)^\gamma_{\alpha_1} \Delta_{\mu_5} N(P) \text{Tr}[i\sigma_{\lambda\rho} \gamma_{\alpha_3} \gamma_{\lambda\rho} \gamma_{\alpha_4}] (TT) \]

\[ S_7 = \bar{N}(P)^\gamma_{\alpha_3} \Delta_2 \gamma_{\alpha_1} \Delta_1 \gamma_{\mu_5} N(P) \text{Tr}[\gamma_\mu_i \gamma_5 p_{\alpha_3} p_{\alpha_4}] (VV + AA) \]

\[ S_8 = \bar{N}(P)^\gamma_{\alpha_3} \Delta_2 \gamma_{\alpha_1} \Delta_1 \gamma_{\mu_5} N(P) \text{Tr}[\gamma_\mu_i \gamma_5 p_{\alpha_3} p_{\alpha_4}] (-AV - VA) \]

\[ S_9 = \bar{N}(P)^\gamma_{\alpha_3} \Delta_{\gamma_{\alpha_1}} N(P) \text{Tr}[i\sigma_{\lambda\rho} \gamma_{\alpha_3} \gamma_{\lambda\rho} \gamma_{\alpha_4}] (TT) \]
F. Leading Form Factor Results

In this appendix, we will present the final results of the expansions introduced in (9.3), (10.3), (11.3). We will deal with the same diagrams as considered in the appendix about leading nucleon diagrams.

We start our presentation with the results concerning the seven diagrams expressed by the electromagnetic current coupling to an up quark.

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_u}{Q^4} \int \frac{[du]}{u_3(u_2 + u_3)^2} \frac{[dv]}{v_3(v_2 + v_3)^2} [(V - A)^2 + 4T^2] \tilde{N}(P')\gamma_\mu N(P)
\]

0

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_u}{Q^4} \int \frac{[du]}{u_2(u_2 + u_3)^2} \frac{[dv]}{v_2(v_2 + v_3)^2} [(V - A)^2 + 4T^2] \tilde{N}(P')\gamma_\mu N(P)
\]

0

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_u}{Q^4} \int \frac{[du]}{u_2u_3(u_2 + u_3)^2} \frac{[dv]}{v_2v_3(v_1 + v_3)^2} [-4T^2] \tilde{N}(P')\gamma_\mu N(P)
\]

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_u}{Q^4} \int \frac{[du]}{u_2u_3(u_2 + u_3)^2} \frac{[dv]}{v_2v_3(v_1 + v_2)^2} [-(V - A)^2] \tilde{N}(P')\gamma_\mu N(P)
\]

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_u}{Q^4} \int \frac{[du]}{u_2u_3(u_1 + u_2)^2} \frac{[dv]}{v_2v_3(v_1 + v_3)^2} [(V + A)^2] \tilde{N}(P')\gamma_\mu N(P)
\]

We continue our presentation with the results concerning the seven diagrams expressed by the electromagnetic current coupling to the down quark.

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_d}{Q^4} \int \frac{[du]}{u_1(u_1 + u_2)^2} \frac{[dv]}{v_1(v_1 + v_2)^2} [2(V^2 + A^2)] \tilde{N}(P')\gamma_\mu N(P)
\]

0

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_d}{Q^4} \int \frac{[du]}{u_1(u_1 + u_2)^2} \frac{[dv]}{v_2(v_1 + v_2)^2} [2(V^2 + A^2)] \tilde{N}(P')\gamma_\mu N(P)
\]

0

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_d}{Q^4} \int \frac{[du]}{u_1u_2(u_1 + u_2)^2} \frac{[dv]}{v_1v_2(v_1 + v_3)^2} [-(V + A)^2] \tilde{N}(P')\gamma_\mu N(P)
\]

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_d}{Q^4} \int \frac{[du]}{u_1u_2(u_1 + u_2)^2} \frac{[dv]}{v_1v_2(v_2 + v_3)^2} [-(V - A)^2] \tilde{N}(P')\gamma_\mu N(P)
\]

\[
\frac{(4\pi\alpha_s)^2}{216} \frac{e_d}{Q^4} \int \frac{[du]}{u_1u_2(u_2 + u_3)^2} \frac{[dv]}{v_1v_2(v_1 + v_3)^2} [4T^2] \tilde{N}(P')\gamma_\mu N(P)
\]

Let us now present the results concerning the seven diagrams described by the isovector axial-vector current coupling to an up quark.
F. Leading Form Factor Results

\[
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_3(u_2 + u_3)^2 v_2(v_2 + v_3)^2} [2(T(V - A) + (V - A)T)] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
0 \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_2(u_2 + u_3)^2 v_3(v_2 + v_3)^2} [2(T(V - A) + (V - A)T)] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
0 \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_2u_3(u_2 + u_3) v_2v_3(v_1 + v_2)} [2(A - V)T] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_2u_3(u_2 + u_3) v_2v_3(v_1 + v_3)} [2T(A - V)] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_2u_3(u_1 + u_2) v_2v_3(v_1 + v_3)} [(V + A)(V + A)] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
\]

At next, we present the results for the seven diagrams given by the isoscalar axial-vector current coupling to an up quark.

\[
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_3(u_2 + u_3)^2 v_3(v_2 + v_3)^2} [(V - A)^2 + 4T^2] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
0 \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_2(u_2 + u_3)^2 v_2(v_2 + v_3)^2} [(V - A)^2 + 4T^2] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
0 \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_2u_3(u_2 + u_3) v_2v_3(v_1 + v_3)} [-4T^2] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_2u_3(u_2 + u_3) v_2v_3(v_1 + v_2)} [-(V - A)^2] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_2u_3(u_1 + u_2) v_2v_3(v_1 + v_3)} [-(V + A)^2] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
\]

At last, we present the results for the seven diagrams given by the isoscalar axial-vector current coupling to the down quark.

\[
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_3(u_1 + u_2)^2 v_1(v_1 + v_2)^2} [2(V^2 + A^2)] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
0 \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_2(u_1 + u_2)^2 v_2(v_1 + v_2)^2} [2(V^2 + A^2)] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
0 \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_1u_2(u_1 + u_2) v_1v_2(v_1 + v_3)} [-(V + A)^2] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_1u_2(u_1 + u_2) v_1v_2(v_2 + v_3)} [-(V - A)^2] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
\frac{(4\pi\alpha_s)}{216} \frac{1}{Q^4} \int \frac{[du]}{u_1u_2(u_2 + u_3) v_1v_2(v_1 + v_3)} [-4T^2] \bar{N}(P')\gamma_\mu\gamma_5 N(P) \\
\]
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