Multilevel Modeling of Fiber-Reinforced Concrete and Application to Numerical Simulations of Tunnel Lining Segments

by

M.Sc. Yijian Zhan

Dissertation

for the degree

Doctor of Engineering (Dr.-Ing.)

Department of Civil and Environmental Engineering
Ruhr University Bochum, Germany

Bochum, November 2016
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Abstract

Aiming at the model-based analysis, design and optimization of engineering structures such as segmental tunnel linings made of fiber-reinforced concrete (FRC), a multilevel modeling framework consisting of a series of model components is developed, which facilitates the investigation of the effect of various design parameters (concrete class, fiber property, fiber distribution, etc.) at different length scales. The fundamental ingredient at the level of single fibers is an analytical model allowing the prediction of the pullout response of a steel fiber, either straight or with hooked end, with or without inclination with respect to the loading direction. For an opening crack in a specific FRC composite, employing the single fiber pullout model, the fiber bridging effect is computed via the integration of the pullout responses of all the fibers intercepting the crack, taking particularly the anisotropic fiber orientation into consideration. Based on the numerically integrated results of crack-bridging effect, the corresponding analytical surrogate function form of the traction–separation relation is derived. For the numerical analysis of fracture processes and failure mechanisms at the structural level, the finite element method using interface solid elements equipped with the traction–separation law obtained from the crack bridging model is used to capture the cracking phenomena and the postcracking ductile behavior of FRC. An implicit/explicit integration scheme and an adaptive mesh-processing technique are implemented to enhance the robustness and to reduce the expense of computation. Having these model components validated, the complete multilevel modeling framework is applied to the simulation of failure behavior of tunnel lining segments made of FRC, demonstrating good performance in following the influence of different design parameters from the single fiber level to the scale of engineering structures. Therefore, the present work constitutes the essential ingredients of a “virtual laboratory” for the design and optimization of FRC materials and structures.
概要

工学博士学位论文
纤维混凝土多层次建模与隧道衬砌管片数值模拟中的应用

作者：占羿箭
导师：Prof. Dr. techn. Günther Meschke

以研发纤维混凝土材料与结构的数值分析、设计与优化的计算平台为目标，建立了一个多层次建模框架，允许捕捉不同空间尺度多项重要参数（混凝土强度、纤维特性、纤维分布等）对整体结构承载力及破坏机理的影响。该框架由以下关键模型构成：首先，在单纤维尺度，提出了一个新的拔出理论模型；特别考虑纤维类型，拔出角度等参数，并预测完整的拔出力-位移关系。之后，在纤维混凝土复合材料开裂的层次，基于单纤维拔出模型，考虑纤维特性与空间分布，尤其是非各同向同性纤维指向，通过计算所有有效纤维的拔出阻力，获得裂缝的接接效应，并以解析函数形式替代数值积分结果以便后续应用。在宏观结构尺度，利用新近提出的一种灵活的界面单元来反映裂缝的生成、扩展、行进等现象，对整体结构的破坏机理进行有限元模拟；程序编写中应用隐式/显式积分以及自适应网格技术以提高总体计算效率。以上模型均通过与实验结果对比而得到验证。最后，将多层次模型整合应用于纤维混凝土制隧道衬砌管片破坏过程的有限元模拟，展示了良好的跨尺度捕捉不同参数影响的能力。综上，基于本文工作，初步建立可用于纤维混凝土复合材料与结构的设计与优化的“虚拟实验室”。

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The thesis reports the main achievements during my research at Institute for Structural Mechanics, Department of Civil and Environmental Engineering, Ruhr University Bochum, starting from the winter of 2010 till the summer of 2016, which has been an extremely fruitful stage of my career.

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Chapter 1

Introduction

1.1 Motivation

The construction of urban as well as inter-city transportation facilities is generally considered as one of the most important constituents of the infrastructure development. In order to meet the increasing demand on mobility, tunnel construction has become an appealing option due to the unique advantages in exploiting the underground space, particularly in the urban regions possessing high density of existing surface buildings [Fig. 1.1(a)], or in the mountain areas where tunnels through hills allow to integrate different districts into high-speed transportation networks [Fig. 1.1(b)].

Figure 1.1: (a) Mechanized tunnel construction carried out under urban conditions (©Crossrail Ltd); (b) tunneling through the base of mountains, connecting different cities into international transportation networks (©Brenner Basistunnel, BBT-SE).
Various techniques in association with equipments have been developed and employed for tunnel construction. As a modern engineering technique, the mechanized shield tunneling conducted using a tunnel boring machine (TBM) has become one of the dominant methods. The use of TBM enables to efficiently excavate through the underground in different regions characterized by distinct geological conditions from soft soil to hard rock with different levels of saturation, and obtain regular tunnel shape with minimized impact on the existing structures and the environment. The evolution of mechanized tunneling techniques has led to modernized devices with increasing excavation capacities such as the machine named as “BERTHA” and used in the tunneling project in Seattle, which is currently known, with the diameter of 17.4 m, as the largest TBM in the world.

1.1.1 Durability of segmental tunnel linings

The typical designed service life of a tunnel is approximately 100 years. This requirement can only be satisfied if the durability of tunnel lining, the main supporting structure maintaining the shape and space of tunnel, is guaranteed. The lining of a tunnel, usually made of concrete and strengthened by different types of reinforcement, can be constructed on-site after the underground is excavated. Alternatively, in mechanized tunneling projects, high quality precast lining segments are delivered and placed by the TBM to the correct position; after one lining ring consisting of several lining segments is installed and integrated into the existing lining system, large thrust forces are imposed by the hydraulic jacks on the front surface of the last ring, which enables the TBM to advance [Fig. 1.2(a)].

![Diagram of TBM excavation and lining placement](image)

**Figure 1.2:** (a) Picture showing the excavation of TBM, driven by hydraulic jack forces on the front surface of the last installed lining ring (©Crossrail Ltd); (b) sketch of possible damage mechanisms in lining segments according to SUGIMOTO (2006); (c) photos showing tunnel linings influenced by damaged segments (GREENHALGH 2010, ROSSLER ET AL. 2012).

It is clear that strict guidelines [e.g. JSCE-TUNNEL ENGINEERING COMMITTEE (2007), GERMAN GEOTECHNICAL SOCIETY (2014) and ITA WORKING GROUP 2 (2016)] are followed on every stage of a tunneling project in order to ensure that a durable lining system is finally obtained. However, due to the rigorous underground environment (such as the high pressures generated by the
soil, rock and water) and the large construction-induced loads (e.g. the thrust forces during excavation) on lining segments, unexpected damages can still occur to different parts of the lining structure. In particular, the load-bearing capacity of segments subjected to the jack forces can be very sensitive to the imperfection or misalignment of installed lining segments, the nonuniform deformation developed between different lining rings, the eccentricity or inclination of jacks while pushing against segments, etc., possibly causing excessive stresses and, consequently, damages in local regions of lining segments when the TBM advances (Fig. 1.2). The typical damage phenomena observed in concrete linings include the cracking in the regions under high tension-dominant stresses, either in the interior or on the circumference of segment, at the segment joints or ring connections, etc. These cracks, if permitted to grow without control, can significantly increase the hazard of corrosion and harm the watertightness, and, consequently, increase the maintenance cost and lead to the deterioration of the serviceability and durability of lining system (BLOM 2002, de WAAL 1999, Sugimoto 2006).

1.1.2 Model-based analysis, design and optimization of lining segments made of fiber-reinforced concrete

As mentioned above, most of the tunnel linings are made of concrete, a heterogeneous material mainly consisting of dry cement and water, sand and aggregates. Plain concrete is characterized by a low tensile strength and a quasi-brittle post-cracking behavior and therefore needs to be particularly strengthened in the case of tension-dominant loading. A common way to enhance the post-elastic ductility of the material is to add steel bars as reinforcement in the regions of concrete structures subjected to high tension. However, opening cracks may considerably limit the durability and therefore the applicability of traditional reinforced concrete (RC) structures. In tunnel linings, where a limitation of crack widths and the demand for material ductility to control construction-induced local damages play an important role, fiber-reinforced concrete (FRC) becomes particularly attractive due to its high ductility, or even strain-hardening behavior, accompanied with distributed cracking at small crack widths not affecting the structural durability (CARATELLI ET AL. 2011, HANSEL AND GUIRGUI 2011, KASPER ET AL. 2008). Since the first applications in 1980s, fibers, either as partial or as complete substitution of conventional rebars, have been proven to be effective in improving the mechanical properties as well as in reducing the total costs of segmental tunnel linings (DE LA FUENTE ET AL. 2012, HANSEL AND GUIRGUI 2011, ITA WORKING GROUP 2 2016, THOMAS ET AL. 2015).

In the last decades the material properties and the structural behavior of FRC have been extensively investigated in the laboratory environment for different loading scenarios (BOULEKBACHE ET AL. 2012, CARATELLI ET AL. 2011, COLOMBO ET AL. 2016, FANELLA AND NAAMAN 1985, KESNER ET AL. 2003, LI ET AL. 1996, MICHELS ET AL. 2013, RAO AND SESHU 2003, SHAH AND NAAMAN 1976). However, laboratory tests are generally expensive and are restricted to specific test configurations; therefore, analytical models and computational tools are widely used for the prediction of material and structural behavior under various loading conditions. In particular, a model-based numerical simulation platform, which takes into account the nonlinear material behavior of all the individual ingredients as well as the mutual interaction between different components,
can serve as a “virtual laboratory” for the analysis, design and optimization of engineering structures such as segmental tunnel linings made of fiber-reinforced concrete in association with other types of reinforcement.

1.2 State of the art

Large efforts have been devoted to the model-based failure analysis of FRC on the material as well as on the structural level. Most of the existing numerical models for the structural analyses of FRC are developed based on the models for plain concrete, by means of modifying the postpeak regime of inelastic constitutive models to represent the enhanced ductility of FRC in terms of an increase of the residual stress and the fracture energy. These models have been formulated in the framework of smeared crack models (Gödde and Mark 2010, Hameed et al. 2013), microplane models (Beghini et al. 2007, Caner et al. 2013), cohesive interface elements (Park et al. 2010, Tailhan et al. 2015) or embedded strong discontinuity approaches (Brighenti and Scorzà 2012, Denneman et al. 2011). As these models include the effect of fibers on a phenomenological level, the enhanced fracture toughness of the specific fiber cocktail and the specific concrete material must be determined a priori from experiments or assumed based upon design codes. However, they do not allow predicting the effect of specific fiber cocktail on the macroscopic material behavior, which needs a resolution of FRC on lower scales. On the meso-scale, models for FRC have been proposed which include explicit representation of individual fibers within representative elementary volumes of FRC samples: In Radtke et al. (2011) fibers are individually represented using embedded enhanced displacements to represent the fiber–concrete slip in the finite element discretization. In Cunha et al. (2012) the contribution of individual fibers is taken into account by the superposition of the stiffness properties derived from the pullout responses of single fibers onto the stiffness of concrete matrix. As an alternative discretization approach on the meso-level, Kang et al. (2014) use lattice models for FRC, in which the fibers are attached to a background lattice model for plain concrete considering the bond–slip effect of fibers as additional components of the spring-like stiffness between the lattice nodes. Meso-scale models allow for a better understanding of the fiber–concrete interaction mechanisms in FRC materials, but are evidently not suitable for the direct use at structural level. To this end, a multiscale modeling framework is required, in which the individual nonlinear behavior of the fiber and the matrix, as well as their mutual interactions involved at different length scales, are represented via a series of sub-models, and the model informations are appropriately transmitted across the scales.

1.2.1 Investigation of single fiber pullout behavior

As a fundamental topic in this research, the pullout behavior of single fibers embedded in concrete matrix has gained much attention. A large number of laboratory tests on single fiber pullout are available, providing a rich data base of various concrete–fiber configurations [see e.g. Leung and Shapiro (1999), Li and Stang (1997) and Robins et al. (2002)]. Alternatively, virtual experiments using finite element models to generate the pullout load–displacement relations have been performed to obtain a more detailed insight into the pullout mechanisms (Breitenbücher et al.)
2014, Chen et al. 2009). Although certain simplifications regarding the concrete–fiber interaction are applied, analytical models are able to predict the main features of the pullout load–displacement relation with good accuracy at minimal computational cost.

As a standard case, for the analysis of straight fibers without inclination with respect to the loading direction, analytical methods typically treat the fiber as a cylinder and focus on the interfacial behavior, characterized by the frictional slip mechanism along the interface between fiber and matrix (Lawrence 1972, Lin and Li 1997, Naaman et al. 1991). Modeling the pullout behavior of inclined fibers involves additional complexities connected with the lateral pressure on the interface, inelastic deformations of the fiber and partial damage of the matrix [see e.g. the experimental observations in Leung and Ybanez (1997) and Leung and Shapiro (1999)]. An extensive literature survey on the pullout behavior of inclined fibers is contained in Laranjeira et al. (2010), where also an analytical model predicting the pullout load–displacement relations of inclined straight fibers is presented. This model is formulated by defining a few key states during the pullout procedure, analyzing the pullout displacement and the corresponding force at each key state and sequentially connecting them as a multilinear relationship.

Besides straight steel fibers, one of the most widely used fiber types are fibers with hooked ends. During the pullout of a hooked-end steel fiber, in contrast to the case of a straight fiber, where the interfacial behavior plays the main role, the resistance of the hook to straightening considerably contributes to the total pullout force (Naaman and Najm 1991, Robins et al. 2002). Only a few analytical models describing the hooked-end fiber pullout without inclination are presented in the open literature (Alwan et al. 1999, Chanvillard 1999, Cunha et al. 2010, Gysel 1999). The inclined situation, with the loading direction deviating from the fiber direction, seems to be only considered in the semi-analytical model proposed in Laranjeira et al. (2010), where the pullout force due to the hooked end is extracted from experimental results and combined with a straight fiber pullout model, and in Soetens et al. (2013) where the anchorage effect of hook is determined via the integration of forces over the differential segments along the fiber axis.

### 1.2.2 Determination of crack bridging effect

At the level of FRC composite containing a number of distributed fibers, the crack bridging effect, i.e. the bridging stress vs. the crack opening displacement relation under tension, is widely considered as the most important post-cracking constitutive behavior (Di Prisco et al. 2013). The crack bridging effect plays a crucial role to link the individual fiber pullout response with the structural failure mechanism and, hence, needs to be determined and incorporated into the structural model. Usually, uniaxial tension tests on notched specimen are performed to obtain the traction–separation relation directly (Barragán et al. 2003, Cunha et al. 2011, Rossi 1997). Alternatively, the bridging law can be obtained via the back analysis from standard three-point bending tests on notched beam (Kooiman et al. 2000, Soetens and Matthyssens 2014, Temmat et al. 2006). Analytical approaches generating the bridging effect via the integration of individual fiber pullout responses are proposed as well. In an analytical crack bridging model, other than those well-recognized parameters (e.g. the properties of single fiber and the content of fiber in the composite), the influence of fiber alignment on the crack bridging stresses is often neglected by assuming...
isotropic fiber orientation (Lee et al. 2011, Lin and Li 1997, Wang et al. 1989). However, laboratory tests have revealed the presence of anisotropic fiber orientation and its impact on the post-cracking ductility of FRC (Barragán et al. 2003, Boulekbache et al. 2010, Şanel and Zihnioğlu 2013).

1.2.3 Numerical modeling of cracking in FRC structures

On the structural level, numerical models using e.g. the finite element method are required, which enables researchers to capture opening cracks and to account for the crack bridging effect of fibers. Concrete cracking may be represented as a damage zone, using continuum-based approaches such as plasticity or damage formulations, rotating or fixed crack models, which, since the mid of the 1980’s, have been enhanced by means of adequate regularization techniques [see, e.g. Hofstetter and Mang (1995), Jiřásek and Bažant (2002) and Mang et al. (2003) for an overview on the smeared representation of cracks]. Alternatively, models allowing the discrete representation of cracks within finite element analyses have been developed by introducing cracks as separate entities directly into the finite element mesh and separating individual finite elements as cracks evolve. This discrete crack approach has been originally developed for linear elastic fracture mechanics problems (Ingraffea and Saouma 1985, Xie and Gerstle 1995) and later also successfully employed for the modeling of cohesive cracks. Amongst these models, zero-thickness interface elements included along the element borders have been demonstrated to be suitable to capture complex cracking phenomena in meso-scale analyses of concrete specimens Carol et al. (2001). Recently, Manzoli et al. (2012) introduced a modification of the classical zero-thickness interface elements by using degenerated solid finite elements with almost zero thickness. This technique can be easily implemented in standard FE programs using conventional solid elements and has been successfully used in capturing complex cracking processes in the structures made of plain- as well as reinforced concrete (Manzoli et al. 2014). A major advantage of this class of models is, that no special procedure for the tracking of evolving cracks is necessary. This is in contrast to the class of embedded strong discontinuity approaches, where the failure kinematics is incorporated directly into the finite element formulation, such as the family of strong discontinuity approaches, based on a local element related enrichment of the approximation space (Linder and Zhang 2014, Mosler and Meschke 2003, Simo et al. 1993) and the models for cohesive cracks formulated in the framework of the extended finite element method (Meschke and Dumstorf 2007, Moës et al. 1999, Sukumar et al. 2015). However, the crack patterns represented via interface elements are inevitably mesh-dependent. This drawback can be alleviated by mesh refinement, at the cost of increased computational expense, which, in turn, can be reduced by pre-defining the interface elements only in the vulnerable regions (Su et al. 2010) or applying an adaptive algorithm for the insertion of interface elements (Pandolfi and Ortiz 2002).

A full integration of all levels (single fiber–matrix interaction, crack bridging behavior and structural failure analysis) does not seem to be accomplished so far. An attempt to include lower scales into a numerical model for engineered cementitious composites made of straight polyvinyl alcohol fibers has been made by Kabele (2007); in the structural analyses, however, the constitutive relation is not yet derived from lower scales, but directly formulated on a macroscopic level.
1.2.4 Analysis of segmental tunnel linings

The serviceability and ductility characteristics of segmental tunnel linings can be inspected by various laboratory tests, including the experiments on real-size specimens for the purpose of inspecting the failure behavior of individual lining segments [see e.g. GETTU ET AL. (2004), POH ET AL. (2009) and CARATELLI ET AL. (2011)], and, due to the high expense, a limited number of full-ring tests conducted to approximate the real lining behavior (BLOM AND VAN OOSTERHOUT 2001, LIU ET AL. 2015, MOLINS AND ARNAU 2011). According to the recommendations and guidelines such as GERMAN TUNNELLING COMMITTEE (DAUB) (2013) and ACI COMMITTEE-544 (2016), the design of tunnel linings requires analytical methods especially for the determination of limit states with respect to the critical load combinations on the cross sections of segment. For the estimation of critical locations and the values of internal force and moment in the lining system in typical load cases, structural design models can be established; in these engineering model, the segments are usually simplified as beams or shells characterized by elastic-plastic behavior under tension, compression or bending, the springs equipped with nonlinear stiffness evolution laws are assumed to represent the joint behavior, and the lining–soil interactions can be considered as pressure on lining and spring-like support (ARNAU AND MOLINS 2012, DE WAAL 1999, DO ET AL. 2013).

However, it is recognized that the durability of the complete lining system can be particularly influenced by the unexpected stress concentration and, consequently, loss of load-bearing capacity of material at a small length-scale (such as cracking and crushing of concrete on the circumference of lining segments, in the joint areas and in the vicinity of bolt connections) that cannot be captured by simplified structural models. Therefore, for the sake of insight into the local damage mechanisms in different loading scenarios, more sophisticated computational tools, such as the finite element methods equipped with appropriate model components describing the individual material behavior and the interaction mechanisms between different ingredients, allow better resolution of problems at detailed levels. Numerical simulations concerning the damages in lining segments subjected to different loading scenarios can be conveniently conducted by employing commercial FEA-software packages; for example, as documented in BURGERS (2006), the cracking in lining segments caused by undesired configuration (eccentricity and inclination) of hydraulic jacks is simulated; CHEN AND MO (2009) investigate the damages in the regions of bolt connections; LIAO ET AL. (2015) report the numerical simulation on the possible cracking caused by the stacking of segments while in storage. In general, those numerical simulations rely on the input of phenomenological material behavior of FRC at the macroscopic level; unless lower-scale models predicting the FRC behavior are incorporated, it is not possible to follow the influence of various low-scale parameters on the failure mechanisms in lining structures.

1.3 Organization of present work

1.3.1 Objectives and highlights of research

The present research aims at the development of a multilevel modeling framework for the nonlinear analyses of steel-fiber–reinforced concrete (SFRC), which allows to follow the influence of various
material parameters through different length-scales of SFRC materials and structures. The proposed multilevel model mainly consists of the submodels corresponding to three different scales as follows (Fig. 1.3):

- Level 1: Modeling of the pull-out behavior of single fibers.
- Level 2: Computation of the crack bridging effect of fiber cocktails.
- Level 3: Structural model including the opening and propagation of cracks considering the fiber reinforcement.

A new analytical model for the pullout behavior of single fibers embedded in a concrete matrix, for different configurations of fiber type, matrix strength and embedment condition is developed. An interface law is proposed for the frictional behavior between fiber and matrix. In the case of inclined fibers, also the plastic deformation of fiber and the local damage of concrete are considered. For hooked-end fibers, the anchorage effect due to the deformed shape of fiber ends is taken into account in the formulation. By combining these submodels, the pullout response of single fibers embedded in a concrete matrix can be predicted. Meanwhile, numerical simulations of pullout tests are performed to obtain insight in the local fiber–concrete interactions and to provide supporting information for the analytical modeling. The single fiber pullout model is successfully validated by means of representative experimental results (Breitenbücher et al. 2014, Zhan and Meschke 2014).

Based on the single fiber pullout model, the bridging effect of an opening crack in a specific SFRC composite is computed by the integration of the pullout resistance of all the fibers intercepting the crack. The anisotropic fiber orientation as a consequence of the casting process and the boundary effect are taken into account. It is demonstrated by selected validation analyses, that this approach is able to reflect the directional preference of fibers in SFRC members and the influence on the post-cracking ductility. The crack bridging response is predicted more realistically as compared to the models where an isotropic fiber orientation is assumed and the boundary effect is neglected. For the purpose of implementing the model in a finite element program, an analytical parameterized surrogate function form is generated based on the numerically obtained traction–separation relations for the specific fiber cocktail (Zhan and Meschke 2016).

At the level of structural finite element analysis, interface solid elements are supplied with the crack bridging model as traction–separation law to represent the opening and propagation of cracks. One advantage of this method is that it does not require specific crack path tracking techniques. The implicit/explicit scheme is implemented for the integration of the (highly nonlinear) post-cracking behavior of interfaces. This strategy has proven to be very efficient as in each load increment only a linear system has to be solved without iteration (Zhan and Meschke 2016). Furthermore, for reducing the overall computational cost, an adaptive mesh-processing technique is implemented, which allows to progressively place the interface solid elements as the crack fronts advance during the structural simulation.

After the validation of FRC model, the potential of employing the complete multilevel computational framework, as a “virtual laboratory” for the simulation and prediction of the load-bearing capacity and failure behavior of tunnel lining segments made of different SFRC composites and subjected to different loading conditions, is demonstrated via selected numerical examples.
1.3. ORGANIZATION OF PRESENT WORK

**PART I**

Experiments
- Hooked-end FRC
- Straight FRC
- Plain Concrete

**Level 1: Single Fiber Pullout Model**

**Level 2: Crack Bridging Model**

**Level 3: Finite Element Model using Interface Solid Elements**

**Application to Simulation of Tunnel Lining Segments**

**Figure 1.3:** Multilevel modeling scheme for FRC materials and structures.
The research is conducted in the framework of subproject B2 “Damage Analyses and Concepts for Damage Tolerant Tunnel Linings” of the Collaborative Research Center (SFB) 837 “Interaction Modeling in Mechanized Tunneling”, funded by the German Research Foundation (DFG) (COLLABORATIVE RESEARCH CENTER 837 2016).

1.3.2 Composition of thesis

The main body of thesis consists of two parts:

Part I: Mechanical modeling of fiber-reinforced concrete materials
- In Chapter 2, an introduction of fiber-reinforced concrete is provided.
- In Chapter 3 and 4, the analytical models predicting the pullout force–displacement relations of single straight and hooked-end steel fibers embedded in concrete matrix are described, respectively.
- The crack bridging model, making use of the single fiber pullout model to generate the traction–separation law for an opening crack within a specific SFRC composite, is presented in Chapter 5.

Part II: Finite element simulation of fiber-reinforced concrete structures
- Chapter 6 contains a brief review of the numerical models for concrete cracking problems; the author’s experience in the embedded strong discontinuity approach is shortly described.
- In Chapter 7, the finite element method using interface solid elements to represent the cracking phenomena in concrete structures is presented. The features of this model is discussed based on a few numerical examples of plain concrete structures.
- The validation of the multilevel SFRC model is demonstrated via the re-analysis of selected laboratory tests on the structures made of SFRC and is presented in Chapter 8.
- The complete and validated multilevel model is finally applied to the numerical simulation of the failure behavior of tunnel lining segments (Chapter 9).

The thesis is closed with concluding remarks and future perspectives (Chapter 10).
Part I

ANALYSES OF MECHANICAL PROPERTIES OF FIBER-REINFORCED CONCRETE MATERIALS
In this chapter, general information concerning fiber-reinforced concrete (FRC) are provided, including the development and unique advantages of FRC, a state-of-the-art overview of the major mechanical properties involved in the failure analyses of fiber-reinforced concrete materials and structures. The efforts made, including the laboratory tests, numerical simulations and analytical methods, on single fiber pullout mechanisms, tensile, compressive and shear responses of FRC composite materials are mentioned briefly.
2.1 Fiber-reinforced concrete (FRC)

Plain concrete

Concrete, a heterogeneous material mainly made of dry cement and water mixed with sand and coarse aggregates (Fig. 2.1), is a stone-like hard material after the ingredients are well bonded during the chemical reactions as well as physical processes among them. Concrete has become one of the most important construction materials; however, it is characterized by a low strength ($f_t$) under tensile stresses, which is typically 1/10 of the strength ($f_c$) under compression, and quasi-brittle behavior after cracking occurs (Fig. 2.1).

![Concrete Specimen and Stress-Strain Response](image)

**Figure 2.1:** Plain concrete: photo of the cross section of concrete specimen (Mehta and Monteiro 2006) and sketch of the typical stress–strain responses of concrete under tensile and compressive loads.

Therefore, plain concrete needs to be strengthened especially under tension-dominant loading conditions. A common way to enhance the ductility of the material is to add steel bars as reinforcement in the regions of structures subjected to tensile stresses. However, since steel-bar reinforcement is generally continuous and is regularly placed at specific locations in the structure, opening cracks can still develop in local areas such as the concrete cover and the corners of structure and considerably limit the durability of traditional reinforced concrete (RC) structures. In rigorous environments, such as in tunnel linings, where a limitation of crack widths and the demand for material ductility to control the localized damages play an important role, fiber-reinforced concrete becomes particularly attractive due to its high post-cracking ductility, or even strain-hardening behavior, accompanied with distributed cracks at small crack widths.

Development of FRC

Even in ancient times, natural fibers were already used to enhance the brittle building materials; for example, straw was used to reinforce the bricks made of clay (ACI Committee-544 2001). The
development of modern FRC can be traced back to the 1960s, when the large potential of adding steel fibers to reduce the brittleness of concrete has been recognized (Romualdi and Batson 1963, Romualdi and Mandel 1964). Subsequent research has expanded and led to FRC composite materials characterized by various types of fiber reinforcement using different fiber materials, size, shapes, surface treatments and fiber contents, etc., in association with different classes of concrete, eventually leading to high performance materials such as high performance fiber-reinforced cement composites (Naaman and Reinhardt 1996) and engineered cementitious composites (Li 1998), with or without the presence of conventional rebars (Moreno et al. 2014).

Figure 2.2: Comparison of cracking phenomena in (a) traditional steel-bar reinforced concrete and (b) steel-bar reinforced concrete with additional fiber reinforcement (Li 2003).

Fiber-reinforced concrete, characterized by enhanced ductility in tension and compression, impact and fatigue resistance, durability due to crack-width control in a small length-scale, has been recognized as a competitive construction material in various types of engineering structures (ACI Committee-544 2001, Di Prisco et al. 2009). In addition to the improved mechanical properties, FRC possesses several important advantages: The structures made of FRC can be much less vulnerable to corrosion than those made of traditional reinforced concrete (RC); FRC is very flexible in terms of fabrication and can be placed in the regions where conventional rebars are not feasible (Fig. 2.2); FRC can be an economic and useful engineering material for either providing additional enhancement or fully replacing the conventional RC material. FRC has already been widely used as cast-in-place material, e.g. for hydraulic facilities (ICOLD Bulletin 1989), highway or airport pavement (Wu and Jones 1987), bridge deck and surface repair (Kunieda and Rokugo 2006), or as shotcrete for slope stabilization and ground support, structure repair and strengthening (Melamed 1985). Alternatively, FRC can be used in quality-demanding precast construction members such as tunnel lining segments (Hansel and Guirguis 2011, Kasper et al. 2008).

Material of fiber

Various raw materials are used for the manufacturing of fibers, leading to single fibers as well as fiber textiles characterized by distinguished properties such as stiffness and strength (ACI Committee-544 2001, Susetyo 2009) (see Table 2.1 for an overview of the commonly available fibers).
Table 2.1: Overview of the properties of commonly available fibers (Susetyo 2009).

<table>
<thead>
<tr>
<th>Material</th>
<th>Diameter (mm)</th>
<th>Density (g/mm³)</th>
<th>Young’s modulus (MPa)</th>
<th>Tensile strength (MPa)</th>
<th>Failure strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.01 - 1</td>
<td>7.8</td>
<td>160,000 - 200,000</td>
<td>345 - 2,070</td>
<td>3.0 - 3.5</td>
</tr>
<tr>
<td>Synthetic</td>
<td>0.001 - 4</td>
<td>0.9 - 1.9</td>
<td>3,450 - 380,000</td>
<td>200 - 3,620</td>
<td>0.5 - 80</td>
</tr>
<tr>
<td>Glass</td>
<td>0.010 - 0.012</td>
<td>2.5 - 2.7</td>
<td>72,000 - 80,000</td>
<td>2,480 - 3,450</td>
<td>3.6 - 4.8</td>
</tr>
<tr>
<td>Natural</td>
<td>0.02 - 0.4</td>
<td>1.1 - 1.5</td>
<td>10,000 - 40,000</td>
<td>115 - 905</td>
<td>3 - 25</td>
</tr>
</tbody>
</table>

Figure 2.3: Straight and hooked-end steel fibers characterized by different lengths and diameters.

Steel fibers are often made through cold wire cutting and can be subjected to further processes, resulting in fibers characterized by different shapes and geometries (Fig. 2.3). As summarized in ACI COMMITTEE-544 (2001), steel-fiber–reinforced concrete features especially the improved flexural toughness, impact resistance, and flexural fatigue endurance. In addition, steel fibers are protected from corrosion by the alkaline environment of the cementitious matrix. In Europe, research as well as application have already approved the validity of using steel fibers to partially or even completely replace the traditional reinforcement in, e.g., tunnel linings (Chiaia et al. 2009, Gettu et al. 2004, Greenhalgh 2010, Kasper et al. 2008, Nehdi et al. 2015).

Synthetic fibers are developed from the petrochemical and textile industries which lead to a variety of fiber forms characterized by rather different physical properties (Table 2.1). The fiber types that have been used in concrete-based materials include acrylic, aramid, carbon, nylon, polyester, polyethylene and polypropylene (ACI COMMITTEE-544 2001). Synthetic fibers are experiencing rapid and continuous development; they can be used in a fairly wide range: for example, the engineered cementitious composites are generally made of polyvinyl alcohol fibers (Li 2008); the expensive carbon fibers are found to be very effective in increasing the tensile strength and stiffness of composites (Toutanj et al. 1994); the macro synthetic fibers are used in shotcrete, allowing less wearing on pumps compared to the case of steel fibers (Yin et al. 2015). Nevertheless, one of the main problems that limits the use of synthetic fibers is the fact that the fibers are generally vulnerable to high temperature (Burratti and Mazzotti 2015).

Glass-fiber–reinforced concrete is an excellent material system producing significant weight saving in non-structural architectural cladding panels and other similar concrete products. Natural fibers
from plant, either unprocessed or processed, can be obtained at low levels of cost and energy consumption in local conditions (ACI COMMITTEE-544 2001). Natural fibers are used for low-cost construction especially in developing countries; nevertheless, critical aspects that limit their industrial application include the low durability and moisture-sensitivity of natural fibers (SAVASTANO JR. ET AL. 2001).

**Geometry of fiber**

Industrial fibers are available in various shapes, in addition to the basic straight, smooth fiber with circular cross section as illustrated in Fig. 2.4. Additional surface treatment or fabrication process are employed in order to obtain improved bonding properties between the fiber and matrix. For example, the whole fiber surface can be roughened via knurling or etching processes, the shape of fiber can be modified with deformed ends, the whole fiber can be crimped or twisted and characterized by a non-circular cross section (NAAMAN 2003). In general, despite the extra cost, these additional crafting processes result in higher bonding strength between the fibers and the matrix.

![Different shapes of fiber.](image)

**2.2 Single fiber pullout behavior**

As a fundamental topic in this research, the pullout behavior of single fibers embedded in the concrete matrix has gained much attention. A large number of laboratory tests have been performed, providing rich data base for various concrete–fiber configurations, which serve as the foundation for analytical and numerical models.

**Laboratory tests**

In general, laboratory tests are performed for obtaining the basic information of the pullout behavior of single fiber embedded in concrete matrix. As illustrated in Fig. 2.5, a fiber is placed in the fresh concrete mixture with the desired embedment length ($L_e$) and inclination angle ($\theta$), after a standard curing period the specimen is placed on a testing device where the concrete matrix is fixed and the fiber is pulled out of the matrix. During the pullout procedure, the pullout force ($F$) and pullout displacement ($u$) are measured.
The simplest case of pullout is a straight fiber embedded in the matrix without any inclination angle w.r.t. the loading direction ($\theta = 0^\circ$), as shown in Fig. 2.5(a). This test is usually conducted for the determination of interfacial friction–slip mechanisms, more specifically, the friction stress ($\tau$) versus the interface slip ($s$) relationship, in association with important interfacial bonding parameters such as the bond strength and the pullout work (NAAMAN AND NAJM 1991).

For the inclined fiber pullout tests, different laboratory setups are considered: As sketched in Fig. 2.5(b), the fiber can be bent in advance, allowing the extruded segment to be clamped by the testing machine (BREITENBÜCHER AND SONG 2014); this pre-bending introduces plastic deformation and can influence the pullout behavior of especially a steel fiber. Alternatively, the fiber can be directly cast in two matrix specimens [Fig. 2.5(c)], allowing better simulation of the real situation of a fiber in composite materials (LEUNG AND SHAPIRO 1999).

The results obtained from single fiber pullout experiments can be remarkably different. As qualitatively sketched in Fig. 2.6, the pullout force–displacement relation is influenced by various parameters:
• The fiber shape has a direct impact on the pullout responses [Fig. 2.6(a)]. In general, the deformed shapes of a steel fiber can contribute extra pullout forces. Nevertheless, in some cases, the excessively enhanced bonding between fiber and matrix may exceed the strength of fiber itself and cause the rupture of fiber during the pullout, which is not an optimal situation considering the total energy dissipation during the complete pullout procedure (Breitenbücher and Song 2014, Feyerabend 1995).

• Larger values of embedment length of fiber are found to be more effective in increasing the pullout forces contributed by a straight fiber than by a deformed one [Fig. 2.6(b)]. This is due to the fact that for a straight fiber, the interface friction along the fiber axis plays the main role, while for a deformed fiber, the elastic-plastic deformation of fiber material contributes a large portion to the pullout resistant forces (Robins et al. 2002).

• The inclination of fiber with respect to the loading direction can change the pullout responses considerably [Fig. 2.6(c)]. It is noticed in the experiments that the fiber inclination does not necessarily harm the pullout behavior; in fact, for a straight or hooked-end steel fiber, a small inclination angle (approximately 30°) is favorable (Breitenbücher and Song 2014, Leung and Shapiro 1999).

• Other important factors affecting the pullout behavior include the fiber size and aspect ratio, the surface roughness, the quality of fiber–matrix interface, etc.

Laboratory tests, although revealing the real behavior and providing the first-hand data of fiber pullout, are generally expensive and time consuming (taking e.g. a few weeks).

Numerical simulations

Virtual experiments, i.e. numerical simulations performed using finite element (FE) software such as ABAQUS, are employed to generate the required pullout load–displacement relations as well as to provide the insight into the pullout mechanisms which are hardly visible in the laboratory condition (Breitenbücher et al. 2014, Chen et al. 2009, Zhan and Meschke 2014). For the FE-analysis of fiber pullout, appropriate models for the concrete matrix and the fiber materials should be selected and the nonlinear material parameters including the softening/hardening behavior in tension and in compression should be assigned; in addition, the fiber–matrix interfacial bond–slip properties should be defined. The results of pullout responses can be obtained at the free end of fiber; the contour plots of concrete damage as well as steel plastic deformation contain valuable information for the understanding of fiber–matrix interactions and pullout mechanisms (Fig. 2.7).

In comparison to laboratory tests, FE simulations can be performed on a personal computer, allowing much faster analysis of single fiber pullout behavior (taking e.g. several hours).

Analytical models

In an analytical model for single fiber pullout, it is essential to take into account the major mechanisms involved in different pullout configurations.

As illustrated in Fig. 2.8(a), for a straight fiber without inclination with respect to the loading direction, analytical methods typically treat the fiber as a cylinder and focus on the interface be-
Figure 2.7: Results of finite element analysis of hooked-end fiber pullout: (a) force–displacement relation; deformation and contour plot of (b) VON MISES stress, (c) tensile damage and (d) compressive damage values.

behavior, characterized by the frictional slip mechanism along the interface between fiber and matrix (LAWRENCE 1972, LIN AND LI 1997, NAAMAN ET AL. 1991).

As shown in Fig. 2.8(b), modeling the pullout behavior of inclined fibers involves additional complexities connected with the lateral pressure on interface, inelastic deformations of fiber and localized damages of matrix [see e.g. LEUNG AND SHAPIRO (1999), LEUNG AND YBANEZ (1997)]. An extensive literature survey on the pullout behavior of inclined fibers is contained in LARANJEIRA ET AL. (2010), where also an analytical model predicting the pullout load–displacement relations of inclined straight fibers is presented. This model is formulated by defining a few key states during the pullout procedure, analyzing the pullout displacement and the corresponding force at each key state and sequentially connecting them as a multilinear relationship.

During the pullout of a hooked-end steel fiber [Fig. 2.8(c)], in contrast to the case of a straight fiber, where the interfacial behavior plays the main role, the resistance of the hook to straightening considerably contributes to the total pullout force (NAAMAN AND NAJM 1991, ROBINS ET AL. 2002). Only a few analytical models describing the hooked-end fiber pullout without inclination are presented in the open literature (ALWAN ET AL. 1999, CHANVILLARD 1999, CUNHA ET AL. 2010, GYSEL 1999). The inclined situation, with the loading direction deviating from the fiber direction, seems to be only considered in LARANJEIRA ET AL. (2010) and SOETENS ET AL. (2013).

In a recent article, the author of thesis presented an analytical model for the pullout behavior
2.3 Elastic properties of FRC composite

In practical situations where the content of fibers are low (merely a few percent or less), the addition of fibers into concrete matrix does not change the elastic behavior of FRC composite significantly. Experimental investigations [e.g. (Williamson 1974)] as well as analytical and numerical methods such as the micromechanics-based models employing the Mori-Tanaka homogenization scheme or multiscale oriented models have been used to evaluate the elastic properties (Dutra et al. 2010, Teng et al. 2004, Zhang et al. 2015); these efforts reveal that in most engineering situations, the elastic moduli of FRC are only marginally higher than plain concrete without any fiber reinforcement.

2.4 Post-elastic behavior of FRC under tension

As mentioned above, plain concrete is characterized by low tensile strength ($f_t$) and post-cracking toughness (energy absorption); therefore, the main purpose of adding fibers into plain concrete is to enhance the post-cracking ductility of material under tension. To this end, the investigation of the tensile behavior of FRC has become one of the central tasks.

With different levels of fiber “reinforcing index” (Fanella and Naaman 1985) (which indicates the overall bonding effect between fibers and matrix that is dependent on several parameters such as the concrete class, fiber material, shape and geometry, and fiber content) the FRC composite material can exhibit considerably different macroscopic behavior [see Fig. 2.9 and references such as Naaman and Reinhardt (1996) and Di Prisco et al. (2009)].
Figure 2.9: Influence of fiber content on the failure of FRC in tension.

- Plain concrete (PC): Without any reinforcement, plain concrete shows the well-recognized quasi-brittle response after the tensile stress ($\sigma$) reaches the tensile strength ($f_t$). Micro cracks initiate at the inherent flaws and weak bonds on the aggregate surfaces and quickly localize into a major crack that penetrates the material body perpendicularly to the tensile direction. Afterwards, the localized crack opens rapidly, accompanied by the fast drop of residual stress; consequently, the plain concrete material fails in a rather brittle manner.

- Normal fiber-reinforced concrete (N-FRC): With the addition of fibers, the post-cracking behavior of material becomes remarkably more ductile. After the localized crack is developed, the fibers intercepting this crack become active in providing resistant forces across the crack. With the opening of crack, the fibers are progressively pullout out of the matrix; the contribution of fibers significantly elevates the level of residual crack bridging stress.

- High-performance fiber-reinforced concrete (H-FRC): If the bonding effect of fibers exceeds a certain threshold (more specifically, the peak of crack bridging stress is larger than $f_t$), then the opening of crack is retarded by the fiber bridging effect. The stresses carried by the fibers at the location of crack plane are transferred to the concrete matrix via the fiber–matrix interactions; this stress transferring effect accumulates with the distance from crack; consequently, at a certain location, the stress level in the matrix exceeds the tensile strength, causing the initiation of new cracks. This process, known as the “multiple-cracking” phenomena, accompanied with the strain hardening behavior of the material from a macroscopic point of view, repeats until cracks are “saturated” in the composite (specifically tailored H-FRC can exhibit such high ductility as metallic materials do (MAALEJ ET AL. 1995)). Finally, after the stress level in one of the parallel cracks exceeds the maximum stress that the crack-bridging fibers can carry, the opening of this crack becomes inevitable and the composite material starts to fail due to the opening of the major crack (FANTILLI ET AL. 2009).

Laboratory tests

Laboratory tests on FRC specimen subjected to tensile loading have been frequently employed to investigate the post-cracking ductility of FRC composite. Among the various types of specimen, the
“dog-bone”-shaped and the notched specimens are frequently used (Fig. 2.10):

- A dog-bone specimen is characterized by the enlarged ends which are well attached to the test machine so that the tensile forces are transferred and a generally homogeneous tensile stress state is obtained in the central area, leading to cracking processes that are limited in this region [see e.g. (Liu 2003, Susetyo 2009, Suwannakarn 2009)]. Depending on the level of fiber reinforcing index, different crack patterns can be obtained: For normal FRC, a major crack perpendicular to the tension direction should develop at an arbitrary position and penetrate the specimen. For high-performance FRC, multiple cracks should be generated before one of them grows into the major crack.

- A notched specimen, usually with a regular (e.g. circular or rectangular) cross section, is attached to the testing machine (Barragán et al. 2003, Laranjeira 2010, Pereira et al. 2012, RILEM Technical Committees 2001, Rossi 1997). The notch (usually produced via saw-cut) is designed to enforce the crack to initiate at the specific location, and to avoid the failure of experiments due to unexpected damages such as the detachment of specimen to the machine. For N-FRC, a major crack emanates from the notch; in this case, the crack bridging effect (i.e. the traction–separation relation for this crack) can be directly measured. In the situation of H-FRC, due to the enhanced fracture toughness, multiple cracks can be obtained in the vicinity of notched cross section.

Note that in general, the crack bridging effect of an opening crack, i.e. the relation between the stress across the crack and the crack opening displacement, which plays a crucial role to link the individual fiber pullout response with the structural behavior of FRC, can be directly measured for N-FRC. However, in the case of H-FRC, since multiple cracks can occur in the range of measuring...
device (e.g. strain gauge), the direct measurement of opening displacement for a specific crack can be a challenging task.

**Analytical methods**

The post-cracking ductile behavior of FRC can also be indirectly obtained by means of *inverse analysis* based on standard three-point bending tests on notched beams, where the linear distribution of crack opening width along the vertical crack path is assumed (Kooiman et al. 2000, Soetens and Matthys 2014, Tlemat et al. 2006).

The analytical approaches for directly computing the crack bridging effect without conducting laboratory tests are proposed as well, where the single fiber pullout model is a necessary ingredient, since the bridging stress has to be computed via the summation of individual fiber pullout responses. The integration can be performed according to e.g. the analytical form used in Wang et al. (1989). Note that usually the isotropic fiber orientation is assumed; i.e. the fibers are considered to be uniformly oriented in all the spatial directions (Lee et al. 2011, Li et al. 1991, Lin and Li 1997). This assumption is revised in the present thesis by taking the anisotropic fiber orientation into consideration (see Chapter 5).

![Crack bridging responses for opening cracks in different FRC composites.](image)

*Figure 2.11:* Crack bridging responses for opening cracks in different FRC composites.

It should be pointed out that, the obtained crack bridging effect, i.e. the \( t - w \) relationship as illustrated in Fig. 2.11, corresponds to the behavior of one single crack in the composite material. Different levels of crack bridging stress can lead to distinct macroscopic behavior of the composite material, as mentioned above. For normal FRC where only one major crack opens and leads to the failure of material, the macroscopic behavior of the composite (i.e. the \( \sigma - \varepsilon \) relation), exhibits a rather similar shape as the crack bridging law (\( t - w \) relation), where the crack opening displacement can be transformed to the cracking strain according to the *crack band theory* (Bazant and Oh 1983). For high performance FRC, as a consequence of the super-critical crack bridging effect, the \( \sigma - \varepsilon \) curve continues to ascend after exceeding the elastic limit stress \( f_t \). During the strain-hardening process, the macroscopic strain can reach a very high level, due to the opening of multiple cracks perpendicular to the loading direction (Fig. 2.9). Attempts have been made to investigate the multiple cracking phenomena which are generally understood to be, in analogy to the situation of
steel-bar reinforced concrete, controlled by the stress transfer mechanisms between the fibers and matrix (Fantilli et al. 2009, Kabele 2007, Li and Leung 1992). However, in the author’s opinion, the existing models need to be enhanced to reflect particularly the situation of FRC made of deformed fibers such as hooked-end steel fibers. As for the determination of the constitutive behavior of high performance FRC, averaging techniques including the micromechanics-oriented homogenization schemes can be used to obtain the relation between the macroscopic stresses and strains [see e.g. Kabele (2007) and Jun and Mechtcherine (2010)]. In order to incorporate the post-cracking behavior of FRC in a structural computational model, either the traction–separation laws or the stress–strain relations can be used, according to the specific assumptions regarding the representation of cracks (see Chapter 6).

2.5 Compressive behavior of FRC composite

As mentioned previously, concrete materials are much more vulnerable to fracturing in tension than to crushing under compression; nevertheless, the compressive behavior of FRC is an important mechanical property of the composite material as well. Similar to the case of plain concrete, the uniaxial compressive test on FRC specimens is frequently selected to obtain the nonlinear stress-strain relation and to determine the impact of different parameters including the fiber material and geometry, fiber content, concrete strength, etc. (Fig. 2.12). It is generally observed that, without any fiber reinforcement, plain concrete specimens fail rapidly (even in an “explosive” manner), showing one or very few macro cracks through the specimen. Due to the presence of fibers, the post-peak response of FRC becomes more ductile, accompanied with the development of a number of distributed cracks intercepted by fibers. The damage mechanisms of FRC under compression involve the splitting and crushing of concrete matrix, the slipping, shear deformation and even buckling of fibers (Budiansky and Fleck 1992, Hsu and Hsu 1994, Niu and Talreja 2000). From the experimental results, it is generally noticed that the addition of fibers into the composite enhances the material ductility by means of slightly increasing the compressive strength $f_c$ and considerably reducing the descending rate in the softening phase, which leads to a significantly enhanced energy absorption of material (ACI Committee-544 2001).

**Figure 2.12:** Compressive behavior of different FRC composites.
Based on the experimental results, various function forms, characterized by different sets of parameters to be calibrated with the test data, are proposed to replicate the hardening-softening responses of FRC (Barros and Figueirias 1999, Fanella and Naaman 1985, Nataraja et al. 1999, Song and Hwang 2004, Thomas and Ramaswamy 2007, Xu and Cai 2010). However, a generally accepted analytical model does not yet exist: based on the comparison among several publications containing experimental data in association with analytical forms for the stress–strain relation under compression, Bencardino et al. (2008) observe that, in every article, the proposed function form generally shows good agreement with the experimental results obtained from the tests performed by the same authors; however, the analytical form used in one paper may not fit the test results reported by other researchers. This is probably due to the fact that different test configurations are considered in different publications; particularly, different fiber–matrix bonding mechanisms (dependent on several factors such as the fiber material, size and shape, fiber content and distribution in the composite) have direct influence on the post-elastic responses of FRC, and not all these factors have been adequately accounted for in any individual model.

2.6 Shear behavior of FRC

Shearing in FRC involves concrete damage, fiber pullout, fiber dowel effect, fiber bending and buckling, etc. There is only limited research focusing on the pure Mode-II fracture (shear) behavior of FRC. In Boulekbache et al. (2012), a “push-off” test is performed on a deep-notched specimen; the test provides the shear stress vs. shear displacement relation of a macro crack in FRC [Fig. 2.13(a)]. In the “shear-panel” tests conducted by Susetyo et al. (2011, 2013), the average constitutive behavior under shear is determined [Fig. 2.13(b)]; numerical simulation is also performed to replicate the tests, by modifying the tensile softening law of FRC.

Figure 2.13: Examples of test on the shear behavior of FRC: (a) “push-off” test (Boulekbache et al. 2012); (b) shear-panel test (Susetyo 2009).
In the context of multiscale oriented computational analyses of fiber-reinforced concrete (FRC) structures, the modeling of single fiber pullout behavior represents the basic constituent to provide traction–displacement relations to be used for the modeling of FRC on a macroscopic scale. This essential ingredient needs to be formulated such that it requires only minimal computational effort. To this end, an analytical model for the pullout behavior of single fibers embedded in a concrete matrix for various configurations of fiber type, matrix strength and embedment condition is proposed. An interface law is developed for the frictional behavior between fiber and matrix. In the case of inclined fibers, also the plastic deformation of fiber and the local damage of concrete are considered. For hooked-end fibers, the anchorage effect due to the deformed topology of the fiber ends is taken into account in the formulation. By combining these sub-models, the pullout response of single fibers embedded in a concrete matrix can be predicted. In addition, numerical simulations of pullout tests are performed to obtain an insight into the local fiber-concrete interactions and to provide supporting information for the analytical modeling. In this chapter, the proposed model for straight fiber pullout is presented. The hooked-end fiber pullout model will be described in the next chapter.
Symbols used in this chapter

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>$A_f$</td>
<td>cross-section area of fiber</td>
<td>mm$^2$</td>
</tr>
<tr>
<td>$A_{sp}$</td>
<td>lateral area of spalling cone</td>
<td>mm$^2$</td>
</tr>
<tr>
<td>$c_f$</td>
<td>volume content of fiber</td>
<td>mm$^3$/mm$^3$</td>
</tr>
<tr>
<td>$d_f$</td>
<td>diameter of fiber</td>
<td>mm</td>
</tr>
<tr>
<td>$\delta$</td>
<td>spalling depth of concrete matrix</td>
<td>mm</td>
</tr>
<tr>
<td>$E_f$</td>
<td>Young’s modulus of fiber</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Young’s modulus of matrix</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>strain along fiber</td>
<td>mm/mm</td>
</tr>
<tr>
<td>$F$</td>
<td>pullout force at the free-end of fiber</td>
<td>N</td>
</tr>
<tr>
<td>$F_0$</td>
<td>pullout force at the end of bonded state</td>
<td>N</td>
</tr>
<tr>
<td>$F_A$</td>
<td>longitudinal force at Point A (fiber segment A-B)</td>
<td>N</td>
</tr>
<tr>
<td>$F_{max}$</td>
<td>maximum pullout force</td>
<td>N</td>
</tr>
<tr>
<td>$f_t$</td>
<td>tensile strength of concrete</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$f_c$</td>
<td>compressive strength of concrete</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$G$</td>
<td>relative bond modulus of fiber–matrix interface</td>
<td>N/mm$^3$</td>
</tr>
<tr>
<td>$G_m$</td>
<td>shear modulus of matrix</td>
<td>N/mm$^2$</td>
</tr>
<tr>
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<td>shear strain</td>
<td>mm/mm</td>
</tr>
<tr>
<td>$\gamma_R$</td>
<td>shear strain at distance $R$ from the fiber axis</td>
<td>mm/mm</td>
</tr>
<tr>
<td>$k$</td>
<td>foundation stiffness (fiber segment B-C)</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>parameter related to fiber–matrix bond</td>
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<tr>
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<td>length of fiber segment A-B</td>
<td>mm</td>
</tr>
<tr>
<td>$L_{BC}$</td>
<td>length of fiber segment B-C</td>
<td>mm</td>
</tr>
<tr>
<td>$L_b$</td>
<td>bonded length of fiber</td>
<td>mm</td>
</tr>
<tr>
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<td>debonding length of fiber</td>
<td>mm</td>
</tr>
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<tr>
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<td>full length of fiber</td>
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</tr>
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<td>$M_B$</td>
<td>bending moment at Point B</td>
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</tr>
<tr>
<td>$\mu$</td>
<td>COULOMB’s friction coefficient</td>
<td>N/N</td>
</tr>
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<td>$\nu_f$</td>
<td>POISSON’s ratio of fiber</td>
<td>-</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>POISSON’s ratio of matrix</td>
<td>-</td>
</tr>
<tr>
<td>$p$</td>
<td>distributed foundation reaction force (fiber segment B-C)</td>
<td>N/mm</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>current inclination angle of fiber segment (fiber segment A-B)</td>
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<tr>
<td>$\psi$</td>
<td>rotation angle of fiber segment A-B with respect to Point B</td>
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<td>$R$</td>
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<tr>
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<td>radius of matrix</td>
<td>mm</td>
</tr>
<tr>
<td>$r_f$</td>
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<tr>
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</tr>
<tr>
<td>$s_{ref}$</td>
<td>reference value of interfacial slip for the sliding phase</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------</td>
<td>--------</td>
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<tr>
<td>σ</td>
<td>tensile stress along fiber</td>
<td>N/mm²</td>
</tr>
<tr>
<td>σ_y</td>
<td>yield stress of steel fiber</td>
<td>N/mm²</td>
</tr>
<tr>
<td>̄σ</td>
<td>average stress in fiber</td>
<td>N/mm²</td>
</tr>
<tr>
<td>T</td>
<td>longitudinal force along fiber</td>
<td>N</td>
</tr>
<tr>
<td>T_b</td>
<td>normal force at the end of bonded fiber section</td>
<td>N</td>
</tr>
<tr>
<td>T_B</td>
<td>longitudinal force at Point B</td>
<td>N</td>
</tr>
<tr>
<td>t</td>
<td>distributed frictional force (fiber segment B-C)</td>
<td>N/mm</td>
</tr>
<tr>
<td>τ</td>
<td>interfacial shear stress</td>
<td>N/mm²</td>
</tr>
<tr>
<td>τ_max</td>
<td>bond strength of the interface</td>
<td>N/mm²</td>
</tr>
<tr>
<td>τ₀</td>
<td>asymptotic frictional stress</td>
<td>N/mm²</td>
</tr>
<tr>
<td>τ_R</td>
<td>frictional stress at distance R from the fiber axis</td>
<td>N/mm²</td>
</tr>
<tr>
<td>θ</td>
<td>inclination angle of fiber</td>
<td>°</td>
</tr>
<tr>
<td>u</td>
<td>pullout displacement at the free-end of fiber</td>
<td>mm</td>
</tr>
<tr>
<td>u₀</td>
<td>pullout displacement at the end of bonded state</td>
<td>mm</td>
</tr>
<tr>
<td>u₁</td>
<td>pullout displacement at the end of debonding stage</td>
<td>mm</td>
</tr>
<tr>
<td>u_b</td>
<td>pullout displacement at the end of bonded fiber section</td>
<td>mm</td>
</tr>
<tr>
<td>u_x</td>
<td>pullout displacement in X-direction</td>
<td>mm</td>
</tr>
<tr>
<td>V_A</td>
<td>transverse force at Point A</td>
<td>N</td>
</tr>
<tr>
<td>V_B</td>
<td>transverse force at Point B</td>
<td>N</td>
</tr>
<tr>
<td>ξ</td>
<td>local coordinate along fiber</td>
<td>-</td>
</tr>
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</table>
3.1 Pullout without fiber inclination

3.1.1 Bond–slip behavior along fiber–matrix interface

As the basic case, straight fiber pullout without inclination with respect to the loading direction has been investigated by a number of researchers, either by means of laboratory tests or by generating analytical methods [see e.g. LI AND STANG (1997) for a review]. It is generally accepted that the pullout procedure can be divided into three stages (Fig. 3.1):

- Bonded state: The surface of the fiber is well connected to the surrounding matrix [Fig. 3.1(a)].
- Debonding stage: Due to the increasing pullout force, the adhesion along the fiber–matrix interface is partially damaged [Fig. 3.1(b)].
- Pulling-out phase: With the interfacial debonding fully developed along the axis, the fiber starts to slide within the fiber channel, with frictional stresses acting on the interface [Fig. 3.1(c)].

According to the shear-lag concept (CHEN ET AL. 2009, LAWRENCE 1972), we consider a fiber with diameter $d_f$, radius $r_f$, cross-sectional area $A_f$ and the elastic modulus $E_f$. A section of the fiber with length $L_e$ is embedded in the concrete matrix characterized by the elastic modulus $E_m$ and the Poisson’s ratio $\nu_m$. The free end of the fiber is subjected to a pullout force $F$ and the corresponding displacement $u$.

$$\tau, F, u$$

Figure 3.1: Straight fiber pullout without inclination: (a) fully bonded state, (b) debonding stage and (c) sliding phase.

Figure 3.2: Fiber pullout without inclination: proposed idealized interface law indicating three stages during the pullout process.
Different types of interfacial friction laws are used in e.g. NAAMAN ET AL. (1991), LIN AND LI (1997), and FANTILLI AND VALLINI (2007). In the present model, the interface law is proposed as follows (Fig. 3.2):

\[
\tau(s) = \begin{cases} 
G_s & \text{if } s \leq s_0, \text{ (fully bonded stage);} \\
\tau_{\text{max}} & \text{if } s_0 < s \leq s_1, \text{ (debonding stage);} \\
\tau_0 + (\tau_{\text{max}} - \tau_0) \exp[(s_1 - s)/s_{\text{ref}}] & \text{if } s > s_1, \text{ (sliding stage).}
\end{cases}
\] (3.1)

Here, \(\tau\) denotes the interfacial shear stress; \(s\) is the slip, i.e. the relative displacement defined as the displacement at a point on the fiber axis with respect to the boundary of the matrix; \(\tau_{\text{max}}\) is the bond strength of the interface and \(\tau_0\) is the asymptotic (residual) value of the frictional stress; \(s_0 = \tau_{\text{max}}/G\) represents the limit value of bonded interface, \(s_1\) is the slip level at the beginning of the sliding stage and \(s_{\text{ref}}\) is the “reference” slip, a parameter controlling the descending branch of the curve.

By considering the interface law proposed above and solving the equations concerning the force equilibrium

\[
dT(\xi) = \pi d f \tau(\xi) \, d\xi \tag{3.2}
\]

and strain compatibility along the fiber axis (denoted by the local coordinate \(\xi\))

\[
\frac{T(\xi)}{A_f E_f} = \varepsilon(\xi) = \frac{d s(\xi)}{d \xi}, \tag{3.3}
\]

the axial force \(T\), the frictional stress \(\tau\) on the interface and the relative displacement \(s\) along the fiber are calculated. The free-end load–displacement relation \((F - u)\) is obtained for the three pullout stages as follows.

**Bonded state**

![Figure 3.3: Bonded state: equilibrium on a differential length \(d\xi\) along the fiber axis.](image)

In the fully bonded state, the \(\tau - s\) relation at any location along the fiber axis \(\xi\) can be written as

\[
\tau(\xi, s) = G_s(\xi). \tag{3.4}
\]
Note that the relative bond modulus $G$ is determined from the analysis of shear deformation of the matrix (BUDIANSKY AND HUTCHINSON 1986, KULLAA 1996): As shown in Fig. 3.3, a differential length $d\xi$ along the fiber is considered. At any point with the radial distance $R$ from the fiber axis, the shear stress $\tau_R$ in the matrix satisfies the equilibrium with the interface friction $\tau$:

$$2\pi rf \tau = 2\pi R\tau_R \implies \tau_R = \frac{\tau}{R}. \tag{3.5}$$

The resulting shear strain at $R$ is

$$\gamma_R = \frac{\tau_R}{G_m}, \tag{3.6}$$

where

$$G_m = \frac{E_m}{2(1 + \nu_m)} \tag{3.7}$$

is the shear modulus of matrix. The relative displacement between the boundary of matrix characterized by the radius $R_m$ and the fiber-matrix interface ($rf$) is written as the integration of shear strain from $rf$ to $R_m$:

$$s = s(rf - R_m) = \int_{rf}^{R_m} \gamma_R \, dR = \frac{\tau R}{G_m} \ln \left(\frac{R}{rf}\right), \tag{3.8}$$

from which the relative bond modulus is obtained as

$$G = \frac{\tau}{s} = \frac{E_m}{d_f(1 + \nu_m) \ln(R_m/rf)}. \tag{3.9}$$

Note that $R_m$ is assumed to satisfy

$$\frac{R_m}{rf} = \frac{1}{\sqrt{c_f}} \tag{3.10}$$

in the environment of fiber-reinforced concrete composite (here $c_f$ is the volume content of fiber).

By introducing $\tau = Gs$ into Eq. (3.2) and considering Eq. (3.3), one obtains the second-order differential equation of axial force $T(\xi)$

$$\frac{d^2 T(\xi)}{d\xi^2} = \lambda^2 T(\xi) \quad \text{with} \quad \lambda^2 = \frac{\pi d_f G}{A_f E_f}, \tag{3.11}$$

which is solved by considering the boundary conditions $T(0) = 0$ and $T(L_e) = F$ to provide the axial force along the fiber

$$T(\xi) = F \frac{\sinh(\lambda L_e)}{\sinh(\lambda L_e)}. \tag{3.12}$$

Recalling that $d T(\xi) = \pi d_f \tau(\xi) \, d\xi$, the distribution of frictional stress is expressed as

$$\tau(\xi) = \frac{F \lambda \cosh(\lambda \xi)}{\pi d_f \sinh(\lambda L_e)}. \tag{3.13}$$
with its maximum value at the free end ($\xi = L_e$):

$$\tau(L_e) = \frac{F\lambda}{\pi d_f} \coth(\lambda L_e).$$  \hspace{1cm} (3.14)

Consequently, from $s = \tau / G$, the relative displacement along the fiber is obtained as

$$s(\xi) = \frac{F\lambda \cosh(\lambda \xi)}{\pi d_f G \sinh(\lambda L_e)}.$$

with its maximum value at $\xi = L_e$ recognized as the free-end displacement

$$u = s(L_e) = \frac{F\lambda}{\pi d_f G} \coth(\lambda L_e).$$  \hspace{1cm} (3.16)

Obviously, a linear relationship between the free-end force $F$ and the displacement $u$ is obtained in fully bonded state.

The limit of bonded state is characterized by $\tau(L_e) = \tau_{\text{max}}$, i.e. the frictional stress at the free end reaches the bond strength of the interface. Recalling Eq. (3.14), we denote the values of pullout force and displacement at the limit state as

$$F_0 = \frac{\pi d_f \tau_{\text{max}}}{\lambda} \tanh(\lambda L_e) \quad \text{and} \quad u_0 = s_0 = \frac{\tau_{\text{max}}}{G}.$$

(3.17)

**Debonding stage**

![Figure 3.4: Debonding stage: high resolution photo of a microcrack, which initiates within the matrix and propagates along the fiber–matrix interface, leading to the gradual damage of the interfacial zone [scanning electron microscopy (SEM) graph by BENTUR ET AL. (1985)].](image)

With $\tau(L_e) = \tau_{\text{max}}$, the interfacial debonding occurs first at the free end of fiber. Debonding is characterized by the initiation and propagation of microcracks within a thin (typically $10 \sim 100 \mu m$) interfacial zone, induced by shear stress (Fig. 3.4). With the increase of pullout load, the debonding region extends towards the fiber tip ($\xi = 0$). Because of the remaining bonded section, the fiber does not slide in the fiber channel yet; therefore, the interfacial slip is still negligible in comparison with the sliding phase to be analyzed later; hence, we assume that the frictional stress $\tau$ within the debonding section $L_d$ uniformly equals to $\tau_{\text{max}}$ (Fig. 3.2). Note that based upon different understanding of the fiber–matrix bonding mechanism, different forms of the frictional stresses in the debonding section are assumed in the literature [see LI AND STANG (1997) for a review].
Starting with $F_0$ and $u_0$, both the free-end load and the displacement increase monotonically with respect to the debonding length $L_d$. At any debonding state characterized by $L_d$, the remaining well bonded section ($\xi \in [0, L_b]$, with the length $L_b = L_e - L_d$) can be analyzed in analogy to the fully bonded fiber state described above; hence, the axial force and the relative displacement at the end of this section ($\xi = L_b$) are

$$T_b = \frac{\pi d_f \tau_{\text{max}}}{\lambda} \tanh(\lambda L_b) \quad \text{and} \quad u_b = s_0. \quad (3.18)$$

Then, considering Eq. (3.2) and taking $T_b$ as the boundary condition for the debonding section ($\xi \in [L_b, L_e]$), by integrating the axial force from $\xi = L_b$ to the free end $\xi = L_e$, we obtain

$$F = T_b + \int_{L_b}^{L_e} dT = T_b + \pi d_f \tau_{\text{max}} L_d. \quad (3.19)$$

Similarly, using Eq. (3.3) and the boundary condition $u_b$ at $\xi = L_b$, the free-end pullout displacement are obtained via the integration of axial strain:

$$u = s_0 + \int_{L_b}^{L_e} ds = s_0 + \frac{\pi d_f \tau_{\text{max}} L_d^2}{2 A_f E_f} + \frac{T_b L_d}{A_f E_f}. \quad (3.20)$$

We notice that during the debonding process, both $F$ and $u$ are dependent on $L_d$. The debonding stage ends when $L_d = L_e$, i.e. the debonding region covers the complete embedded length of fiber. At this state, one obtains the peak pullout force and the corresponding displacement as

$$F_{\text{max}} = \pi d_f \tau_{\text{max}} L_e \quad \text{and} \quad u_1 = s_0 + \frac{\pi d_f \tau_{\text{max}} L_e^2}{2 A_f E_f}. \quad (3.21)$$

Pull-out phase

*Figure 3.5: Fiber pullout phase: scanning electron micrograph of the fiber channel after the fiber is pulled out (MARCHESI and MARCHESE 1993).*

During the pullout phase, the interfacial zone along the fiber is gradually damaged due to the increasing interfacial slip, leading to a release of shear stresses along the interface. Fig. 3.5 shows the scanning electron microscopy (SEM) graph of the fiber channel after the fiber is pulled out. The
damage along the interfacial zone is clearly observed. We assume that the interfacial shear stress $\tau$ decays from the bond strength $\tau_{\text{max}}$ and approaches an asymptotic value $\tau_0$ (Fig. 3.2):

$$\tau(s) = \tau_0 + (\tau_{\text{max}} - \tau_0) \exp \left( -\frac{s_1 - s}{s_{\text{ref}}} \right).$$ (3.22)

In this stage, the whole fiber slides through the fiber channel as a rigid body along the remaining embedded length $L_d$. The existing deformation in the fiber is assumed to be “frozen”, considering the fact that the sliding displacement is much larger. Therefore, one has $s(\xi) = u$, i.e. the relative slip along the fiber uniformly equals to the free-end displacement. Consequently, the free-end force-displacement response during the pullout phase is obtained as

$$F(u) = \pi d_f \tau(u) L_d,$$ (3.23)

where

$$\tau(u) = \tau_0 + (\tau_{\text{max}} - \tau_0) \exp \left( -\frac{u_1 - u}{s_{\text{ref}}} \right)$$ (3.24)

is the decaying frictional stress; $s_1$ and $s(\xi)$ in Eq. (3.22) are replaced by $u_1$ and $u$, respectively, and $L_d = L_e - u + u_1$ is the remaining length of fiber embedded in the matrix.

### 3.1.2 Model verification and parameter determination

The pullout model is verified by means of experimental results reported in Leung and Shapiro (1999). In these laboratory tests, the concrete matrix has a compressive strength of $f_c = 36.5$ MPa, from which the elastic modulus $E_m$ and the tensile strength $f_t$ can be calculated according to German
Standard (DIN 1045-1 2000). Steel fibers with different values of yield strength, varying from 275 MPa to 1,171 MPa and different cases regarding inclination angle w.r.t. to the loading direction have been investigated experimentally. From the results of the pullout tests of fibers without inclination, the essential interface parameters ($\tau_{\text{max}}$, $\tau_0$ and $s_{\text{ref}}$) are determined. All parameters used for the re-analysis are listed in Table 3.1.

Remark: Unless otherwise stated, throughout the present thesis, $\nu_m = 0.2$ is considered for concrete matrix; $E_f = 210,000$ MPa and $\nu_f = 0.3$ are used for steel fiber.

As an example, Fig. 3.6(a) shows a comparison of the load–displacement relationship obtained from the laboratory test and the proposed model for the pullout test on low-strength fiber ($\sigma_y = 275$ MPa); the results of other fiber strengths are rather similar, because the fiber yield stress does not play an important role in the pullout of an aligned straight fiber. An excellent correlation is observed for the complete pullout behavior including the slope of the $F - u$ curve during the pullout phase. The nonlinear characteristics of the ascending branch reflecting the fully bonded and debonding stages is enlarged and plotted in the figure as well.

Having computed $T(\xi)$ for the different pullout stages, one can also compute the average fiber stress $\bar{\sigma}$:

$$\bar{\sigma} = \frac{1}{L_e} \int_{L_e}^{L_e} \sigma(\xi) \, d\xi, \quad \text{with} \quad \sigma(\xi) = \frac{T(\xi)}{A_f}.$$  \hspace{1cm} (3.25)

Fig. 3.6(b) shows the development of the average fiber stress during pullout as a function of the free-end displacement. This value can be useful in a model at a higher length-scale, e.g. a micromechanics-oriented model for the homogenized constitutive properties of FRC composite materials.
3.2 Pullout of inclined straight fiber

For the development of an analytical model describing the pullout of a single straight steel fiber embedded in a concrete matrix with an inclination angle with respect to the loading direction, local inelastic deformations of the fiber, the lateral pressure imposed onto the concrete and local damage of the concrete have to be considered. These aspects are also considered in recently proposed analytical models (FANTILLI AND VALLINI 2007, LARANJERA ET AL. 2010). Adopting similar concepts, we make use of the interface law described above, in conjunction with an algorithm to solve the governing equations of all sub models involved and to generate the free-end load–displacement relations during the pullout of an inclined straight steel fiber in a concrete matrix.

![Image](image_url)

**Figure 3.7:** Straight fiber pullout with inclination: (a) illustration of deformed state indicating two sections along the fiber; (b) representation of Section A-B as cantilever; (c) representation of Section B-C as beam on elastic foundation.

In what follows, we define the X-axis pointing in the direction of the pullout load assumed to be perpendicular to the crack plane. The idealized geometrical situation of an inclined steel fiber at a certain pullout state corresponding to a pullout displacement $u_x$ in the loading direction is illustrated in Fig. 3.7. In Fig. 3.7(a), $\theta$ indicates the initial embedment inclination angle w.r.t. the X-direction and $\delta$ is the concrete spalling depth; Point O represents the initial intersection of the fiber axis with the crack plane, Point C refers to the tip of the embedded fiber, Point B is the current exit point of the fiber from the matrix and Point A is the symmetric center of Section B-O. Note that the geometrical configuration shown in Fig. 3.7(a) is adapted to the experiments in LEUNG AND SHAPIRO (1999), where the concrete spalling can only occur at one side of the crack plane. In the present work, we also consider the general situation of concrete spalling on both sides [see e.g. LARANJERA ET AL. (2010)] for applications of this model to structural analyses.

For a certain pullout state characterized by an inclination $\theta$, a displacement $u_x$ and a spalling depth $\delta$, the fiber–concrete interaction is represented via two sub-models referred to as the cantilever model A-B and the beam on elastic foundation model B-C.
3.2.1 Fiber segment as cantilever

Because of the matrix spalling (as will be described later), Section A-B of the fiber is considered to be exposed without interaction with the matrix; therefore, it is treated as a cantilever with Point B fixed. From the geometrical situation shown in Fig. 3.8, the current inclination angle of this segment is determined as

$$\varphi = \arctan \left( \frac{\delta \tan \theta}{\delta + u_x} \right),$$

and the magnitude of its rotation w.r.t. Point B is

$$\psi = \theta - \varphi.$$  \hspace{1cm} (3.27)

The transverse force $V_A$ at Point A can be already obtained as a result of $\psi$, considering the elastic-plastic deformation at Point B (see Appendix A for the full details):

$$V_A = V_A(\psi).$$  \hspace{1cm} (3.28)

Subsequently, by analyzing the equilibrium on this section, the relations connecting the tensile force $T_B$, shear force $V_B$ and bending moment $M_B$ of the fiber cross section at point B, to the transverse force $V_A$ and the unknown longitudinal force $F_A$ at Point A are

$$T_B = F_A \cos \psi - V_A \sin \psi, \quad V_B = F_A \sin \psi + V_A \cos \psi, \quad M_B = V_A L_{AB},$$

with the length of Section A-B as

$$L_{AB} = \frac{\delta \tan \theta}{2 \sin \varphi}.$$  \hspace{1cm} (3.30)

3.2.2 Fiber segment as beam on elastic foundation

The fiber segment remaining embedded in the matrix is idealized as an elastic beam resting on an elastic foundation (Fig. 3.9). As described in Appendix B, the transverse deflection along the fiber axis and the induced distributed transverse forces $p(\xi)$ at the concrete–steel interface are calculated from solving the governing differential equation, with the boundary conditions provided by $V_B$ and
3.2. PULLOUT OF INCLINED STRAIGHT FIBER

\[ T_B \]

\[ V_B \]

\[ p(\xi) \]

\[ \tau(\xi) \]

\[ M_B \]

\[ B \]

\[ C \]

Figure 3.9: Straight fiber pullout with inclination: representation of Section B-C as beam on elastic foundation.

\[ M_B \] obtained above. Local crushing of the concrete is considered by means of a reduction of \( p(\xi) \) depending on the compressive strength of the concrete. In a second step, the distributed frictional forces \( t(\xi) \) generated along the fiber–concrete interface, depending on the (uniform) axial slip

\[ s = 2L_{AB} - \frac{\delta}{\cos \theta} \]  \hspace{1cm} (3.31)

and \( p(\xi) \) via Coulomb’s law, are determined. Finally, the integration of \( t(\xi) \) along Section B-C with length

\[ L_{BC} = L_e - 2L_{AB} \]  \hspace{1cm} (3.32)

gives the tensile force \( T_B \) at the current exit point B:

\[ T_B = \int_{L_{BC}} t(\xi) \, d\xi. \]  \hspace{1cm} (3.33)

This value should be identical to the value obtained in Eq. (3.29) from Section A-B.

3.2.3 Spalling of concrete matrix

The previous two sub-models are accompanied with the determination of the matrix spalling size \( \delta \) as follows.

From the experimental observations [Fig. 3.10(a)], it is observed that the matrix spalling is caused by the concentrated stresses at the exit point of fiber from matrix. In the present work, the matrix spalling volume is idealized as a cone, with the vertex located at the current exit point and the base sitting on the crack surface. The “drop”-like base is the combination of a triangle-(1) and a semicircle-(2) [Fig. 3.10(b)]. The lateral area of the cone is computed as

\[ A_{sp}(\delta) = \delta^2 \left( \frac{\pi \cos \theta}{2 \sin^2 \theta} + \sqrt{1 - \cos^2 \theta \sin^2 \theta} \sin \theta \cos \theta \right) \]  \hspace{1cm} (3.34)
The consideration of spalling is based on the assumption of uniform tensile stress distribution in the lateral area $A_{sp}$ of the cone:

$$V_B \leq f_t A_{sp}(\delta).$$

(3.35)

### 3.2.4 Algorithm for computing fiber pullout responses

With the formulations prepared previously, we propose a numerical algorithm to solve the governing equations for each of the sub-models and generate the complete pullout load–displacement relationship of single inclined straight fibers.

1. **A priori**, the geometries and material properties of the fiber and matrix, as well as the embedment conditions need to be known. From laboratory testing of the pullout behavior of a straight fiber without inclination, determine $\tau_{max}$, $\tau_0$ and $s_{ref}$ as input for the interface law in Eq. (3.1).

2. For an inclined fiber with the initial embedment length $L_e$ and angle $\theta$ in the matrix, the spalling size $\delta$ is initialized and set to a small value ($\delta^0 = r_f \sin \theta$). Also the longitudinal force $F_A$ at Point A is initialized as $F_A^0 = F_{max} \cos \theta$, with $F_{max}$ the peak pullout force for straight fiber without inclination.

3. The pullout process is driven by the crack opening displacement $u_x$ that is applied in increments: $u_{x,n+1} = u_{x,n} + \Delta u_x$, where $n$ denotes the increment number. For each increment $[n, n + 1]$, determine the pullout force $F_{n+1}$ as follows:

   a) Using the given pullout displacement $u_{x,n+1}$, together with the estimated spalling size $\delta_{n+1}^i$ (start with $\delta_n$ as the first estimation; $i$ indicates the iteration number of $\delta$), the geometrical configuration of the fiber is determined (Fig. 3.7); we compute the transverse force $V_A^{i,n+1}$ by analyzing the Section A-B according to Fig. 3.8 and Eq. (3.28).

   b) For the current geometrical situation characterized by $u_{x,n+1}$, $\delta_{n+1}^i$, together with the force $V_A^{i,n+1}$, search the corresponding value of force $F_A^{i,n+1}$. 

---

**Figure 3.10:** Matrix spalling during the pullout of inclined fiber: (a) experimental observation (cut-view) (LEUNG AND SHAPIRO 1999); (b) idealized shape of the matrix spalling (cut-view and front view).
i. Using the trial value $F_{A,n+1}$, calculate the tensile force $T_{B,n+1}^{i,j}$, shear force $V_{B,n+1}^{i,j}$ and bending moment $M_{B,n+1}^{i,j}$ at point B, from Eq. (3.29).

ii. Solve the beam on elastic foundation problem with the boundary conditions $V_{B,n+1}^{i,j}$ and $M_{B,n+1}^{i,j}$ at Point B; calculate again the tensile force $T_{B,n+1}^{i,j}$ at Point B using Eq. (3.33).

iii. Check if the value of $T_{B,n+1}^{i,j}$ obtained for Section B-C using Eq. (3.33) is equal to the corresponding value of $T_{B,n+1}^{i,j}$ calculated from Section A-B using Eq. (3.29), up to a tolerance (tol):

$$\left| T_{B,n+1}^{i,j,(A-B)} - T_{B,n+1}^{i,j,(B-C)} \right| \leq \text{tol}. \quad (3.36)$$

If the tolerance is exceeded, the update $F_{A,n+1}^{i,j+1} = F_{A,n+1}^{i,j} + \Delta F_A$ is performed by means of a bisection method. The steps i to iii are repeated until Eq. (3.36) is satisfied.

c) The force $F_{A,n+1}$ is determined. Meanwhile, the transverse force $V_{B,n+1}^{i,j}$ is also obtained and used to check the spalling criterion Eq. (3.35); if it is violated, increase the spalling depth by a small value $\delta_{n+1}^{i+1} = \delta_{n+1}^i + \Delta \delta$ and goto a); repeat the steps a) to c) to determine the admissible $\delta_{n+1}$ and the corresponding $F_{A,n+1}$ and $V_{A,n+1}$.

d) With the correct values of $\delta_{n+1}$, $F_{A,n+1}$ and $V_{A,n+1}$ obtained for the current increment, calculate the pullout force

$$F_{n+1} = F_{A,n+1} \cos \varphi + V_{A,n+1} \sin \varphi. \quad (3.37)$$

4. Increase the fiber end displacement $u_x$, until the remaining embedded length $L_{BC}$ becomes zero, i.e. the fiber is fully pulled out; the complete $F - u_x$ diagram is generated.

### 3.2.5 Model validation

The analytical model described above is validated by means of experimental results reported in Leung and Shapiro (1999). In the laboratory tests, five different fiber types with the strength ranged from 275 MPa to 1,171 MPa (denoted as Fiber-A to Fiber-E, respectively) have been used. For every fiber type, three different inclination angles, i.e. 0°, 30° and 60°, have been considered.

In the present work, three out of the five types of fiber are re-analyzed using the proposed analytical model:

- Fiber-A: low strength with $\sigma_y = 275$ MPa,
- Fiber-C: normal strength with $\sigma_y = 635$ MPa,
- and Fiber-E: high strength with $\sigma_y = 1,171$ MPa.

The material and the interface parameters are determined from standard fiber pullout tests without inclination for the specific fiber type according to Section 3.1. In addition to the parameters contained in Table 3.1, the parameters listed in Table 3.2 have been used.

Fig. 3.11 contains the pullout load–displacement diagrams of the model applied to different combinations of fiber strength and inclination angle. The figure demonstrates, that the proposed
Table 3.2: Parameters used for the modeling of straight fiber pullout with inclination w.r.t. the loading direction.

<table>
<thead>
<tr>
<th>Steel fiber</th>
<th>Yield stress $\sigma_y$</th>
<th>275 / 635 / 1,171 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber–matrix interaction</td>
<td>Embedment length $L_e$</td>
<td>10 mm</td>
</tr>
<tr>
<td></td>
<td>Inclination angle $\theta$</td>
<td>$0^\circ / 30^\circ / 60^\circ$</td>
</tr>
<tr>
<td></td>
<td>Bond strength $\tau_{\text{max}}$</td>
<td>3.18 MPa</td>
</tr>
<tr>
<td></td>
<td>Asymptotic frictional stress $\tau_0$</td>
<td>0.64 MPa</td>
</tr>
<tr>
<td></td>
<td>Reference slip $s_{\text{ref}}$</td>
<td>0.25 mm</td>
</tr>
<tr>
<td></td>
<td>Friction coefficient $\mu$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

From the comparison among the results, we come with several interesting observations as follows (note that the fibers have the same geometry and embedment length):

• **Bonded and debonding stages:** In each case, the first nearly vertical ascending branch up to the peak represents the bonded and debonding stages. Concerning the level of peak force ($F_{\text{max}}$), the following aspects are noticed [see Fig. 3.12(a)]:
  – For the same fiber strength with different inclination angles, the maximum peak force is found in the case of $30^\circ$ which is slightly larger than that in $0^\circ$, due to the additional frictional forces as a result of the lateral reactions on the fiber–matrix interface. With $60^\circ$ inclination, the peak force $F_{\text{max}}$ is significantly lower than those in the other two cases, because the high values of inclination angle and spalling depth causes a large portion of fiber (Section A-B) to be exposed, and the length of the remaining embedded fiber segment B-C that can carry the pullout force is reduced.
  – Without inclination, the peak force is only related to the interfacial bond strength $\tau_{\text{max}}$ (see Section 3.1); therefore, the difference in fiber strength does not have any influence, since the plastic deformation of fiber is not involved in this case.
  – With the same inclination angle, either $30^\circ$ or $60^\circ$, the peak force only increases marginally regardless of the considerably enhanced fiber strength (from 275 MPa to 1,171 MPa), suggesting that the plastic deformation of fiber and the corresponding contribution to the pullout resistant force is not the major mechanism in this situation.

• **Matrix spalling:** The matrix spalling mainly occurs before the pullout force reaches the first peak. As compared in Fig. 3.12(b), the spalling depth $\delta$ increases with respect to the inclination angle. The change of fiber strength does not show remarkable effect on the spalling size.

• **Pulling-out phase:** After reaching the peak forces, the curves start to descend with different rates depending on the inclination angle (Fig. 3.11). Regardless of the difference between the results for different values of fiber strength, the following tendencies of pullout responses are observed as a result of the change of inclination angle:
- For the aligned fibers (0°), the steep descending branch consumes most (approximately 80%) of the peak force, followed by a slow decaying residual force associated with the sliding of fiber out of the matrix.
- With a small inclination angle (30°), the descending curve reduces the pullout force by approximately half of the peak force. Afterwards, the curve follows a slow decline, however, maintaining a residual force level considerably larger than that in the situation of 0°.
- With a large inclination (60°), after a rather limited decrease, the values of residual pull-
out forces remain on a considerable level. The reason can be understood as follows (see Fig. 3.7): Although the fiber is being pulled out and the remaining embedded fiber segment (B-C) is becoming shorter, the exposed fiber section (A-B) is getting longer and the geometrical condition is changing, which leads to an increase in the lateral reaction forces at the exit (Point B) as well as along Section B-C; consequently, the total resistance does not decay significantly.

- **Late period of pulling-out:** For inclined fibers, the increase of the pullout force shortly before the fiber is fully pulled out of the concrete matrix is attributed to the increased bending resistance of the remaining small portion of the fiber, leading to a significant increase of frictional stresses along the fiber–matrix interface. Therefore, we notice that with the increase of either inclination angle or fiber strength, the level of the second peak force increases. Afterwards, the forces drop rapidly to zero. (The delay of the predicted second peak of the pullout force prior to the complete failure is possibly caused by the idealization of the geometrical configuration of the inclined fiber in the model, which is able to transfer stresses until the fiber leaves the fiber channel. In reality, however, the damaged fiber channel allows the remaining fiber segment to slide out of the matrix even before the fiber tip has reached the exit point.)

- **Pullout toughness:** The pullout “toughness” is calculated as the area under the pullout curve, which indicates the amount of energy dissipation during the complete pullout procedure. From Fig. 3.12(c), we observe that aligned fibers ($0^\circ$) generally exhibit the same level of toughness; the inclined fibers, especially with $30^\circ$ angle, consume much higher energy than those without inclination ($0^\circ$); further increase of inclination angle to $60^\circ$ does not show additional effect. Regarding the influence of fiber strength we notice that the increase of fiber strength from low (275 MPa) to medium (635 MPa) considerably enhances the toughness; using high-strength (1171 MPa) fiber only show marginal further enhancement.

According to the straight steel fiber pullout behavior discussed above, the following conclusions can be drawn:
• The interface bond–friction mechanism plays the major role.
• Fibers with small inclination (approximately $30^\circ$) exhibit the most favorable pullout response, considering both the peak force and the pullout toughness.
• The increase of fiber strength enhances the pullout responses, however, not proportionally.

### 3.3 Finite element simulation

![Experimental range vs. Simulation](a)

![Contour plot of concrete scalar damage variable](b)

**Figure 3.13:** Results of numerical simulation of low strength fiber with inclination angle $30^\circ$: (a) comparison between the computed pullout load–displacement relations and the range of the experimental results; (b) contour plot of the concrete scalar damage variable at the end of pullout.

To obtain a better insight into the pullout mechanisms of inclined fibers, finite element analyses have been performed, using the commercial finite element program ABAQUS. In the numerical simulations, an ideal elastoplastic material model using the Von Mises yield criterion is employed for the steel fiber, and the concrete damaged plasticity model implemented in ABAQUS is used as the constitutive model for the concrete matrix; along the fiber–matrix interface, cohesion and frictional contact conditions are specified. Cracking and compressive crushing of concrete as well as large deformations are considered. Fig. 3.13 shows the force–displacement diagram as well as the contour plot of concrete damage variable obtained from the pullout simulation of a low strength fiber with inclination angle of $30^\circ$ [the full details are contained in ZHANG (2011)].

In Fig. 3.13(a), we notice nearly the same characteristic stages as discussed for the analytical model results: first an almost vertical ascending branch for bonded and debonding stages, then a steep descent due to the damage induced along the fiber–matrix interface zone during pullout stage and an increasing pullout force prior to full fiber pullout connected with the increasing bending resistance in the last stage is observed. In Fig. 3.13(b), the spalling depth $\delta$ obtained from the simulation as $\delta \approx 1.1$ mm corresponds reasonably well with the respective value from the analytical model ($\delta = 0.81$ mm); the damaged region on the upper-left of the current exit point clearly corresponds to the concrete damage due to the very short fiber segment in the last stage.
In comparison to straight fibers, steel fibers with deformed geometry usually exhibit even higher ductility during the pullout procedure. One of the most widely used types of deformed steel fibers is the hooked-end fiber, characterized by the hook on each end. During the pullout procedure, the resistance of the hooked end to straightening often contributes, as an anchorage effect, to the main portion of the total pullout force, in comparison to the case of a straight fiber, where the interfacial behavior plays the main role (Robins et al., 2002). The interaction between hooked-end fibers and the concrete during pullout has been investigated experimentally by a number of researchers, see e.g. Htut and Foster (2007), Marchese and Marchese (1993), Markovich et al. (2001), and the references therein. In the present work, with the supporting information from finite element simulation, an analytical approach is proposed, with the analysis of the effect of the hooked end considered as the central task in this situation. This sub-model is connected with the model for straight fibers as described in the previous chapter, providing an analytical model for the pullout response of a hooked-end fiber with an inclined orientation with respect to the loading direction. The full details of model are presented in this chapter.
List of symbols used in this chapter

- $A_f$: cross-section area of fiber, $\text{mm}^2$
- $\alpha$: angle of the Arc b-c, $^\circ$
- $d_f$: diameter of fiber, mm
- $E_m$: YOUNG’s modulus of matrix, N/mm$^2$
- $F_{\text{hook}}$: pullout force contributed by the hooked end, N
- $F_{\text{str}}$: residual friction force contributed by the straightened hooked end, N
- $f_1$: concentrated transverse reaction force on Point b, N
- $f_2$: concentrated transverse reaction force on Point d, N
- $f_3$: concentrated transverse reaction force on Point c, N
- $f_c$: compressive strength of concrete, N/mm$^2$
- $f_t$: tensile strength of concrete, N/mm$^2$
- $g_1$: reaction force at the middle of Section b-c, N
- $g_2$: reaction force at the middle of Section d-e, N
- $g_3$: reaction force at the middle of straightened Section d-e, N
- $l_{\text{h}1}$: length of Section a-b, mm
- $l_{\text{h}2}$: length of Section c-d, mm
- $l_{\text{h}arc}$: length of Arc b-c (or d-e), mm
- $L_{BC}$: length of fiber segment B-C (Chapter 3), mm
- $L_e$: embedment length of fiber in matrix, mm
- $M$: bending moment in fiber, N-mm
- $M_{\text{el}}$: elastic limit of $M$, N-mm
- $M_{\text{pl}}$: fully plastic value of $M$, N-mm
- $M_c$: bending moment at Point c, N-mm
- $M_d$: bending moment at Point d, N-mm
- $M_e$: bending moment at Point e, N-mm
- $\mu$: COULOMB’s friction coefficient, N/N
- $\varphi$: current inclination angle of Section A-B (Chapter 3), $^\circ$
- $r_b$: radius of the Arc b-c, mm
- $r_f$: radius of fiber, mm
- $\sigma_y$: yield stress of steel fiber, N/mm$^2$
- $s_{\text{ref}}$: reference value of interfacial slip, mm
- $\bar{\sigma}$: average stress in fiber, N/mm$^2$
- $T$: longitudinal force in the fiber, N
- $T_{\text{el}}$: elastic limit of $T$, N
- $T_B$: longitudinal force at Point B (Chapter 3), N
- $T_c$: longitudinal force at Point c, N
- $T_d$: longitudinal force at Point d, N
- $T_e$: longitudinal force at Point e, N
- $t$: distributed frictional force (Chapter 3), N/mm
- $\tau_{\text{max}}$: fiber–matrix bond strength, N/mm$^2$
| Symbol | Description | Unit/
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_0)</td>
<td>asymptotic fiber–matrix frictional stress</td>
<td>N/mm²</td>
</tr>
<tr>
<td>(\theta)</td>
<td>inclination angle of fiber</td>
<td>°</td>
</tr>
<tr>
<td>(u)</td>
<td>pullout displacement at the free end of fiber</td>
<td>mm</td>
</tr>
<tr>
<td>(V)</td>
<td>transverse force in fiber</td>
<td>N</td>
</tr>
<tr>
<td>(V^\text{el})</td>
<td>elastic limit of (V)</td>
<td>N</td>
</tr>
<tr>
<td>(V_c)</td>
<td>transverse force at Point c</td>
<td>N</td>
</tr>
<tr>
<td>(V_d)</td>
<td>transverse force at Point d</td>
<td>N</td>
</tr>
<tr>
<td>(V_e)</td>
<td>transverse force at Point e</td>
<td>N</td>
</tr>
<tr>
<td>(V^\text{sup})</td>
<td>lateral supporting capacity of matrix</td>
<td>N</td>
</tr>
<tr>
<td>(\xi_1)</td>
<td>local coordinate parallel to fiber</td>
<td>-</td>
</tr>
<tr>
<td>(\xi_2)</td>
<td>local coordinate perpendicular to fiber</td>
<td>-</td>
</tr>
</tbody>
</table>
4.1 Finite element simulation

To support the development of analytical model, a finite element model for the hooked-end fiber pullout behavior has been established using ABAQUS, in analogy to the analysis of straight fiber pullout in the preceding chapter [for model details, see ZHANG (2011)].

![Figure 4.1: Numerical simulation of the hooked-end fiber pullout: (a) load–displacement diagram; contour plots (in deformed configuration) of (b) VON MISES stress, (c) tensile damage variable and (d) compressive damage variable during pullout.](image)

Fig. 4.1 contains the results from the numerical simulation of the pullout of a normal-strength ($\sigma_y = 1,100$ MPa) hooked end steel fiber embedded in a high-strength concrete matrix ($f_c = 84$ MPa) without inclination w.r.t. the loading direction: Fig. 4.1(a) exhibits the results of pullout load-displacement relation. The contour plot in Fig. 4.1(b) illustrates the distribution of VON MISES stresses, which indicates the considerable plasticization of the fiber material during the straightening of the hooked end in those portions of the fiber where the MISES stress exceeds the yield strength. Fig. 4.1(c) represents the damage zones in the concrete matrix induced by tensile stresses. A comparison with Fig. 4.1(d), which contains the contour plot of the crushing damage caused by the compressive stresses along the fiber–matrix interfaces, however, shows that crushing damage is dominant compared to tensile spalling. These local damaging mechanisms in the vicinity of the hooked end and the plastic deformation of fiber are the major differences between the pullout of a hooked-end fiber and a straight fiber.
4.2 Analytical modeling

4.2.1 Anchorage effect of the hooked end

An analytical approach is proposed to directly calculate the anchorage force induced by the hooked fiber end during the pullout process. This sub-model is then combined with the straight fiber pullout model described in the previous section to finally generate an analytical model for the prediction of the pullout force–displacement relations of hooked end steel fibers embedded in a concrete matrix with arbitrary inclination angle w.r.t. the loading direction.

As shown in Fig. 4.2, the shape of the hooked end is idealized as being composed of four segments:

- a straight section a-b, with the length $l^h_1$,
- the first curved section b-c with radius $\rho^h$, angle $\alpha^h$ and length $l^h_{arc} = \rho^h \alpha^h$,
- the second straight section c-d with length $l^h_2$
- and the second curved section d-e.

At the very beginning of pullout, the whole fiber behaves elastically. With the increase of pullout force, the steel in the curved segments of the hooked end starts to yield. When both curved segments of the hooked end are in a plastic state, the fiber starts to slide in the fiber channel. An other favorable factor contributing to sliding of the fiber is the deformation of the surrounding matrix due to the transverse forces on the interface which lead to a widening of the fiber channel.

![Figure 4.2: Hooked-end fibers: idealized geometry of the hooked end.](image)

![Figure 4.3: Hooked-end fibers: key states (KS) during the pullout procedure.](image)
In the proposed analytical model, the anchorage effect of the hooked end is represented by a multilinear load–displacement relation, capturing a sequence of so-called “key states (KS)” as shown in Fig. 4.3. A similar concept has been proposed in LARANJEIRA ET AL. (2010), where the anchorage forces are extracted from the experimental results.

For every key state, the equilibrium of forces and moments in the segments of the hooked end is analyzed and the resulting anchorage force is calculated, taking into account the yielding of steel and the damage of concrete. According to Fig. 4.4, distributed interface stresses induced along the curved contact surfaces are represented by reaction forces. For example, as illustrated in the contour plots (Fig. 4.1), the regions where the concrete suffers high compressive damage suggest the positions of concentrated transverse reaction forces, denoted by $f_1 \sim 3$, $g_1$, and $g_2$ in the schematic diagrams in Fig. 4.4. Note that in Fig. 4.4, all arrows assumes the positive forces and bending moments.

**Key state-1**

This state is considered as the elastic limit state of the fiber, characterized by the initiation of plasticity at Point c and e. The hooked end is divided into two sections: a-c from the current fiber tip to Point c in the fiber channel, and c-e from Point c to e (Fig. 4.5).

For Section a-c, the equilibrium of forces in horizontal (denoted by $\xi_1$ as shown in Fig. 4.2) and
vertical (ξ2) directions and the rotational moment w.r.t. Point c is written as follows:

\[
T_c \cos \alpha^h = \mu f_1 + V_c \sin \alpha^h, \\
T_c \sin \alpha^h + V_c \cos \alpha^h = f_1, \\
f_1 \rho^h \sin \alpha^h = \mu f_1 (1 - \cos \alpha^h) \rho^h + M_c. \tag{4.1}
\]

Meanwhile, we consider the yield condition

\[
\frac{V}{V^{el}} + \frac{T}{T^{el}} + \frac{M}{M^{el}} = 1 \tag{4.2}
\]

on the fiber cross section at Point c, with

\[
V^{el} = \frac{1}{\sqrt{3}} \sigma_y A_f, \quad T^{el} = \sigma_y A_f \quad \text{and} \quad M^{el} = \frac{1}{4} \pi \sigma_y r_f^3 \tag{4.3}
\]

the shear force, tensile force and the bending moment for the fiber cross section at the elastic limit state, respectively. From these four equations [Eq. (4.1)-(4.2)], the unknown forces and the moment (f1, Tc, Vc and Mc) are determined.

For Section c-e, the equilibrium of forces in ξ1 and ξ2 directions and rotation moments w.r.t. Point e gives

\[
T_e + V_c \sin \alpha^h = T_c \cos \alpha^h + f_2 \sin \alpha^h + \mu f_2 \cos \alpha^h, \\
f_2 \cos \alpha^h = \mu f_2 \sin \alpha^h + T_c \sin \alpha^h + V_c \cos \alpha^h + V_e, \\
M_c + (T_c + \mu f_2) (1 - \cos \alpha^h) \rho^h + V_c (\frac{l_h}{2} + \rho^h \sin \alpha^h) + M_e = f_2 \rho^h \sin \alpha^h. \tag{4.4}
\]

Considering the yield condition [Eq. (4.2)] at Point e allows to solve for the unknown forces f2, Te, Ve and the moment Me. As a result,

\[
F^{\text{hook}} = T_e \tag{4.5}
\]

represents the contribution of the anchorage effect of the hooked end to the total pullout force at this state.
Key state-2

The maximum anchorage force is reached when the slip of the hooked end approaches approximately half of the length of a curved segment (Fig. 4.6), i.e., \( u = \frac{l_h \text{arc}}{2} \). For this state, we consider the same two sections as KS-1; however, the section a-c has become shorter due to the slip.

![Figure 4.6: Hooked-end fiber pullout: key state-2.](image)

From the experimental observation and numerical simulation, we notice that the very short fiber segment a-c becomes rather stiff and the surrounding matrix deforms significantly. As mentioned before, the widening of the fiber channel due to the crushing damage is also favorable for the mobilization of the hooked end. Therefore, similar to the yield condition used above, we introduce a “slip condition”, which is now applied at Point c as

\[
\frac{V}{V^{\text{el}}} + \frac{V}{V^{\text{sup}}} + \frac{T}{T^{\text{el}}} + \frac{M}{M^{\text{pl}}} = 1. 
\]  
(4.6)

Here, the second term is a simplified form to take into account phenomenologically the effect of channel widening induced by the distributed transverse forces on the fiber–matrix interface along the hooked end;

\[
V^{\text{sup}} = f_c \cdot d_f \left( \frac{l_h}{l_1} + \frac{l_h \text{arc}}{2} \right) \]  
(4.7)

represents the supporting capacity of the surrounding matrix corresponding to the length of this fiber section. In the fourth term, \( M^{\text{pl}} = 4 \sigma_y r_f^3 / 3 \) is the fully plastic bending moment for the fiber cross section. Using this condition at Point c, together with the equilibrium equations (4.1), the unknown forces at the Section a-c are solved. Subsequently, \( T_e \) is obtained in a similar way to KS-1, by means of applying now at Point e the fully plastic condition, which is slightly different from Eq. (4.2):

\[
\frac{V}{V^{\text{el}}} + \frac{T}{T^{\text{el}}} + \frac{M}{M^{\text{pl}}} = 1. 
\]  
(4.8)

The anchorage force at this state is obtained as

\[
F^{\text{hook}} = T_e + F^{\text{str}}. 
\]  
(4.9)

Here \( F^{\text{str}} \) indicates the remaining frictional force induced by the curved section d-e which is being straightened and is entering the straight fiber channel. It is determined by separately treating the incompletely straightened section d-e as a simply supported beam with the length \( l_h \text{arc} \) (Fig. 4.7).
4.2. ANALYTICAL MODELING

Figure 4.7: Hooked-end fiber pullout: incompletely straightened arc in the straight fiber channel as a simply supported beam.

Considering the bending moment $M^{pl}_{pl}$ at the midspan, a lateral reaction force of the matrix

$$g_3 = \frac{4M^{pl}_{pl}}{l_{arc}} \tag{4.10}$$

at the midspan is calculated. Consequently, we obtain the frictional force

$$F^{str} = \mu g_3 = \frac{4\mu M^{pl}_{pl}}{l_{arc}}. \tag{4.11}$$

The remaining part of the hooked end (from Point a to d) is not taken into account here, because the straight fiber channel is already damaged as Section d-e passes.

**Key state-3**

The fiber tip slides towards Point b (Fig. 4.8).

By analyzing equilibrium on section b-c according to:

$$g_1 \sin \frac{\alpha^{h}}{2} + T_c \cos \alpha^{h} = V_c \sin \alpha^{h} + \mu g_1 \cos \frac{\alpha^{h}}{2},$$

$$T_c \sin \alpha^{h} + V_c \cos \alpha^{h} = g_1 \cos \frac{\alpha^{h}}{2} + \mu g_1 \sin \frac{\alpha^{h}}{2},$$

$$g_1 \rho h \sin \frac{\alpha^{h}}{2} = M_c + \mu g_1 \rho \left(1 - \cos \frac{\alpha^{h}}{2}\right). \tag{4.12}$$
and applying the slip condition [Eq. (4.6)] at Point c, the unknown forces and the moment \((g_1, T_c, V_c, M_c)\) are determined. Subsequently, the section c-e is analyzed and the resulting anchorage force \(F^\text{hook}\) is obtained in analogy to KS-2.

**Key state-4**

At this state, the fiber tip reaches Point c. The remaining fiber segment c-e is now divided into two: Section c-d and Section d-e (Fig. 4.9).

![Figure 4.9: Hooked-end fiber pullout: key state-4.](image)

We first check the equilibrium on Section c-d considering the *slip condition* at Point d:

\[
V_d = f_3, \quad T_d = \mu f_3, \quad M_d = f_3 l_h^2, \quad \frac{V_d}{V^{\text{sup}}} + \frac{V_d}{V^{\text{el}}} + \frac{T_d}{T^{\text{el}}} + \frac{M_d}{M^{\text{pl}}} = 1, \quad (4.13)
\]

with \(V^{\text{sup}} = f_c d_f l_h^2\). The unknown forces and the moment \((f_3, T_d, V_d, M_d)\) are solved and used in the equilibrium equations for Section d-e:

\[
T_e + V_d \sin \alpha^h = T_d \cos \alpha^h + g_2 \sin \frac{\alpha^h}{2} + \mu g_2 \cos \frac{\alpha^h}{2},
\]

\[
g_2 \cos \frac{\alpha^h}{2} = \mu g_2 \sin \frac{\alpha^h}{2} + T_d \sin \alpha^h + V_d \cos \alpha^h + V_e, \quad (4.14)
\]

\[
M_d + T_d (1 - \cos \alpha^h) \rho^h + V_d \rho^h \sin \alpha^h + \mu g_2 (1 - \cos \frac{\alpha^h}{2}) M_e = g_2 \rho^h \sin \frac{\alpha^h}{2}.
\]

Considering the fully plastic condition [Eq. (4.8)] at Point e, the forces and moment \((g_2, T_e, V_e, M_e)\) and, subsequently, the anchorage force \(F^\text{hook}\) are determined.

**Key state-5**

The tip reaches Point d (Fig. 4.10). The unknowns \((g_2, T_e, V_e, M_e)\) are computed in the same way as Section b-c in KS-3. The anchorage force is obtained from Eq. (4.9).

**Key state-6**

The whole hooked end is straightened and pulled out of the initial hooked channel, and starts to slide in the straight channel, with the remaining contribution \(F^\text{hook} = F^\text{str}\) to the total pullout force. (}
4.3 Model validation

The model is validated by means of the experimental results reported in BREITENBÜCHER ET AL. (2014) and BREITENBÜCHER ET AL. (2014). In these laboratory tests, various configurations of pullout are systematically investigated. The major material variables involve:

- two different concrete classes: normal-strength concrete with compressive strength $f_c = 44$ MPa and high-strength concrete with $f_c = 84$ MPa;

This residual force $F_{str}$ is also observed in laboratory tests, see e.g. SONG AND BREITENBÜCHER (2012).

The anchorage forces corresponding to every key state are now computed. By connecting these results, a multilinear relationship of anchorage force vs. axial slip relation is obtained.

### 4.2.2 Pullout of inclined hooked-end fiber

We combine the sub-model to consider the anchorage effect of the hooked end with the analytical model for straight fiber pullout described in Chapter 3. The contribution of the hooked fiber end is taken into account as an additional axial force on the embedded fiber section B-C, by modifying Eq. (3.33) as

$$T_B = \int_{L_{BC}} t(\xi_1) \, d\xi_1 + F_{\text{hook}}. \quad (4.15)$$

Adopting the numerical algorithm described in Chapter 3, the free end pullout load–displacement relation of hooked-end steel fibers embedded in a concrete matrix, with and without inclination angle w.r.t. the loading direction can now be computed. Meanwhile, the average fiber stress $\bar{\sigma}$ can be computed as

$$\bar{\sigma} = \frac{1}{L_e} \int_{L_e} \sigma(\xi_1) \, d\xi_1$$

$$= \frac{1}{L_e A_f} \left\{ \int_{L_{AB}} [T(\xi_1) \cos \varphi + V(\xi_1) \sin \varphi] \, d\xi_1 + \int_{L_{BC}} [T(\xi_1) \cos \theta + V(\xi_1) \sin \theta] \, d\xi_1 \right\}. \quad (4.16)$$

## Figure 4.10

Hooked-end fiber pullout: key state-5.
Table 4.1: Parameters used for the modeling of hooked-end fiber pullout.

<table>
<thead>
<tr>
<th>Concrete matrix</th>
<th>Normal-strength</th>
<th>High-strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus $E_{m}$</td>
<td>33,538 MPa</td>
<td>41,605 MPa</td>
</tr>
<tr>
<td>Compressive strength $f_c$</td>
<td>44 MPa</td>
<td>84 MPa</td>
</tr>
<tr>
<td>Tensile strength $f_t$</td>
<td>3.27 MPa</td>
<td>4.75 MPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Steel fiber</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress $\sigma_y$</td>
<td>1,100 MPa (normal) / 2,200 MPa (high)</td>
<td></td>
</tr>
<tr>
<td>Diameter $d_f$</td>
<td>0.75 mm</td>
<td></td>
</tr>
<tr>
<td>Hook geometry $l_h^1, l_h^2, \alpha^h, \rho^h$</td>
<td>1.8 mm, 1.3 mm, 45°, 1.4 mm</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interaction</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Embedment length $L_e$</td>
<td>20 mm</td>
<td></td>
</tr>
<tr>
<td>Inclination angle $\theta$</td>
<td>0° / 15° / 30° / 45° / 60°</td>
<td></td>
</tr>
<tr>
<td>Bond strength $\tau_{max}$</td>
<td>1.70 MPa</td>
<td></td>
</tr>
<tr>
<td>Asymptotic frictional stress $\tau_0$</td>
<td>0.42 MPa</td>
<td></td>
</tr>
<tr>
<td>Reference slip $s_{ref}$</td>
<td>0.38 mm</td>
<td></td>
</tr>
<tr>
<td>Friction coefficient $\mu$</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

- two different strengths of fiber: normal-strength hooked-end steel fiber (Dramix® RC-80/60-BN) with yield stress $\sigma_y = 1,100$ MPa and high-strength fiber (Dramix® RC-80/60-BP) with $\sigma_y = 2,200$ MPa;
- five different inclination angles: 0°, 15°, 30°, 45° and 60°.

The parameters used in modeling are listed in Table 4.1. The obtained model results together with the experimental data are discussed in different pullout scenarios as follows.

### 4.3.1 Pullout of hooked-end fiber without inclination

Fig. 4.11 contains comparisons of the experimental results with the model results for different combinations of concrete strength and fiber yield stress, obtained from the pullout of hooked-end fiber embedded in concrete matrix without inclination w.r.t. the loading direction ($\theta = 0°$).

A remarkably good correlation is observed in all cases. In each diagram, the first nearly vertical ascending branch corresponds to the elastic state of hooked end; the further ascent of curve leads to the peak of pullout force which is reached when both of the arcs of hook become fully plastic and the hooked end starts to enter the straight part of fiber channel; afterwards, the pullout force decreases, accompanied with the progressive straightening and sliding of the segments of hooked end; finally, when the whole hook is straightened and sliding in the straight fiber channel, the residual pullout force decays gradually. As an example, the corresponding key states are marked in the lower-right diagram, where in addition to the maximum pullout force, the two plateaus associated with the sliding of the partially straightened hooked end in the curved fiber channel and the fully straightened hooked end sliding in the straight fiber channel, respectively, are predicted in very good agreement with the test results.
4.3. MODEL VALIDATION

Based on the comparison among the pullout responses obtained in these four cases, we have the following observations:

- The use of high-strength steel fiber leads to significant increase of pullout forces, with the peak force approximately 75% higher than using normal-strength fiber.
- In the situation of high-strength steel fibers, it appears effective to use high-strength concrete instead of normal-strength concrete, as the level of pullout force increases noticeably (20%).
- For normal-strength steel fibers, the increase of concrete strength does not change the pullout response remarkably, which implies that the normal-strength concrete matrix already provides sufficient support for the straightening of hooked end.

4.3.2 Pullout of inclined hooked-end fiber in high-strength concrete

The pullout tests of inclined hooked-end fibers were all conducted using high-strength concrete. All the model results and the respective test results are plotted in Fig. 4.12 and Fig. 4.13, from which the following phenomena are observed.
Figure 4.12: Model validation for the pullout of hooked-end fiber embedded in high-strength concrete: model results vs. experimental results of the load–displacement relation for different combinations of fiber strength and inclination angle; horizontal axis: pullout displacement (mm); vertical axis: pullout force (N).
4.3. MODEL VALIDATION

General response

With the increase of inclination angle $\theta$, the pullout force–displacement curves incline towards the horizontal axis, which indicates that the characteristic pullout status (key states) are postponed with respect to the pullout displacement: The situation $\theta = 15^\circ$ does not show remarkable difference from the case of $\theta = 0^\circ$; the pullout force–displacement characteristics change considerably when the inclination angle is increased from $30^\circ$ to $60^\circ$. Particularly, in the case of $60^\circ$, instead of a steep ascending branch prior to the peak force, the pullout force continuously increases and reaches its maximum at a rather high level of pullout displacement, followed by a flat descending branch. In this case ($60^\circ$), the first portion of the pullout process is governed by the interfacial debonding, elastic-plastic deformations in the fiber and the concrete matrix spalling; the spalling changes the geometrical configuration at the exit point of fiber from matrix significantly and leads to a large value of displacement at the free end, despite that the interfacial slip along the fiber axis is still limited.

Note that in the tests discussed here, with the increase of inclination, the fibers tend to be more vulnerable to rupture, especially in the case of normal-strength fiber. This is probably caused by the manual pre-bending of fibers while preparing the specimens, i.e. the fibers were bent to the required angle before being placed into the fresh concrete. In real engineering conditions where all fibers are directly cast with concrete matrix, rupturing may not occur frequently.

Maximum pullout force and pullout toughness

![Graphs showing pullout force, spalling depth, and toughness vs. inclination angle](image)

Figure 4.13: Comparison of the model results for different pullout cases of hooked-end steel fiber in high-strength concrete matrix: (a) maximum pullout force, (b) spalling depth and (c) pullout toughness with respect to the inclination angle.

The pullout toughness is calculated by the integration of the area under the pullout curve, upto 10 mm pullout displacement, which indicates the amount of energy absorption during the pullout procedure. As illustrated in Fig. 4.13(a) and (c), with the growing inclination angle, both the maximum pullout force and the pullout toughness increase; this tendency maintains until $45^\circ$ and starts
to decrease afterwards. The use of high-strength fiber results in considerable enhancement (by approxi-
mately 70%) to the maximum force and the toughness.

**Matrix spalling**

As shown in Fig. 4.13(b), the matrix spalling size increases with respect to the inclination angle. The use of high-strength fiber generally results in approximately 20% growth of spalling size.

**Optimal inclination angle**

Based on the above observations on the pullout behavior of inclined hooked-end fibers, similar conclusions as those in the situation without inclination can be drawn. Furthermore, concerning the influence of inclination angle, it is noticed that, generally speaking, the optimal inclination angle is 30°. When $\theta > 30^\circ$, the negative effect of the postponed peak force and the increased spalling depth becomes more remarkable.
Chapter 5

Model for Crack Bridging Effect

The analytical models presented in the preceding two chapters allow for the prediction of different single fiber pullout responses depending on various pullout configurations. These models consist the fundamental block for the multiscale oriented modeling framework for FRC materials and structures. At a higher length scale, i.e. in the FRC composite materials, the crack bridging effect, which plays a crucial role linking single fiber pullout mechanisms with the failure behavior of FRC structures, needs to be determined. In the present work, we select the analytical integration form proposed in Wang et al. (1989) as the starting point of crack bridging model. The anisotropic fiber orientation obtained with the consideration of casting process and boundary effect is taken into account. Employing the previously developed analytical model for single fiber pullout, the bridging effect is obtained via the integration of the pullout responses of all the fibers intercepting the crack. Based on the numerically computed crack bridging results, an analytical surrogate function form of the traction–separation law is obtained, which is later used conveniently in the structural simulation of FRC.
### List of symbols used in this chapter

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_f$</td>
<td>cross-section area of fiber</td>
<td>mm$^2$</td>
</tr>
<tr>
<td>$a$, $b$, $c$</td>
<td>semi-axes of the ellipsoid representing fiber orientation</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_N$</td>
<td>orientation factor with respect to N-direction</td>
<td>-</td>
</tr>
<tr>
<td>$c_1$</td>
<td>coefficient in the crack bridging law</td>
<td>mm/mm</td>
</tr>
<tr>
<td>$c_2$</td>
<td>coefficient in the crack bridging law</td>
<td>1/mm</td>
</tr>
<tr>
<td>$c_f$</td>
<td>volume content of fiber</td>
<td>mm$^3$/mm$^3$</td>
</tr>
<tr>
<td>$d_f$</td>
<td>diameter of fiber</td>
<td>mm</td>
</tr>
<tr>
<td>$E^*$</td>
<td>YOUNG’s modulus of the composite material</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$E_c$</td>
<td>YOUNG’s modulus of concrete</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$E_f$</td>
<td>YOUNG’s modulus of fiber</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$E_m$</td>
<td>YOUNG’s modulus of matrix</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$\eta_{\text{wall}}$</td>
<td>distance to the boundary of mold</td>
<td>mm</td>
</tr>
<tr>
<td>$F$</td>
<td>force</td>
<td>N</td>
</tr>
<tr>
<td>$F_{\text{sp}}$</td>
<td>single fiber pullout force</td>
<td>N</td>
</tr>
<tr>
<td>$f_c$</td>
<td>compressive strength of concrete</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$f_t$</td>
<td>tensile strength of concrete</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$f_{t*}$</td>
<td>tensile strength of the composite material</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$G_F$</td>
<td>tensile fracture energy</td>
<td>N/mm</td>
</tr>
<tr>
<td>$G_{F0}$</td>
<td>base value of fracture energy</td>
<td>N/mm</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>fiber orientation profile ($= [a, b, c]^T$)</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{1-4}$</td>
<td>fiber orientation profile in different areas of cross section</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{\text{cast}}$</td>
<td>fiber orientation profile as a result of casting process</td>
<td>-</td>
</tr>
<tr>
<td>$L_f$</td>
<td>full length of fiber</td>
<td>mm</td>
</tr>
<tr>
<td>$L_R$</td>
<td>size of cross section in R-direction</td>
<td>mm</td>
</tr>
<tr>
<td>$L_S$</td>
<td>size of cross section in S-direction</td>
<td>mm</td>
</tr>
<tr>
<td>N-R-S</td>
<td>local coordinate system of crack</td>
<td>-</td>
</tr>
<tr>
<td>$p$</td>
<td>probability density function</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>average probability density function in the cross section</td>
<td>-</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>rotational angle with respect to N-axis</td>
<td>rad</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>yield stress of steel fiber</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$t$</td>
<td>total crack bridging stress</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$t_1$</td>
<td>parameter in the crack bridging law</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$t_2$</td>
<td>parameter in the crack bridging law</td>
<td>N/mm$^3$</td>
</tr>
<tr>
<td>$\tau_{\text{coh}}$</td>
<td>cohesive stress in crack due to concrete matrix</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$\tau_{\text{fib}}$</td>
<td>fiber bridging stress across the crack</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>inclination angle of fiber</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_N$</td>
<td>fiber inclination with respect to N-direction</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_{\text{wall}}$</td>
<td>lower limit of fiber inclination angle with respect to the wall</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_R$</td>
<td>fiber inclination with respect to R-direction</td>
<td>rad</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>$\theta_{\text{wall}}^R$</td>
<td>lower limit of fiber inclination with respect to the wall perpendicular to R-direction</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_S$</td>
<td>fiber inclination with respect to S-direction</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_{\text{wall}}^S$</td>
<td>lower limit of fiber inclination with respect to the wall perpendicular to S-direction</td>
<td>rad</td>
</tr>
<tr>
<td>$V$</td>
<td>volume of the hemi-ellipsoid representing fiber orientation</td>
<td>-</td>
</tr>
<tr>
<td>$w$</td>
<td>crack opening displacement</td>
<td>mm</td>
</tr>
<tr>
<td>$w_{\text{ref}}$</td>
<td>reference value of crack opening</td>
<td>mm</td>
</tr>
<tr>
<td>$w_u$</td>
<td>ultimate value of crack opening</td>
<td>mm</td>
</tr>
<tr>
<td>$\tilde{x}$</td>
<td>distance of fiber center to crack plane</td>
<td>mm</td>
</tr>
<tr>
<td>X-Y-Z</td>
<td>global CARTESIAN coordinate system</td>
<td>-</td>
</tr>
</tbody>
</table>
5.1 Crack bridging effect of fibers

Using the analytical pullout model described in the preceding chapters, the fiber bridging effect is obtained via the integration of the pullout response of all fibers intercepting the crack, taking an anisotropic orientation of fibers into consideration.

\[ t_{\text{fib}}(w) = \frac{c_f}{A_f} \int_{\tilde{x}=0}^{L_f/2} \int_{\theta=0}^{\arccos(2\tilde{x}/L_f)} F_{\text{sfp}}(\tilde{x},\theta,w) p(\theta) \, d\theta \, p(\tilde{x}) \, d\tilde{x}. \]  

(5.2)

In Eq. (5.2), \( c_f \) is the volume fraction of the fibers and \( A_f \) is the cross-section area of one fiber. The single fiber pullout force \( F_{\text{sfp}}(\tilde{x},\theta,w) \) is dependent on \( \tilde{x} \) and \( \theta \) and is provided by the analytical model described in the preceding chapters. The spatial dispersion characteristics \( p(\theta,\tilde{x}) \) of the fiber cocktail in the composite are represented by the probability density \( p \) as a function of the inclination angle \( \theta \) and the position \( \tilde{x} \) of the fiber and can be determined as follows.
5.1. CRACK BRIDGING EFFECT OF FIBERS

5.1.1 Anisotropic fiber orientation as a consequence of casting procedure

The spatial orientation of fibers is often assumed to be random, i.e. the probability that an arbitrary fiber points in any spatial direction is identical for all directions; such an isotropic orientation can be graphically represented by a sphere (AVESTON and KELLY 1973, WANG ET AL. 1989). However, the assumption of isotropy of the post-cracking response often does not hold in practice. It is observed, that for the same fiber composition, the level of post-cracking stresses in a crack, which is perpendicular to the casting direction, is usually significantly lower than the stress level in a crack oriented parallel to the casting direction. As a consequence of the casting process of FRC structures, less fibers tend to align with the casting direction. Fibers with a larger inclination angle with respect to the crack plane generally contribute a lower pullout resistance (BARRAGÁN ET AL. 2003, CUNHA ET AL. 2011, SOETENS AND MATTHYS 2014).

Figure 5.2: Ellipsoids representing fiber orientation: (a) isotropic orientation; (b) anisotropic orientation as a result of the casting process.

Therefore, in order to capture the directional difference, we use an ellipsoid, characterized by the semi-axes \(a\), \(b\) and \(c\) to represent the fiber orientation in the 3D configuration:

\[
\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1.
\]  

(5.3)

The fiber orientation profile \(\lambda = [a, b, c]^T\) reflects the spatial preference of the fiber cocktail in the global coordinate system (X-Y-Z). As illustrated in Fig. 5.2, if \(\lambda = [0.33, 0.33, 0.33]^T\), the ellipsoid becomes a sphere and an isotropic orientation is recovered. For \(\lambda = [0.40, 0.40, 0.20]^T\), more fibers are aligned to the horizontal (X-Y) plane.

Remark 1: In the present work, the latter is assumed to be the “standard” orientation profile in conventional SFRC structures as a consequence of the casting process: \(\lambda^{\text{cast}} = [0.40, 0.40, 0.20]^T\). In fact, the final state of orientation profile \(\lambda\) in the FRC member is a result of the production process associated with many factors, including the rheological property of fresh mixture, the casting and flow direction, the mold shape and size, the method and duration of vibration, etc. (ABRISHAMBAF ET AL. 2015, LEE ET AL. 2011, SOROUSHIAN AND LEE 1990). Various methods can be used to determined the fiber orientation: In the laboratory environment, employing e.g. stereological analysis on the image of the cross sections of FRC specimens, the remarkable preference of fiber alignment...
in different spatial directions can be determined [see e.g. BABUT (1986)]; non-destructive monitoring using e.g. magnetic device (FERRARA ET AL. 2012) or with X-ray computer tomography (CT) equipment (BORDelon AND ROesler 2014) are used as well. Alternatively, analytical approaches based on fiber flow analysis [e.g. FOLGar AND TUCKer (1984) and LARANJeira ET Al. (2012)], can be considered.

Remark 2: The fiber orientation can be described in different forms. In principle, the complete and general description of fiber orientation requires a distribution function w.r.t. all the possible spatial directions; however, such a function can be cumbersome for practical use (FERRARA ET AL. 2011). Therefore, simplified representations of fiber orientation are proposed, using e.g. the orientation tensor (ADVANI AND TUCKer 1987) or the orientation factor (SOROUshIAN AND LEE 1990).

5.1.2 Determination of probability–inclination relation

Based on a given fiber orientation profile \( \lambda = [a, b, c]^T \), in conjunction with the direction of (potential) crack plane, the probability density \( p(\theta) \) can be computed as a function of the inclination angle \( \theta \) with respect to the crack. As illustrated in Fig. 5.3, it is assumed that the axes of the ellipsoid are aligned with the crack plane (R-S) and the normal direction (N) to the crack plane, respectively. This assumption is generally valid in most engineering problems.

As illustrated in Fig. 5.3, for any value of \( \theta \), the corresponding probability density \( p \) is obtained by computing the differential volume \( dV(\theta) \) of the ellipsoid between the two conical surfaces cor-
responding to $\theta$ and $\theta + d\theta$:

$$p(\theta) = \frac{d V(\theta)}{V d\theta},$$

(5.4)

with $V = 2\pi abc/3$ as the total volume of the hemi-ellipsoid characterized by $\theta \in [0, \pi/2]$ and $\varphi \in [0, 2\pi]$.

![Probability density graph](image)

**Figure 5.4:** Examples of probability density $p$ as a function of $\theta$ for different fiber orientation profiles.

As shown in Fig. 5.4, examples of different $p - \theta$ curves for a vertical crack are obtained from different orientation profiles:

- For the case $\lambda = [0.33, 0.33, 0.33]^T$, the resulting isotropic orientation is identical to the analytical form $p(\theta) = \sin \theta$ as considered in Wang et al. (1989).
- For $\lambda = [0.25, 0.50, 0.25]^T$, more fibers tend to align to the normal direction of crack, which is favorable for the post-cracking capacity.
- $\lambda = [0.40, 0.20, 0.40]^T$ corresponds to the case that more fibers tend to align to the crack plane, which is an adverse situation for crack bridging.

**Remark:** It should be emphasized that the $p - \theta$ relation is dependent on fiber orientation as well as the direction of crack. The $p - \theta$ relation obtained using Eq. (5.4) corresponds to a unit area of crack. The $p(\theta)$ result at a specific location (e.g. in the central region of FRC member) can be different from the result at another position (e.g. in the vicinity of boundaries, as will be discussed in the following).

### 5.1.3 **Boundary effect on fiber orientation**

As illustrated in Fig. 5.5, in the vicinity of boundaries, the possible spatial orientation of any fiber is limited; consequently, the fibers tend to align parallel to the boundary surfaces (Dupont and Vandewalle 2005, Laranjeira et al. 2012, Soroushian and Lee 1990). The *boundary effect* is dependent on the fiber length and the dimension of the mold. In the present work, this effect is considered following Dupont and Vandewalle (2005) to determine an average $\tilde{p}(\theta)$ relation for the calculation of the crack bridging tractions.
Figure 5.5:Boundary effect: fibers close to the wall of mold are forced to align to the boundary.

Figure 5.6:Different areas in a vertical rectangular cross section of FRC specimen characterized by different fiber orientations with or without the influence of boundaries.

As shown in Fig. 5.6, we consider a rectangular cross section of FRC specimen as the potential crack plane characterized by a width of \( L_R \) and a height of \( L_S \). Depending on the influence of the boundary, the cross section is divided into four different areas.

- **Central Area-1:** The orientation profile in the central area is assumed uniformly as \( \lambda_1 = \lambda^{\text{cast}} \), because none of the fibers belonging to this region can touch any boundary and the walls of the mold do not affect the fiber orientation.

- **Areas influenced by one wall (e.g. Area-2):** In a boundary area such as Area-2, at an arbitrary location (as denoted by the “◦”-mark in Fig. 5.7) with a distance \( \eta_{\text{wall}} \in [0, L_f/2] \) from the vertical boundary, the possible inclination angle \( \theta_R \) of any fiber with respect to the R-direction must satisfy
  \[
  \theta_{\text{wall}} \leq \theta_R \leq \frac{\pi}{2},
  \tag{5.5}
  \]
  with the smallest possible angle
  \[
  \theta_{\text{wall}} = \arccos\left(\frac{2\eta_{\text{wall}}}{L_f}\right).
  \tag{5.6}
  \]

Therefore, the boundary effect on fiber orientation is accounted for by means of excluding the volume of the ellipsoid contained in the cone characterized by \( \theta_{\text{wall}} \) (Fig. 5.7) while computing the \( p - \theta_N \) relation using Eq. (5.4). Note that \( \theta_N \) is the inclination angle with respect to the normal (N) direction of the (potential) crack plane.
5.1. CRACK BRIDGING EFFECT OF FIBERS

\( \theta_{R}^{\text{wall}} \leq \theta_{R} \leq \frac{\pi}{2}, \quad \theta_{S}^{\text{wall}} \leq \theta_{S} \leq \frac{\pi}{2} \). \hspace{1cm} (5.7)

Here, \( \theta_{S} \) is the possible inclination angle of any fiber with respect to the horizontal boundary and \( \theta_{S}^{\text{wall}} \) indicates the lower limit of \( \theta_{S} \).

It is noticed that, considering the boundary effect, the fiber orientation and the resulting \( p - \theta_{N} \) relations are dependent on the local coordinate (denoted as \( r \) and \( s \)) in the crack plane. Therefore, the final average \( \bar{p} - \theta_{N} \) relation for the whole cross section is computed as

\[
\bar{p}(\theta_{N}) = \frac{1}{L_{R}L_{S}} \int_{-L_{R}/2}^{L_{R}/2} \int_{-L_{S}/2}^{L_{S}/2} p(r, s, \theta_{N}) \, ds \, dr.
\] \hspace{1cm} (5.8)

The \( \bar{p} - \theta_{N} \) relation is used in Eq. (5.2) for the integration of the fiber bridging effect for the (potential) crack plane.

Remark 1: A further operation on \( p - \theta_{N} \) relation leads to a scalar value \( \alpha_{N} \)

\[
\alpha_{N} = \int_{0}^{\pi/2} p(\theta_{N}) \cos \theta_{N} \, d\theta_{N}
\] \hspace{1cm} (5.9)

which is frequently referred to as the “orientation factor” (with respect to N-direction). As an example, considering \( L_{f} = 51 \) mm, \( L_{R} = 152 \) mm, \( L_{S} = 152 \) mm as reported in SOROUSHIAN AND LEE (1990), the final orientation factor calculated is \( \alpha_{N} = 0.63 \), which agrees very well with the experimentally measured result as 0.62. This agreement provides an example for the validity of the approach proposed in the present work.

Remark 2: The situation described above is considered as a “standard” case for the analysis of boundary effect. More specific situations, where, for example, the dimension of cross section is too small, the FRC specimen is notched or the cross section is a circle, can be dealt with based on the same principles.

**Figure 5.7:** Boundary effect on fiber orientation: in the vicinity of wall of mold, the possible fiber orientation is restricted.
5.1.4 Other factors influencing the fiber bridging effect

The distribution of fibers is considered to be homogeneous within the width of the representative volume, i.e. \( \tilde{x} \in [0, L_f/2] \); therefore, a constant function of

\[
p(\tilde{x}) = \frac{2}{L_f}
\]

is used (WANG ET AL. 1989).

An additional aspect to be considered is the observation from (limited) experimental observations, that only 50\% - 90\% of hooked-end steel fibers are active, due to the loss of efficiency as a consequence of the "group-effect": In the FRC composite containing a large number of distributed fibers, the fibers are too close to each other for the matrix to provide sufficient support for the straightening of the hooked ends of every fiber (BARRAGÁN ET AL. 2003, CUNHA 2010, CUNHA ET AL. 2011). In the present work, a proportion of 70\% is assumed for low-content FRC (with less than 1\% fibers by volume) and applied to the fiber content \( c_f \) while computing the bridging stresses using Eq. (5.2). It is (qualitatively) clear that with the increase of fiber content, lower fiber effectiveness ratio will be observed; however, a substantial investigation of the group-effect, addressing only the fiber content but also other important parameters such as fiber-matrix bonding properties into consideration, would be helpful for the determination of this reduction factor more accurately.

5.2 Results of crack bridging law

5.2.1 Total crack bridging response

It is generally accepted that the total bridging effect for an opening crack is obtained as the total contribution of concrete matrix and fiber (LI ET AL. 1993, STRACK 2007):

\[
t(w) = t^{\text{coh}}(w) + t^{\text{fib}}(w),
\]

Here \( t^{\text{coh}} \) represents the cohesion of concrete matrix, and \( t^{\text{fib}} \) indicates the fiber bridging stress.

The cohesive law in plain concrete materials is frequently expressed in an exponential softening law:

\[
t^{\text{coh}} = f_t \exp \left( -\frac{w}{w_{\text{ref}}} \right),
\]

with the reference value of crack opening displacement calculated as

\[
w_{\text{ref}} = \frac{G_F}{f_t},
\]

here, \( G_F \) is the tensile fracture energy and \( f_t \) is the tensile strength of plain concrete. The values of the plain concrete parameters are usually available from standard laboratory tests.

Remark: In the case that only the compressive strength of plain concrete \( f_c \) is provided, the following formulas can be used to determine the Young’s modulus and the tensile strength (DEUTSCHES
5.2. RESULTS OF CRACK BRIDGING LAW

\[ E_c = 9500 \cdot f_c^{1/3} \]

\[ f_t = \begin{cases} 
0.3 \cdot (f_c - 8)^{2/3} & \text{for } f_c < 58 \text{ N/mm}^2 \\
2.12 \cdot \ln(0.1 \cdot f_c + 1) & \text{for } f_c \geq 58 \text{ N/mm}^2 
\end{cases} \]  

(5.14)

The tensile fracture energy \( G_F \) can be calculated according to INTERNATIONAL FEDERATION FOR STRUCTURAL CONCRETE (1993):

\[ G_F = G_{F0} \left( \frac{f_c}{f_{c0}} \right)^{0.7} \]  

(5.15)

with \( f_{c0} = 10 \text{ N/mm}^2 \), and the base value of fracture energy \( G_{F0} = 0.025 \text{ N-mm/mm}^2 \), 0.03 N-mm/mm\(^2\) or 0.058 N-mm/mm\(^2\) for the maximum aggregate size 8 mm, 16 mm or 32 mm, respectively. Alternatively, one may also refer to more sophisticated forms such as the one proposed by BAŽANT AND BECQ-GIRAUDON (2002).

5.2.2 Analytical surrogate function of crack bridging law

Since the single fiber pullout responses are computed by means of a numerical algorithm (ZHAN AND MESCHKE 2014), the fiber bridging stress [Eq. (5.2)] and the resulting total bridging effect in the crack [Eq. (5.11)] are obtained numerically as well.

For the convenient use of traction–separation laws in the structural simulations, an analytical surrogate model is employed to replace the \( t - w \) relation within a finite element structural model. To this end, the following parameterized function is selected:

\[ t(w) = (f_t^* - t_1) \exp \left( -\frac{w}{w_u} \right) + t_1 \frac{w_u - w}{w_u} + t_2 w \exp(c_1 - c_2 w). \]  

(5.16)

Here \( t_1 \) (MPa), \( t_2 \) (MPa/mm), \( c_1 \) (mm/mm) and \( c_2 \) (1/mm) are coefficients determined from fitting to the numerical evaluation of Eq. (5.11), \( w_u = L_f/2 \) represents the ultimate crack opening and \( f_t^* \) is the (slightly increased) tensile strength of the FRC composite (\( f_t^* \approx 1.02 f_t \) for 1% steel fiber) approximated as

\[ f_t^* = f_t \frac{E^*}{E_m}. \]  

(5.17)

\( E^* \) is the effective elastic modulus of the composite calculated from continuum micromechanics homogenization based on the MORI-TANAKA scheme. It can be expressed in the simplified format (TENG ET AL. 2004) as

\[ E^* = E_m \frac{1 + 2.5 \beta cf}{1 - \beta cf}, \quad \text{with } \beta = \frac{E_f/E_m - 1}{E_f/E_m + 2.5}. \]  

(5.18)

Remark: The selected function form consists of three terms. The first term is correlated with the traction of plain concrete; the second component is associated with the interface frictional responses during the whole fiber pullout procedure; the third term is selected to capture the increasing branch of bridging stress induced by the anchorage effect of hooked ends. Therefore, Eq. (5.16) can be conveniently used to describe the crack bridging law in different fiber cocktails (Fig. 5.8).
5.3 Validation of crack bridging model

5.3.1 Uniaxial tension test on a notched specimen

Description of problem

The first validation example investigates the quality of the model in describing the bridging effect of fibers across a single horizontal crack. To this end, uniaxial tension tests have been performed on notched prismatic specimens as illustrated in Fig. 5.9 and reported in Putke et al. (2014). The prism has the dimension of $150 \times 150 \times 300$ (mm) with a 30 mm notch at the middle height along each of the four faces. When subjected to a tensile force $F$, a horizontal crack emanates from the notches and eventually cuts through the complete cross section. The crack bridging stress is computed as $t = F/(90 \times 90 \text{mm}^2)$; the crack opening displacement $w$ is measured across the notch.

The SFRC composites are made of high-strength concrete ($f_c = 84 \text{ MPa}$). Two different fiber cocktails have been investigated:

- “L60”: SFRC containing 0.77% (60 kg/m$^3$) long hooked-end fibers (Dramix® RC-80/60-BN).
Table 5.1: Material Parameters used for the notched prism.

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Concrete matrix</strong></td>
<td>Elastic modulus</td>
<td>$E_m = 41,605$ MPa</td>
</tr>
<tr>
<td></td>
<td>Compressive strength</td>
<td>$f_{c} = 84$ MPa</td>
</tr>
<tr>
<td></td>
<td>Tensile strength</td>
<td>$f_{t} = 4.75$ MPa</td>
</tr>
<tr>
<td></td>
<td>Tensile fracture energy</td>
<td>$G_F = 0.13$ N/mm</td>
</tr>
<tr>
<td><strong>Steel fiber</strong></td>
<td>Fiber type</td>
<td>hooked-end, straight</td>
</tr>
<tr>
<td></td>
<td>Yield stress</td>
<td>$\sigma_y = 1,100$, $2,200$ MPa</td>
</tr>
<tr>
<td></td>
<td>Length</td>
<td>$L_f = 60$, $13$ mm</td>
</tr>
<tr>
<td></td>
<td>Diameter</td>
<td>$d_f = 0.75$, $0.19$ mm</td>
</tr>
</tbody>
</table>

- “L30S30”: SFRC made of a mixture containing 30 kg/m$^3$ long hooked-end fibers and 30 kg/m$^3$ short straight fibers.

The material parameters used in the analysis are listed in Table 5.1.

The specimens are produced in a “standing” mold; hence, the horizontal crack is perpendicular to the casting direction (Fig. 5.9). The fiber orientation profile $\lambda_{\text{cast}} = [0.40, 0.40, 0.20]^T$ is considered for the determination of the $p - \theta$ relation. Note, that due to the saw-cut notch, the wall effect has been eliminated in this case; consequently, the result of Case III in Fig. 5.4 can be used directly in Eq. (5.2) for the computation of fiber bridging effect.

**Discussion of results**

![Figure 5.10](image)

**Figure 5.10**: Results of re-analysis of the uniaxial tension tests on notched prism: crack bridging stress across a horizontal crack obtained from the experiments, the original numerical crack bridging model according to Eq. (5.11) and the analytical surrogate model according to Eq. (5.16) for (a) Case L60 and (b) Case L30S30.

Fig. 5.10 shows the results of crack bridging stress vs. the crack opening displacement using a
numerical evaluation of Eq. (5.11) (red dotted lines) and the analytical surrogate form according to Eq. (5.16) (blue solid lines). The range of experimental results is included in grey. For comparison, also the model response for plain concrete is included (black dash-dotted lines).

In general, the relations between the bridging stress and the crack opening are well captured by the proposed crack bridging model. The coefficients for the analytical form of the traction–separation law [Eq. (5.16)], are obtained from fitting to the original numerical model as follows:

- For L60, a \( t_{L60}(w) \) function is obtained with the coefficients \( t_1 = 0.77, t_2 = 0.45, c_1 = 0.50 \) and \( c_2 = 0.64 \). (Note that \( w_u = 30 \) mm for long hooked-end fibers.)

- For L30S30, the short straight fibers are first considered independently, assuming a content of 60 kg/m\(^3\), which results in a \( t_{S60}(w) \) relation with \( t_1 = 0.40 \) and the remaining coefficients equal to zero. (Note that \( w_u = 6.5 \) mm for short straight fibers.) Afterwards, the traction–separation law is obtained as the weighted average of them, i.e. \( t_{L30S30} = 0.5t_{L60} + 0.5t_{S60} \).

As can be seen in Fig. 5.10, the original (numerically evaluated) model and the analytical surrogate model yield almost identical results.

It is observed in Fig. 5.10(a) that, for Case L60, after the crack initiates, the bridging stress drops rapidly to approximately 0.8 MPa. However, in contrast to plain concrete, where the post-cracking stress continues decaying quickly, the SFRC exhibits a fairly ductile behavior, including an increasing portion (with the peak stress approximately equal to 1.2 MPa) associated with the anchorage forces of the hooked ends of fibers, followed by a slow decreasing curve connected with the pulling out of fibers. For Case L30S30, similar tendency of traction–separation relation is noticed [Fig. 5.10(b)]: Due to the presence of short straight fibers, the post-peak stress decays to approximately 0.6 MPa; subsequently, the stress recovers slightly to a level of 0.8 MPa, followed by a slow descend of the bridging stress. If one compares this case (L30S30) with the previous one (L60), one can discover that the use of short straight fibers, despite their high strength, leads to a lower post-cracking ductility due to the lack of anchorage effect contributed by deformed shapes such as hooked ends. Of course, these “micro fibers” may have other advantages; for instance, they can be more flexibly cast in a small scale where the use of long fibers is hardly possible.

One may notice, especially in Fig. 5.10(b), that the experimental values of crack opening displacement corresponding to the post-peak descending branch are significantly larger than the values obtained from the model. This is due to the fact that it was difficult in the laboratory environment to obtain a perfectly simultaneous development of cracks from every side; consequently, the homogeneous opening of the horizontal major crack could not be guaranteed, which leads to an overestimation of the crack mouth opening displacement.

Recalling the discussion on fiber orientation, this case corresponds to an unfavorable situation for the post-cracking capacity. A favorable case is discussed in the following validation example.

### 5.3.2 Uniaxial tension test on a “dog-bone” specimen

**Description of problem**

In the second example, a uniaxial tension test on “dog-bone” specimens reported in SUSETYO (2009) is analyzed. The experimental setup is illustrated in Fig. 5.11(left). The thickness of the specimen
(in Z-direction) is 70 mm. In the test, a crack perpendicular to the tensile direction (X) opens approximately along the symmetry line of the specimen. The FRC composite is made of normal-strength concrete ($f_c = 50$ MPa) and contains 0.5% or 1.0% hooked-end steel fibers (Dramix® RC-80/50-BN). The material parameters are listed in Table 5.2.

![Diagram of uniaxial tension test on "dog-bone" specimen](image)

**Figure 5.11:** Setup of the uniaxial tension tests on “dog-bone” specimen (left). Results of $\bar{p} - \theta$ relation with and without consideration of the influence of casting process and boundary effect (right).

In contrast to the previous example, the specimen has no predefined notch, the boundary effect, leading to a preferred orientation of the fibers in the vicinity of the free surfaces, should be taken into account: For the determination of the average $\bar{p} - \theta$ relation for the (potential) crack plane, the cross section of specimen ($L_y = 100$ mm and $L_z = 70$ mm), which is perpendicular to the tensile force, is divided into four different areas with and without the influence of boundaries (see Fig. 5.6), and the boundary effect in each area is evaluated. Fig. 5.11(right) shows the resulting average $\bar{p} - \theta$ relation in the cross section considered in this test. It is observed that the casting process in conjunction with the boundary effect has a remarkable impact on the orientation of fibers. In comparison with the situation where the isotropic fiber orientation is assumed and the boundary effect is neglected, more fibers tend to align (or with a small angle) to the normal direction of the crack, which will result in a higher post-cracking ductility.

**Discussion of results**

According to Fig. 5.12 the test results are characterized by a relatively large scatter (grey area). Without surprise, the FRC exhibits remarkable post-cracking ductility, which is well captured by the present model. Both the test results and the crack bridging model show, after an initial drop, an increase of the crack bridging stress, in which the hooked ends are activated, is followed by a decreasing branch associated with the successive pullout of fibers. The coefficients of the analytical surrogate model are determined as $t_1 = 0.81$, $t_2 = 0.82$, $c_1 = 1.14$ and $c_2 = 1.03$ for the FRC containing 0.5% fibers [Fig. 5.12(a)] and $t_1 = 1.38$, $t_2 = 1.20$, $c_1 = 1.29$ and $c_2 = 1.08$ for the FRC containing 1.0% fibers [Fig. 5.12(b)].

If one compares the result of FRC containing 0.5% fibers obtained from this “dog-bone” test with respect to the case L60 in the previous example, one may notice that the concrete has a lower
Table 5.2: Material parameters used in the analysis of the “dog-bone” test.

<table>
<thead>
<tr>
<th>Concrete matrix</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus $E_c$</td>
<td>34,998 MPa</td>
</tr>
<tr>
<td>Compressive strength $f_c$</td>
<td>50 MPa</td>
</tr>
<tr>
<td>Tensile strength $f_t$</td>
<td>3.62 MPa</td>
</tr>
<tr>
<td>Tensile fracture energy $G_F$</td>
<td>0.09 N/mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Steel fiber</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber type</td>
<td>hooked-end</td>
</tr>
<tr>
<td>Yield stress $\sigma_y$</td>
<td>1,100 MPa</td>
</tr>
<tr>
<td>Length $L_f$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Diameter $d_f$</td>
<td>0.62 mm</td>
</tr>
</tbody>
</table>

Figure 5.12: Results of re-analysis of the uniaxial tension tests on “dog-bone” specimens: crack bridging stress across a single vertical crack obtained from the experiments, the original numerical crack bridging model according to Eq. (5.11) and the analytical surrogate model according to Eq. (5.16) for the FRC containing (a) 0.5% fibers and (b) 1.0% fibers.

strength and, although a similar type of fiber is used, the fiber content is 35% lower. Surprisingly, a significantly higher level of post-cracking stresses is observed in this case [Fig. 5.12(a)]: The peak stress of the hardening branch (approximately 1.7 MPa) is nearly 50% higher as compared to the previous example [see Fig. 5.10(a)]. This result demonstrates the impact of the casting process and the wall effect on the fiber orientation and, consequently, on the post-cracking capacity of SFRC.

During the re-analysis of the FRC with 1.0% fibers, an overestimation of the crack bridging stress was noticed [see the red short-dashed line in Fig. 5.12(b)], if assuming that 70% of the hooked-end steel fibers are active (as in the previous cases). As mentioned in Section 5.1, due to the increased group-effect in FRC containing high content of fibers, the portion of hooked-end fibers that are active during the crack opening decreases. Therefore, the crack bridging effect is re-calculated by assuming an active ratio of 60%, which leads to a result that is in good agreement with the experimental range.
[see the red long-dashed line in Fig. 5.12(b)] and, subsequently, the surrogate analytical form of crack bridging law (the blue solid curve).

In SUSETYO (2009), the FRC with 1.5% fibers is tested as well. Assuming 60% fiber efficiency, the peak bridging stress is calculated by the present crack bridging model as 4.2 MPa, which falls well in the range measured during the experiments. However, due to the super-critical crack bridging effect, the specimens exhibit strain-hardening behavior accompanied with multiple-cracking phenomena. In such a situation, the measurement of the opening displacement of one single crack becomes rather inaccurate, since several cracks can be contained in the range of measuring device. Therefore, the crack bridging effect for this case is not graphically presented here.
Part II

SIMULATIONS OF DAMAGE PHENOMENA IN FIBER-REINFORCED CONCRETE STRUCTURES
Chapter 6

Numerical Models for Cracking in Concrete: Short Review

In the last decades, a large number of numerical models for concrete cracking, aiming at the reliable prognoses of the fracture processes of concrete structures with or without reinforcement, have been developed. Cracks can be represented as a damage zone, using continuum-based approaches such as plasticity or damage formulations, rotating or fixed crack models. Alternatively, models allowing for a discrete representation of cracks within finite element analyses have been developed by introducing cracks as independent entities directly into the finite element mesh, separating individual finite elements or even incorporating the failure kinematics into the finite element formulations as cracks evolve [see e.g. Hofstetter and Meschke (2011) for an overview of the popular types of numerical model for concrete cracking]. Based on the models for plain concrete, numerical models for structural analyses of FRC are developed, by means of modifying the post-peak regime of inelastic constitutive models to represent the enhanced ductility of FRC in terms of an increase of the fracture energy. In this chapter, the author attempts to provide a brief review of concrete cracking models, by means of introducing the essential concepts and governing equations, the advantages and drawbacks of different models.
**List of symbols used in this chapter**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>internal variable for the inelastic behavior of material</td>
</tr>
<tr>
<td>$B$</td>
<td>strain operator (standard finite element method)</td>
</tr>
<tr>
<td>$C_e$</td>
<td>elastic stiffness of material (4th-order tensor)</td>
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<tr>
<td>$D$</td>
<td>surface energy dissipation rate N/(mm·s)</td>
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<tr>
<td>$D_{0}$</td>
<td>initial stiffness matrix of discrete crack</td>
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<tr>
<td>$d$</td>
<td>damage variable</td>
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<td>$\delta_{\Gamma}$</td>
<td>DIRAC’S delta on $\Gamma$</td>
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<td>boundary of domain/strong discontinuity surface</td>
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<td>$\Gamma_u$</td>
<td>DIRICHLET boundary (subjected to displacement conditions)</td>
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<td>separation-mode matrix for finite element with crack</td>
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<td>HEAVISIDE step function on $\Gamma$</td>
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<td>2nd-order unity tensor</td>
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<tr>
<td>$I_1$</td>
<td>first principal invariant of stress N/mm²</td>
</tr>
<tr>
<td>$J_2$</td>
<td>second principal invariant of deviatoric stress $\mathbf{s}$ (N/mm²)²</td>
</tr>
<tr>
<td>$J_3$</td>
<td>third principal invariant of deviatoric stress $\mathbf{s}$ (N/mm²)³</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>inelastic multiplier</td>
</tr>
<tr>
<td>$L_{1,2}$</td>
<td>length of measurement device mm</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>n</td>
<td>normal vector of surface or crack</td>
</tr>
<tr>
<td>N-R-S</td>
<td>local coordinate system of crack</td>
</tr>
<tr>
<td>ν</td>
<td>POISSON’s ratio of material</td>
</tr>
<tr>
<td>Ω</td>
<td>domain occupied by the material body</td>
</tr>
<tr>
<td>p</td>
<td>hydrostatic stress N/mm²</td>
</tr>
<tr>
<td>Q</td>
<td>stress projection operator</td>
</tr>
<tr>
<td>q</td>
<td>decaying limit stress of material (softening law) N/mm²</td>
</tr>
<tr>
<td>ψ</td>
<td>HELMHOLTZ free energy of unit surface N/mm</td>
</tr>
<tr>
<td>s</td>
<td>deviatoric stress (tensor)</td>
</tr>
<tr>
<td>σ</td>
<td>stress (scalar) N/mm²</td>
</tr>
<tr>
<td>σᵧ</td>
<td>yield stress N/mm²</td>
</tr>
<tr>
<td>σ ₁,₂,₃</td>
<td>principal stresses N/mm²</td>
</tr>
<tr>
<td>σ̄</td>
<td>equivalent stress N/mm²</td>
</tr>
<tr>
<td>T</td>
<td>coordinate transformation matrix</td>
</tr>
<tr>
<td>t</td>
<td>traction (scalar) N/mm²</td>
</tr>
<tr>
<td>t</td>
<td>traction (vector)</td>
</tr>
<tr>
<td>tₙ,s</td>
<td>normal/tangential component of t N/mm²</td>
</tr>
<tr>
<td>ṭ</td>
<td>equivalent traction N/mm²</td>
</tr>
<tr>
<td>ṭbulk</td>
<td>traction (vector) computed from the bulk material</td>
</tr>
<tr>
<td>ṭcrack</td>
<td>traction (vector) computed in the crack</td>
</tr>
<tr>
<td>ṭ*</td>
<td>prescribed traction on the boundary</td>
</tr>
<tr>
<td>u</td>
<td>displacement field (scalar) mm</td>
</tr>
<tr>
<td>u</td>
<td>displacement field (vector)</td>
</tr>
<tr>
<td>u*</td>
<td>prescribed displacement on the boundary</td>
</tr>
<tr>
<td>uₑ</td>
<td>nodal displacement vector of finite element</td>
</tr>
<tr>
<td>ū</td>
<td>continuous part of displacement field</td>
</tr>
<tr>
<td>ēu</td>
<td>discontinuous part of displacement field</td>
</tr>
<tr>
<td>[u]</td>
<td>displacement discontinuity (scalar) mm</td>
</tr>
<tr>
<td>[u]</td>
<td>displacement discontinuity (vector)</td>
</tr>
<tr>
<td>[u]ₙ,r,s</td>
<td>normal/tangential component of [u] mm</td>
</tr>
<tr>
<td>Vₑ</td>
<td>volume of finite element mm³</td>
</tr>
<tr>
<td>w</td>
<td>crack opening (scalar) mm</td>
</tr>
<tr>
<td>w</td>
<td>crack opening (vector)</td>
</tr>
<tr>
<td>wₙ,s</td>
<td>normal/tangential component of w mm</td>
</tr>
<tr>
<td>X</td>
<td>position vector of a material point</td>
</tr>
<tr>
<td>X-Y-Z</td>
<td>global CARTESIAN coordinate system</td>
</tr>
<tr>
<td>0</td>
<td>zero (vector or tensor)</td>
</tr>
</tbody>
</table>
6.1 Cracking in concrete materials

Cracking processes in quasi-brittle materials

Concrete materials, either with or without reinforcement, contain a number of natural flaws such as voids and weak interface layers between the aggregates and cement paste. As illustrated in Fig. 6.1(a), a macroscopic crack in concrete structure usually initiates at the location suffering highly concentrated tensile stresses. A developing macro crack consists of different fracture zones characterized by different cracking states. Concerning a representative volume element (RVE) of material at a specific location on the macroscopic crack path, with the increase of tensile loads and the propagation of macro crack, this RVE can experience different cracking stages [see Fig. 6.1(b) and, e.g., BAŽANT AND OH (1983)]:

- Stage-I: Micro cracks initiate from the inherent flaws when the tensile stress acting on the RVE approaches the tensile strength ($f_t$) of concrete.
- State-II: As soon as the limit stress ($f_t$) is reached, some of the micro cracks start to merge into a macroscopic crack that penetrates the RVE, which is known as the crack localization phenomenon.
- State-III: With further increase of tensile loads, the localized crack opens rapidly, accompa-
6.1. CRACKING IN CONCRETE MATERIALS

Nied by the softening of material, i.e. the descending of residual stress carried by the remaining micro-structures such as aggregates that still connect both sides of the macro crack. Meanwhile, the neighboring micro cracks are unloaded.

The macroscopic behavior of the RVE experiencing cracking can be described by means of stress vs. strain formulation [Fig. 6.1(c)]. In engineering practice, the micro-cracking stage (Stage-I) is usually neglected; as a result, the ascending branch of stress–strain relation associated with the responses of intact material and the micro-cracking process is often simplified and represented by the linear elastic stage characterized by the Young’s modulus $E$. Alternatively, one may use a traction ($t$) vs. separation ($w$) relation to represent the softening behavior of RVE [Fig. 6.1(d)]; here the separation $w$ is the total crack opening width in the tensile direction contributed by all the micro cracks and the localized crack in the RVE. From the $t - w$ relation, an important quantity $G_F$, i.e. the fracture energy consumed (per unit area of the plane perpendicular to the tensile direction) during the opening of all the micro cracks as well as the localized crack can be computed as (BAŽANT AND OH 1983, HILLERBORG ET AL. 1976)

$$G_F = \int_{t=f}^{0} t(w) \, dw,$$

which can be graphically interpreted as the area under the softening branch of $t - w$ curve. This value is generally accepted as an inherent material property that is only dependent on the composition of concrete.

![Figure 6.2](image)

**Figure 6.2:** “Size effect” on the softening behavior of concrete under tension: (a) concrete specimen containing a crack band (assuming that the deformation is measured in two different ranges, i.e. $L_1$ and $L_2$); (b) almost identical results of $t - w$ relations obtained from $L_1$ and $L_2$; (c) remarkably different results of the softening branch of $\sigma - \varepsilon$ relation derived from the measurements in $L_1$ and $L_2$.

It is worth pointing out that due to the existence of a crack band (characterized by the width $h_c$) along the macroscopic crack path [Fig. 6.1(a)], to which the occurrence of cracking is limited, the experimentally determined softening behavior of concrete can be dependent on the range of measurement. As illustrated in Fig. 6.2, this “size effect” (BAŽANT AND OH 1983) is understood as follows: In the laboratory tests, as long as the measurement device covers the crack band ($L \geq h_c$), the softening behavior of the concrete RVE on the macroscopic crack path can be obtained as $t - w$...
relation; the \( t - w \) results should be nearly identical, regardless of the difference in the range of measurement [Fig. 6.2(b)]; this is due to the fact that the post-peak softening behavior of concrete is governed by the total crack opening displacement \( w \) in the crack band. On the contrary, if the softening behavior is described as \( \sigma - \varepsilon \) relations (where \( \sigma = t \) and \( \varepsilon = w/L \)), the results can be influenced by the length \( L \) of measurement device [Fig. 6.2(c)], because the cracking-induced deformation that is restricted in the crack band is “smeared” (averaged) in the range of measurement by assuming uniform deformation. This size effect has to be taken into account in some types of concrete cracking model to obtain objective results from structural simulations (see Section 6.3).

**Cracking as boundary value problems**

![Cracking in concrete structure as a boundary value problem.](image)

The concrete cracking process can be formulated as a *boundary value problem* (Fig. 6.3), starting with the standard *linear elastic structural mechanics* formulations as briefly described in the following. Note that in the present work, the *quasi-static* and *small-strain* assumptions are considered.

The boundary (denoted as \( \Gamma \)) of the domain (\( \Omega \)) occupied by the material body is divided into two parts: the *DIRICHLET* boundary (\( \Gamma_u \)) subjected to displacement-controlled conditions and the *NEUMANN* boundary (\( \Gamma_\sigma \)) under stress-type loading conditions:

\[
\partial \Omega = \Gamma = \Gamma_u \cup \Gamma_\sigma; \quad \Gamma_u \cap \Gamma_\sigma = \emptyset.
\]  

(6.2)

The *DIRICHLET* boundary condition can be written in the general form as follows:

\[
u(X) = u^*(X) \quad \forall X \in \Gamma_u.
\]  

(6.3)

Here, the position vector \( X \) represents the original coordinate of a material point in the domain and \( u^* \) is the prescribed displacement on the boundary. The *NEUMANN* boundary condition imposes the equilibrium of stresses

\[
\sigma(X) \cdot n = t^*(X) \quad \forall X \in \Gamma_\sigma,
\]  

(6.4)

with \( \sigma \) the *CAUCHY* stress (in tensor form), \( n \) the unit vector of the outward normal direction and \( t^* \) the applied traction at \( X \) on the boundary. The stresses at an arbitrary point is associated with the
strains via the constitutive law
\[
\sigma = C_e : \varepsilon \quad \forall X \in \Omega,
\]
with $C_e$ the linear elastic stiffness tensor of material. The strain tensor $\varepsilon$ is obtained as the symmetric part of the displacement gradient, under the assumption of small-strain kinematics:
\[
\varepsilon = \nabla^{\text{sym}} u \quad \forall X \in \Omega.
\]
Without considering the body force (induced by, e.g., the gravity), the quasi-static (neglecting the dynamic effect) balance of momentum has to be satisfied at any material point:
\[
\text{div} \sigma = 0 \quad \forall X \in \Omega.
\]

The equations above define a set of differential equations for the linear elastic boundary value problem. Considering the presence of cracks (Fig. 6.3), additional equations describing the constitutive behavior of cracks and the crack-induced kinematics need to be supplied.

In principle, all those equations describing the BVP have to be satisfied at any point in the domain; unfortunately, an analytical solution providing accurate results is merely valid for a rather limited simple situations but generally not available for practical engineering problems. To this end, numerical methods, in particular the finite element methods (FEM) are developed to obtained approximate solutions to the BVP [see e.g. ZIENKIEWICZ AND TAYLOR (2005)]. In the last decades, particularly with the development of powerful computing devices, a variety of numerical methods have been proposed to take the cracks into account while solving the BVP of structures made of concrete-based materials. In these methods, cracks can either be considered as exposed surfaces subjected to tractions depending on the crack opening, or as internal materials characterized by degraded stiffness; the cracking induced kinematics can be smeared within a small range, or be concentrated into a discrete surface in the domain; if the concrete contains reinforcing ingredients, the reinforcement can be mixed with the concrete matrix or considered separately.

### 6.2 Discrete crack models

As early as the 1960s, NGO AND SCORDELIS (1967) and NILSON (1968) already introduced the separation of finite elements to model the interfacial damage along predefined paths on the borders of elements. The inevitable mesh-dependency of crack paths and, consequently, the simulation results, can be reduced by employ e.g. the remeshing techniques, at the cost of increased complexity of algorithm and computational expense (INGRAFFEA AND SAOUMA 1985, XIE AND GERSTLE 1995). Nevertheless, discrete cracks placed as separate entities between bulk elements enable, if equipped with an appropriate constitutive model describing the post-cracking behavior, the numerical simulation of various types of interface damage induced failure problems, such as the delamination in composite materials, the static and dynamic fracturing in (quasi-) brittle heterogeneous materials (e.g. rock and concrete) (BALZANI AND WAGNER 2008, CAROL ET AL. 2001, GENS ET AL. 1989, XU AND NEEDLEMAN 1994).
Constitutive model for a discrete crack

As described in the preceding section, after the crack initiation the softening behavior accompanied with the opening of crack can be formulated as traction (stress) vs. separation (displacement) relation, based on the concept of fictitious crack model (HILLERBORG ET AL. 1976). The softening behavior of discrete cracks are described by various cohesive crack models, characterized by different ways of coupling the normal and tangential responses of crack (CABALLERO ET AL. 2008, CAMACHO AND ORTIZ 1996, STANKOWSKI ET AL. 1993).

![Figure 6.4: Softening behavior of a discrete crack: (a) traction vector \( t \) induced by the separation vector \( w \); (b) softening law, describing the decaying limit stress as the crack opens.](image)

The cohesive model for a discrete crack can be formulated based on the principles of thermodynamics [see e.g. OLIVER (2000) and JIRÁSEK AND ZIMMERMANN (2001)]. The surface density of free energy during the degradation process of the crack can be written as

\[
\psi(w, \gamma) = \frac{1}{2\gamma} w \cdot D_{\text{ref}} \cdot w, \tag{6.8}
\]

where \( w = [w_n, w_s]^T \) is the local crack opening vector [see the 2D illustration in Fig. 6.4(a)]; the damage-like variable \( \gamma \in (0, \infty) \) reflects the degradation of the cohesive material in the crack; \( D_{\text{ref}} \) is a reference stiffness matrix that can be selected to be corresponding to a specific damage state (JIRÁSEK AND ZIMMERMANN 2001). The energy dissipation of any process must satisfy the following inequality:

\[
D = t \cdot \dot{w} - \dot{\psi} \geq 0, \tag{6.9}
\]

with \( D \) denoting the dissipation rate per unit area of crack.

The constitutive relation which links the local traction vector \( t = [t_n, t_s]^T \) with the separation vector across the crack can be written as

\[
t = \frac{\partial \psi}{\partial w} = \frac{1}{\gamma} D_{\text{ref}} \cdot w. \tag{6.10}
\]
The elastic domain in the traction (stress) space is defined for the admissible traction as

$$\mathcal{E} \equiv \{ t \mid f(t, \alpha) \leq 0 \},$$  \hspace{1cm} (6.11)

with $f$ the loading function defined in e.g. the following format

$$f(t, \alpha) = \tilde{t} - q(\alpha) \leq 0;$$  \hspace{1cm} (6.12)

here, $\tilde{t}(t)$ is the equivalent traction which is an indicator of the stress level that can be calculated from $t$ in different forms. The softening law

$$q = q(\alpha)$$  \hspace{1cm} (6.13)

describes the relation between the decaying limit stress $q$ and the displacement-like internal parameter $\alpha$ [Fig. 6.4(b)]; the internal parameter

$$\alpha = \alpha(w)$$  \hspace{1cm} (6.14)

is dependent on the opening history of crack. The value of the damage-like variable $\gamma$ indicating the deterioration of interface can be determined from the value of internal parameter $\alpha$

$$\gamma = \gamma(\alpha).$$  \hspace{1cm} (6.15)

The evolution of the internal parameter $\alpha$ coincides with the inelastic multiplier $\dot{\lambda}$ representing the rate of the inelastic process in the crack:

$$\dot{\alpha} = \dot{\lambda}.$$  \hspace{1cm} (6.16)

With the Kuhn-Tucker loading/unloading conditions

$$\dot{\lambda} \geq 0, \quad f \leq 0, \quad \dot{\lambda}f = 0$$  \hspace{1cm} (6.17)

and the consistency condition

$$\dot{\lambda}f = 0,$$  \hspace{1cm} (6.18)

the model is complete.

Note that for the cohesive model described above, the interface degradation is only considered after the crack is initiated; the initiation of crack can be detected by inspecting the stress state in the bulk material (Camacho and Ortiz 1996, Jirasek and Zimmermann 2001, Xu and Needleman 1994). Such a model is sometimes referred to as “extrinsic” (Song et al. 2006); its counterpart, an “intrinsic” interface model, contains an initial stiffness describing the linear elastic behavior of crack and is formally similar to a continuum damage model (as will be mentioned in Section 6.3). In an intrinsic interface model, the free energy can be written as

$$\psi(w, d) = \frac{1}{2}(1 - d)w \cdot D_0 \cdot w;$$  \hspace{1cm} (6.19)

here, the damage variable $d \in [0, 1]$ indicates the degradation of interface material and $D_0$ represents the (very high) initial stiffness of the interface. Consequently, the constitutive relation between the traction and the separation becomes

$$t = (1 - d)D_0 \cdot w.$$  \hspace{1cm} (6.20)

The remaining part of the model can be formulated in analogy to the extrinsic cohesive model.
Discrete cracks as interface elements

In finite element implementations, discrete cracks can be represented by means of zero-thickness interface elements, which are placed along the predefined crack paths or the whole discretized domain prior to the numerical simulation [Fig. 6.5(a)]. Alternatively, the interface elements can be generated progressively in specific regions (Pandolfi and Ortiz 2002, Su et al. 2010), either before or during the computation progress, in order to reduce the total computational cost required for the simulation of structural failure processes. Recently, Manzoli et al. (2012) propose an alternative interface modeling technique: The classical zero-thickness interface element is replaced by degenerated solid finite elements with very high aspect ratio [Fig. 6.5(b)]. This technique can be easily implemented in standard FE programs using conventional solid elements and has been successfully used in capturing complex crack patterns in e.g. concrete structures (Manzoli et al. 2014) and drying soils (Sánchez et al. 2014). This approach will be discussed in Chapter 7 in detail.

The discrete crack model using interface elements can be used to capture the fracture and failure behavior of structures made of fiber-reinforced concrete: In Park et al. (2010), cohesive interface elements equipped with a trilinar softening law are placed along the predefined crack path; in Tailhan et al. (2015), interface elements, with the post-cracking softening behavior assumed to follow a linear descending curve, are placed in the whole domain without enforcing the crack path prior to the numerical simulation.

Figure 6.5: Finite element methods using (a) zero-thickness interface elements or (b) with degenerated solid elements to capture cracks.

The discrete crack model using interface elements can be used to capture the fracture and failure behavior of structures made of fiber-reinforced concrete: In Park et al. (2010), cohesive interface elements equipped with a trilinar softening law are placed along the predefined crack path; in Tailhan et al. (2015), interface elements, with the post-cracking softening behavior assumed to follow a linear descending curve, are placed in the whole domain without enforcing the crack path prior to the numerical simulation.

6.3 Smeared crack models

In the context of “smearing” cracks in finite elements, cracks can be represented as a damage zone, using continuum-based formulations in association with plasticity or damage theories. The smeared crack models, originated from Rashid (1968), with the introduction of the crack band theory (Bazant and Oh 1983), have been enhanced by means of adequate regularization techniques [see, e.g. de Borst et al. (1998), Hofstetter and Mang (1995), Jirásek and Bazant (2002), Rots (1988) and Mang et al. (2003) for an overview on smeared representations of cracks].
Concrete fracturing in tension

In a smeared crack model, the cracking-induced softening behavior of concrete is expressed as stress-strain relations based on the continuum mechanics concepts. As sketched for the 1D (uni-axial) stress–strain responses in Fig. 6.6(a), with the increasing tensile stress and strain within the representative volume element (RVE), the material exhibits firstly an almost linear elastic stress–strain response that can be described using the standard HOOKE's law (in 3D form):

\[ \sigma = C_\varepsilon : \varepsilon; \quad \text{(6.21)} \]

here \( C_\varepsilon \) represents the 4th-order elastic stiffness tensor dependent on the YOUNG's modulus \( E \) and POISSON's ratio \( \nu \) of material.

The elastic state is terminated by the initiation of cracking that can be detected from a strain-based loading criterion such as the one proposed in MAZARS AND PIJAUDIER-CABOT (1989):

\[ f(\varepsilon, \alpha) = \tilde{\varepsilon}(\varepsilon) - \alpha \leq 0; \quad \text{(6.22)} \]

here \( \tilde{\varepsilon} \) is the equivalent strain calculated based on the strain tensor according to the specific loading criterion; the strain-like internal variable \( \alpha = \alpha(\varepsilon) \) is dependent on the history of strain such as \( \alpha = \max(\tilde{\varepsilon}) \) (the maximum equivalent strain that the RVE has experienced).

In the post-cracking regime, it is generally observed that the concrete material suffers the loss of load-carrying capacity; in other words, the stiffness of material decreases as the tensile load increases. If unloaded at a certain state, the RVE can recover the original dimension, without exhibiting residual stress or strain [Fig. 6.6(a)]. To this end, the post-cracking behavior of concrete in tension is frequently described in the framework of continuum damage mechanics (KACHANOV 1958, 1986, MAZARS AND PIJAUDIER-CABOT 1989) as

\[ \sigma = (1 - d)C_\varepsilon : \varepsilon. \quad \text{(6.23)} \]
Here the scalar value $d$, named as the “damage variable”, indicates the (isotropic) loss of material integrity as the crack opens. The damage variable $d$ can be explicitly computed using a predefined function reflecting the softening behavior of concrete:

$$d = d(\alpha).$$  \hfill (6.24)

The loading/unloading conditions require that for any process

$$f \leq 0, \quad \dot{\alpha} \geq 0 \quad \text{and} \quad f \dot{\alpha} = 0.$$  \hfill (6.25)

**Remark:** As mentioned in Section 6.1, the cracking in concrete is usually restricted to the crack bands; the width of crack band is an inherent material property independent of the selection of representative volume element (RVE). In the finite element implementation of a smeared cracking model, the continuum constitutive law, i.e. the stress–strain relation, is assigned to every integration point (corresponding to a RVE). For smearing the localized cracks in the region of RVE corresponding to a specific integration point, a suitable regularization technique considering the ratio between the crack band width and the characteristic length of finite element is necessary to link the strain-based softening law with the actual displacement-controlled softening behavior of concrete-based materials (BAŽANT AND OH 1983, CERVENKA AND PAPANIKOLAOU 2008, FEENSTRA 1993, GÖDDE AND MARK 2015, LEE AND FENVES 1998, OLIVER 1989). This allows to eliminate the mesh-size sensitivity of the computed post-cracking softening behavior of concrete and to obtain objective results at the structural level.

**Concrete crushing in compression**

In comparison with the fracturing process in tension, the crushing of concrete material under compressive loads is significantly different [Fig. 6.6(b)]: The linear elastic response under compression is usually limited to a proportion of the compressive strength (approximately $f_c/2$, depending on the class of concrete). With the increasing load, the inherent flaws gradually develop into micro cracks that propagate in a stable manner, accompanied with the hardening-like response of material. After reaching the compressive strength $f_c$, the grow of cracks becomes unstable, leading to a limited number of macro cracks parallel to the loading direction, accompanied by even explosive failure of material. Furthermore, it is observed that the value of limit elastic stresses in one direction can be significantly dependent on the stress level of other two directions; after concrete experiences crushing, irrecoverable deformations are observed, eventually accompanied by the increase of the volumetric strain. Therefore, to address these failure phenomena, constitutive models making use of different yield criteria [such as the yield surfaces proposed by MOHR (1900a,b), DRUCKER AND PRAGER (1952) and MENÉTREY AND WILLAM (1995), and the modified forms based on them], in conjunction with the plasticity theory, are proposed to describe the post-elastic behavior of concrete under compression-dominant loading conditions (CERVENKA AND PAPANIKOLAOU 2008, FEENSTRA AND DE BORST 1995, LEE AND FENVES 1998, MESCHKE ET AL. 1998).

The essential ingredients of a plasticity-based constitutive model for concrete under compression are as follows [see e.g. SIMO AND HUGHES (1998) and NETO ET AL. (2008)]: The strain tensor
is additively resolved into an elastic component $\varepsilon^e$ and a plastic term $\varepsilon^p$ [as illustrated by the 1D sketch in Fig. 6.6(b)]:

$$\varepsilon = \varepsilon^e + \varepsilon^p.$$  \hspace{1cm} (6.26)

The stress is dependent on the elastic strain; hence, the constitutive law is written as

$$\sigma = C^e : \varepsilon^e = C^e : (\varepsilon - \varepsilon^p).$$  \hspace{1cm} (6.27)

The plastic loading function defines the evolving admissible stress domain:

$$f(\sigma, \alpha) = \tilde{\sigma}(\sigma) - q(\alpha) \leq 0.$$  \hspace{1cm} (6.28)

Here $\tilde{\sigma}$ is the stress-like indicator computed from the stress tensor $\sigma$, according to the selected yield criterion. To reflect the realistic failure surface of concrete under biaxial and triaxial stress conditions and to account for the dependence on hydrostatic pressure, $\tilde{\sigma}$ can contain the stress invariants such as

$$I_1(\sigma) = \text{tr} \sigma, \quad J_2(\sigma) = \frac{s : s}{2} \quad \text{and} \quad J_3(\sigma) = \det s.$$  \hspace{1cm} (6.29)

Here $s = \sigma - I_1/3 \mathbf{I}$ denotes the deviatoric stress tensor ($\mathbf{I}$ is the 2nd-order unity tensor).

In the post-elastic regime, with the increase of plastic loads on the material, the evolving magnitude of the limit stress is determined from the hardening/softening law $q(\alpha)$, with $\alpha$ the strain-like internal variable depending on the history of plastic deformation. Note that the equations above correspond to the isotropic hardening/softening of material; other cases can be found in e.g. SIMO AND HUGHES (1998). The evolution of plastic strain and the internal parameter is controlled by the consistency parameter (plastic multiplier) $\dot{\lambda} \geq 0$ indicating the rate of plastification process, according to the flow rules:

$$\dot{\varepsilon}^p = \dot{\varepsilon}^p(\dot{\lambda}), \quad \dot{\alpha} = \dot{\alpha}(\dot{\lambda}).$$  \hspace{1cm} (6.30)

Depending on the specific form of plastic potential, the flow rule for the plastic strain can be classified as associative or non-associative.

The loading/unloading conditions require

$$f \leq 0, \quad \dot{\lambda} \geq 0 \quad \text{and} \quad f \dot{\lambda} = 0$$  \hspace{1cm} (6.31)

for any process.

Remark: In analogy to the tensile situation, when the concrete enters post-peak softening regime after reaching $f_c$, the crushing zone localizes into a “crushing band”, leading to (more or less) identical stress vs. post-peak inelastic displacement relations, yet different stress vs. post-peak strain relations calculated from the measurements covering different lengths of the specimen in the loading direction (BAŽANT AND XIANG 1997, JANSEN AND SHAH 1997, NAKAMURA AND HIGAI 2001, VAN MIER 1986). Therefore, it is also essential to introduce a proper regularization technique into the finite element implementation for the simulation of structures experiencing concrete crushing due to excessive compressive stresses, so that objective results can be obtain independently on the mesh sizes while using continuum constitutive laws (FEENSTRA AND DE BORST 1995, ČERVENKA AND ČERVENKA 2014, WINKLER ET AL. 2004).
Coupled models

Among the continuum-based constitutive models for concrete, a number of them contain the effort to merge both the tensile fracturing and compressive crushing into a unified formulation. Such a coupled model may require more than one loading criteria, characterized by a multisurface elastic domain $\mathcal{E}$ in the stress space [see e.g. Feenstra and de Borst (1995), Meschke et al. (1998) and Cervenka and Papanikolaou (2008)]:

$$\mathcal{E} \equiv \{ \sigma \mid f_i(\sigma, q_i) \leq 0 \},$$

(6.32)

with $i = 1, \ldots, m$ indicating the specific yield function and $q_i$ the corresponding hardening/softening law. These $m$ yield functions should include those defining the tensile limit (Rankine 1857) and those for compressive limit [e.g. the Drucker-Prager criterion, see Fig. 6.7(a)]. In the case that more than one criteria are violated, a set of nonlinear equations need to be solved.

An additional sense of “coupling” refers to the combination of damage and plasticity theories in a thermodynamically consistent formulation, which leads to an additive splitting of the inelastic deformations [see e.g. Lubliner et al. (1989), Meschke et al. (1998), Simo and Ju (1987) and Fig. 6.7(b)]:

$$\varepsilon^{pd} = \varepsilon^p + \varepsilon^d.$$  

(6.33)

Here, the inelastic strain component $\varepsilon^{pd}$, which can be obtained from the flow rule in analogy to the plasticity theory, is split into two terms corresponding to the irreversible deformation and the degradation of material, respectively.

The smeared crack models can be directly incorporated, as the local constitutive law at the integration points of finite element, into the standard finite element framework. This type of models
have been used for the numerical simulation of concrete structures with and without reinforcement, including FRC structures; for instance, in HAMEED ET AL. (2013), a damage value is used to describe the evolving fiber-induced residual stress in a crack; GÖDDE AND MARK (2015) account for the cracks in the framework of elasto-plastic damage theory, where the post-cracking softening behavior of FRC is governed by the plastic strain which is linked to the discrete crack opening width via regularization. Nevertheless, since the crack opening is smeared in the finite element, the loss of explicit representation of the cracking-induced kinematics motivates an other type of concrete cracking models, where the discrete crack is directly embedded in the interior of finite elements. This type of models are introduced in the following sections.

6.4 Embedded strong discontinuity approach

The embedded strong discontinuity approach (ESDA), originated in the early 1990s (SIMO ET AL. 1993), where after the initiation of a crack within individual finite elements, the crack-induced kinematics are enhanced locally to accommodate the discontinuity (displacement jump) in the element. The ESDA has experienced rapid development in the last two decades (JIRÁSEK AND ZIMMERMANN 2001, LINDER AND ARMERO 2009, LINDER AND ZHANG 2014, MOSLER AND MESCHKE 2003, OLIVER 1996, REGUEIRO AND BORJA 1999). The increasing popularity of ESDA is based on the fact that a discrete crack equipped with an appropriate cohesive law can be directly incorporated into the bulk finite elements, independently on the finite element mesh, allowing to avoid the mesh-dependency of numerical results obtained using the interface elements placed on the element boundaries. Furthermore, the governing equations can be solved locally in every finite element, without introducing additional degrees of freedom at the structural level.

\[
\Gamma^- \Omega^+ + \Omega^\text{discontinuous}
\]

Figure 6.8: (a) Domain with strong discontinuity; (b) 1-dimensional sketch of the displacement and strain fields.

The kinematics in a domain \( \Omega \) possessing a strong discontinuity surface \( \Gamma \) (as illustrated in Fig. 6.8) can be described as follows [see e.g. OLIVER (2000)]: The displacement field \( \mathbf{u} \) in \( \Omega \) is written as the summation of a continuous part \( \mathbf{\bar{u}} \) and a discontinuous component \( \mathbf{\hat{u}} \):

\[
\mathbf{u} = \mathbf{\bar{u}} + \mathbf{\hat{u}} = \mathbf{\bar{u}}^\text{continuous} + H_{\Gamma} \| \mathbf{u} \| \text{ discontinuous}; \tag{6.34}
\]
here, $H_\Gamma$ is the Heaviside step function on $\Gamma$ and $[u]$ is the displacement jump. The resulting strain field is written as the summation of a bounded part $\bar{\varepsilon}$ and an unbounded term $\hat{\varepsilon}$:

$$\varepsilon = \nabla^{\text{sym}} u = \bar{\varepsilon} + \hat{\varepsilon} = \bar{\varepsilon} + \delta_\Gamma ([u] \otimes n)_s,$$

(6.35)

with $\delta_\Gamma$ the Dirac's delta on $\Gamma$.

### 6.4.1 Finite element with embedded strong discontinuity

For a finite element that already experienced the crack initiation and, hence, separated by a discrete crack (the strong discontinuity), the conventional linear elastic element formulations need to be enhanced with a set of kinematic and equilibrium equations. These enhanced formulations are described in the following, taking the constant strain tetrahedral element illustrated in Fig. 6.9 as an example. Note that the formulations below are written in analogy to those documented in Jirášek and Zimmermann (2001).

**Kinematics**

The nodal displacement vector of element in the global coordinate system (X-Y-Z) takes the following form:

$$u^e = [u_1^x, u_1^y, u_1^z, u_2^x, u_2^y, u_2^z, u_3^x, u_3^y, u_3^z, u_4^x, u_4^y, u_4^z]^T. \quad (6.36)$$

The local crack opening vector, i.e. the displacement discontinuity

$$[u] = [[u]_n, [u]_r, [u]_s]^T, \quad (6.37)$$

contains $[u]_n$, $[u]_r$, and $[u]_s$ that represent the normal and tangential components of the displacement jump $[u]$ in the local coordinate system (N-R-S) of the crack (Fig. 6.9). The continuum part
of displacement becomes
\[ \bar{\mathbf{u}} = \mathbf{u}^e - \mathbf{\tilde{u}} = \mathbf{u}^e - \mathbf{H} \cdot \mathbf{[u]}, \] (6.38)
with the “separation-mode” matrix
\[ \mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & T^T \end{bmatrix}^T \] (6.39)
reflecting that the crack separates the local node 4 from nodes 1, 2 and 3, according to the situation illustrated in Fig. 6.9. Here \( \mathbf{0} \) is a \( 3 \times 3 \) matrix and \( \mathbf{T}^T \) transforms the crack opening vector \( \mathbf{J} \mathbf{u} \) from the local COS to the global COS (see Appendix C). The continuum strain in the bulk of element is obtained as
\[ \mathbf{\varepsilon} = \mathbf{B} \cdot \bar{\mathbf{u}}. \] (6.40)

**Remark:** The separation-mode matrix \( \mathbf{H} \) is dependent on the local topology of cracks, i.e. the position and orientation of the crack path in a specific finite element. For example, if the crack separates the nodes 3 and 4 from the nodes 1 and 2, \( \mathbf{H} \) becomes \( \begin{bmatrix} 0 & 0 & T^T & T^T \end{bmatrix} \). Since the separation-mode of the element nodes directly affects the local enhancing mode of displacement and strain, it is a crucial task to determine the correct crack topology when the cracks propagate. This issue will be discussed later.

### Static conditions

As usual, the stress in the bulk material is computed by means of linear elasticity as (in vector notation)
\[ \mathbf{\sigma} = \mathbf{C}_e \cdot \mathbf{\varepsilon}, \] (6.41)
with \( \mathbf{C}_e \) the \( 6 \times 6 \) elasticity matrix, which allows the computation of the internal nodal force vector
\[ \mathbf{f}^{\text{int}} = \mathbf{V}_e \mathbf{B}^T \cdot \mathbf{\sigma}, \] (6.42)
with \( \mathbf{V}_e \) the volume of element.

The bulk stress is projected onto the crack plane and further transformed to the local coordinate system as [see the 2D configuration described in JIRÁSEK and ZIMMERMANN (2001)]
\[ \mathbf{Q} \cdot \mathbf{\sigma} = \mathbf{t}^{\text{bulk}}, \] (6.43)
with
\[ \mathbf{Q} = \begin{bmatrix} n_x^2 & n_y^2 & n_z^2 & 2n_xn_y & 2n_xn_z & 2n_yn_z \\ n_xn_y & n_y^2 & n_z^2 & n_xn_y + n_z^2 & n_xn_z & n_y^2 + n_z^2 \\ n_xn_z & n_yn_z & n_z^2 & n_xn_z + n_y^2 & n_xn_y + n_z^2 & n_y^2 + n_z^2 \end{bmatrix}. \] (6.44)

Taking into account the traction \( \mathbf{t}^{\text{crack}}(\mathbf{[u]}) \) transmitted by the cohesive crack itself, the static condition requires
\[ \mathbf{t}^{\text{bulk}}(\mathbf{[u]}) = \mathbf{t}^{\text{crack}}(\mathbf{[u]}), \] (6.45)
where $t^{\text{crack}}([u])$ can be determined based on the constitutive model describing the behavior of a discrete crack in the quasi-brittle material (Section 6.2). In the framework of finite element implementation, this local equilibrium equation system can be solved by means of Newton-Raphson iterative solution scheme; the unknown value of $[u]$ which records the crack opening status in the element can be obtained.

**Remark:** The above described formulation (including the element enrichment and the equilibrium conditions), is classified as the “statically and kinematically optimal nonsymmetric formulation” (SKON), which employs the advantages of a natural traction continuity condition in conjunction with a proper representation of kinematics even at the late stages of crack opening process, and, consequently, leads to an optimal numerical performance (Jirásek 2000).

### 6.4.2 Determination of crack topology

Unlike the smeared representation of crack-induced kinematics (always considered at the integration points) within the finite elements, or the discrete modeling of cracks by means of activating the interface elements placed a priori along the edges of bulk elements in advance, specific techniques are required for the determination of crack paths while using the embedded strong discontinuity approach to perform structural simulations. The necessity of a crack tracking algorithm is due to the fact that the topology (position and orientation) of a crack, which is not known in advance, directly influences the node-separation mode and consequently the kinematics in the element with crack.

**Crack locking**

![Crack path determination](image)

**Figure 6.10:** Crack path determined by (a) directly placing the elemental crack at the centroid (integration point, marked as “o”) or (b) employing tracking algorithm.

As illustrated in Fig. 6.10(a), when the crack initiation is detected in an element, one may naturally place the crack (perpendicular to the normal direction $n$) at the integration point (e.g. the centroid of constant strain triangular element). However, the crack lines in two neighboring elements may lead to a conflicting node-separation mode of the common edge (i.e. the edge connecting Node-2 and 3 in the figure): According to the topology of existing crack in Element-1, this edge should
be separated by the crack; however, it is not the case considering Element-2. Such inappropriately predicted crack paths, known as the “crack-locking” phenomenon, can cause severe problems during the numerical simulation of structures (Jirásek and Zimmermann 2001). In order to avoid crack locking in ESDA, various techniques have been proposed in the last decade.

**Crack tracking techniques**

In addition to the conventional program for the solution of local equations in every individual finite element and the global equilibrium of structure, a crack path tracking algorithm should be included in the numerical implementation to follow the crack propagation on the structural level. A crack tracking algorithm should address the following essential considerations [see e.g. Cervera et al. (2010)]: A structural crack should initiate at a “root-element” (usually located on the boundaries of domain); the crack initiation criterion (e.g. Rankine criterion) for a new local crack in a finite element should provide the direction of crack; the propagation of crack path should be determined by enforcing the continuation across neighboring elements, so that the conflict in the node-separation mode can be avoided [Fig. 6.10(b)]; cracking in the elements outside the crack path is not allowed.

The development of ESDA is accompanied with the advance of crack tracking techniques; these tracking algorithms, in addition to the necessary ingredients mentioned above, contain different features designed to achieve the reliable prediction of crack paths during structural simulations. Since the crack topology determined by a tracking algorithm is generally fixed during the structural simulation, the accuracy of the predicted propagation direction plays an important role. As is well known, due to the high stress concentration at the crack tip, the accuracy of stress (strain) field evaluated in the finite element (usually low order element, e.g. constant strain element) directly located at the crack tip may not be guaranteed; consequently, the normal direction of crack predicted from e.g. the maximum principal stress (strain) can deviate significantly from the “correct” direction; this error can accumulate and lead to unacceptable results of crack path. The problem can be alleviated by means of the nonlocal averaging of stresses (strains) in a limited region containing a number of elements in the vicinity of crack tip, in order to predict the crack normal directions more accurately (Feist 2004, Feist and Hofstetter 2007, Jirásek and Zimmermann 2001). Alternatively, a delayed crack fixation can be considered: The crack direction in a newly cracked element is not fixed until the crack opening width exceeds a certain threshold, so that the crack direction can be “adapted” instead of being fixed immediately (Cervera et al. 2010, Jirásek and Zimmermann 2001, Sancho et al. 2007). With regard to the region involved while tracking the crack path, a tracking algorithms can be classified as local- (Alfaiate et al. 2002), partial domain- (Feist and Hofstetter 2006) or global tracking (Oliver et al. 2004). Concerning the representation of macroscopic crack pattern within the structure, one may explicitly connect the crack lines (surfaces in 3D) from one element to another and store the extra topology information element-wise. Alternatively, one may define an additional field variable in the domain; the crack should pass the locations where zero values of this field is expected, and the crack normal direction should correspond to the gradient of this field (Oliver et al. 2004).

In general, crack tracking algorithms perform well in the 2D finite element simulation of concrete structures: A unique and continuous crack path can be obtained by connecting the crack lines
from one element to another. However, one may face additional challenges while attempting to determine the crack surfaces in 3D, depending on the specific tracking technique used [see Jaeger et al. (2008) for a review]: With the propagation of crack front, a number of elements may violate the cracking criterion simultaneously; furthermore, one may observe that an element, in which a new crack is initiated, is surrounded by more than one elements that already experienced cracking. As a result of the excessive conditions known from the surrounding elements, the crack surface predicted by the tracking algorithm can be either non-planar or planar yet non-unique (Feist 2004, Feist and Hofstetter 2007, Gasser and Holzapfel 2006). Another important issue that limits the application of tracking algorithm in complex 3D structural simulations is that, it demands considerable programming effect and requires additional computational cost, particularly in the situation where a big number of macroscopic cracks need to be followed simultaneously.

Non-tracking techniques

Without employing any tracking algorithm, the crack topology can be determined locally in every individual element. For example, a “crack adaptation” concept is proposed in Sancho et al. (2007): When a crack initiates in a certain element (as soon as the maximum principal stress exceeds the tensile strength of concrete), the crack normal direction is considered as identical to the direction of maximum principal stress. The position of crack is not explicitly determined; on the contrary, the local crack topology is implicitly reflected by the node-separation mode [and the resulting $H$ matrix in Eq. (6.39)] which is selected in such a way that the gradient of the shape function for the solitary node is as parallel to the crack normal direction as possible. Furthermore, the newly determined crack is allowed to “adapt” itself by means of the delayed fixation of node-separation mode. The performance of this technique has been demonstrated via 2D as well as 3D examples (Sancho et al. 2007). Nevertheless, according to the experience of the author of present thesis, it seems that this method can not completely eliminate the conflict of node separation modes between two neighboring finite elements, since the crack path continuation is not enforced. Another form of ESDA is proposed by Mosler (2005) and Radulovic et al. (2011), where multiple intersecting cracks are allowed within one finite element, which may allow to avoid crack locking completely; however, solving the equations involving multiple non-orthogonal cracks may lead to computational instabilities, similar to the situation in a smeared crack model such as de Borst and Nauta (1985).

6.4.3 Numerical example

The embedded strong discontinuity approach can be employed for the numerical simulation of the failure behavior of structures made of plain- or fiber-reinforced concrete (Brighenti and Scorza 2012, Denneman et al. 2011). As shown in the following example, a notched beam subjected to three-point bending is investigated by the author of present thesis. The specimens are made of three different fiber-reinforced concrete composites. More details regarding the experimental settings and material parameters can be found in Putke et al. (2014). In the finite element model, the constitutive behavior of embedded discrete cracks, i.e., the $t^{\text{crack}}([u])$ relation, is defined according
to the traction–separation law obtained from the crack bridging model (Chapter 5); therefore, the post-cracking softening behavior of cracks in the FRC structure is well represented.

Figure 6.11: Results of numerical simulation using ESDA for the FRC notched beam under three-point bending: comparison of (a) crack patterns and (b) structural responses obtained from experiments and simulations.

Fig. 6.11 clearly shows the capability of ESDA in capturing the structural failure responses of concrete-based materials. From Fig. 6.12, it is clearly observed that the major cracking phenomena, that a structural crack initiates at the tip of notch and propagates vertically towards the top surface of the specimen, is well captured by using the ESDA in conjunction with crack path determination approaches in the 2D configuration. Nevertheless, one may notice different results of the crack topol-
ogy obtained by allowing crack adaptation and by employing the tracking technique in association with nonlocal averaging: For the former [Fig. 6.12(a)], since the determination of crack topology in every element along the macro crack path does not rely on any information from the neighboring elements, a discontinuous crack pattern is observed. In the latter case [Fig. 6.12(b)], the crack line in every newly damaged element is determined based on the known geometrical information of the existing crack in the adjacent element; hence, a continuous crack path is obtained.

Remark 1: In the formulation and example presented above, the displacement jump $J_u$ is assumed to be uniform in every individual finite element; therefore, even though the crack path is continuous, the crack opening is not conformable across element boarders. This problem can be alleviated by means of incorporating nonuniform interpolation of $[u]$ in the finite element (LINDER AND ARMERO 2007).

Remark 2: The example described above is merely aimed at demonstrating the performance of an ESDA. This problem will be re-analyzed using FEM with interface solid elements, which is finally selected as the appropriate numerical tool for FRC structural simulation (see Chapter 8).

### 6.5 Extended finite element method

In analogy to the embedded strong discontinuity approach (ESDA) described in the preceding section, an alternative way to incorporate the failure kinematics directly into the finite element formulation is known as the extended finite element method (XFEM), first introduced in MOËS ET AL. (1999). Similar to the ESDA, the XFEM allows strong discontinuities to be placed arbitrarily in the finite elements and enables to capture the fracture phenomena in quasi-brittle materials without the modification of mesh. In the last years, the XFEM has been employed in a variety of fracture problems (CHEN ET AL. 2012, MESCHKE AND DUMSTORFF 2007, MESCHKE AND LEONHART 2015, ZI AND BELYSCHKO 2003). For a state-of-the-art review of XFEM, one may refer to SUKUMAR ET AL. (2015).

In the framework of XFEM, for the approximation of displacement field, the crack is modeled via enriched displacement modes, using different types of node-based shape function depending on the geometrical relation between the specific node and the crack [Fig. 6.13(a)]. Note that, for the ESDA, the enriching modes have an elemental support; i.e. the additional degrees of freedom (DOF) representing the displacement jump are connected with those individual elements crossed by the discontinuity; these additional DOF can be condensed at the elemental level during the solution of local equilibrium equations and thus do not substantially increase the structural computation. In the XFEM, however, the enrichment has a nodal support, and the additional DOF are attached to those regular nodes belonging to the elements crossed by the discontinuity, introducing additional DOF into the structural equation system when the macro crack propagates.

For the determination of crack path in XFEM, the level-set method is introduced to represent the crack topology in 2D (STOLARSKA ET AL. 2000) and 3D (GRAVOUIL ET AL. 2002, MOËS ET AL. 2002). The level-sets provide an implicit way to store the evolving crack topology by representing the crack path as the zero level contour of a signed distance function. For instance, two sets of signed distance function are required to define a crack in 2D [Fig. 6.13(b)]: the function value
6.6 Meso-scale models for FRC

In general, the cracking models described above are originally developed for plain concrete. Based on these plain concrete models, attempts are made to simulate the failure behavior of structures made of fiber-reinforced concrete, by means of directly modifying the post-cracking responses in order to take into account the toughening effect of fibers. This approach relies on the input of a phenomenological (macroscopic) description of the post-cracking response of FRC which is usually obtained directly from experiments.

In order to allow the prediction of FRC structural behavior based on a given set of basic material parameters (such as concrete strength, single fiber properties and fiber content), the analysis and modeling of FRC on lower scales are necessary. On the meso-scale, models for FRC have been proposed which include explicit representations of individual fibers within representative elementary volumes (REV) of FRC samples. In Radtke et al. (2011) and Fang and Zhang (2013) fibers are individually represented using embedded enhanced displacements to represent the fiber–concrete slip in the finite element discretization. A similar model is proposed in Oliver et al. (2012), where the slip is derived from an additional displacement field corresponding to the fiber phase. In Cunha et al. (2012) the contribution of individual fibers is taken into account by the superposition of the stiffness properties derived from the experimental pullout responses of fibers onto the stiffness.
of concrete matrix. As an alternative discretization approach on the meso-scale level, Bolander and Saito (1997) and Kang et al. (2014) use lattice models for FRC, in which the fibers are attached to a background lattice model for plain concrete considering the bond–slip effect of fibers as additional components of the spring stiffness between the lattice nodes. In Schlauffert and Cusatis (2012) and Schlauffert et al. (2012), a similar “lattice discrete particle model”, based on the discrete consideration of both concrete matrix and fibers is proposed, where the aggregates in concrete are represented via spherical particles linked with each other by means of cohesion and the additional fiber bonding effect. For the generation of individual fibers in the REV, specific statistical information regarding the fiber distribution and fiber orientation, which significantly influence the damage phenomena in FRC (Fig. 6.14), needs to be considered (Viejo et al. 2014, Zhan et al. 2014).

![Figure 6.14](image-url)

**Figure 6.14:** Example of the meso-scale finite element analysis of FRC, showing the influence of fiber orientation on the damage pattern in the RVE (Mohseni 2013, Zhan et al. 2014).

Meso-scale models allow for a good understanding of the fiber-concrete interaction mechanisms of FRC materials, but are evidently inappropriate for the directly application in the simulations of a large FRC structure containing huge number of fibers. Therefore, the fracturing phenomena and failure responses of FRC material are investigated in a multiscale modeling framework, starting with the very fundamental problems at the single fiber level, providing the ductile post-cracking behavior of FRC by means of a crack bridging model (see Part I of the present thesis), and subsequently used in the concrete cracking models at the structural level.
In the preceding chapter, the widely used types of numerical model for cracking in concrete structures are briefly introduced. Among these models, the discrete crack models, where the cracking-induced kinematics are directly represented by means of degrading interfaces between finite elements, are highly compatible with the post-cracking ductile response of a specific FRC predicted by the models described in Part I of the thesis and allow to capture complex cracking phenomena in concrete and fiber-reinforced concrete structures, without the demand for a crack path tracking algorithm. In this chapter, the interface solid element introduced by MANZOLI ET AL. (2012) that is selected to represent the fracturing phenomena in concrete materials and structures, is described in detail. This technique uses degenerated solid finite elements, which can be easily adapted based on a standard FEM program, to approximate the interfacial damage mechanisms in concrete materials. This method is developed based upon the principles of continuum strong discontinuity approach (OLIVER 2000) for the representation of cracks in finite elements, making use of the strong discontinuity kinematics and continuum constitutive laws in conjunction with conventional finite elements characterized by a very high aspect ratio.
### Symbols used in this chapter

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>internal parameter for interface</td>
<td>mm</td>
</tr>
<tr>
<td>$\alpha^\text{ex}$</td>
<td>explicit estimation of the internal parameter</td>
<td>mm</td>
</tr>
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<td>$\beta$</td>
<td>weighting factor of shear strength w.r.t. tensile strength</td>
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<td>$C_e$</td>
<td>elastic stiffness of material (4th-order tensor)</td>
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<td>$c_1$</td>
<td>coefficient in the interface softening law</td>
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<td>$\delta_\Gamma$</td>
<td>DIRAC’s delta on $\Gamma$</td>
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<td>$E^*$</td>
<td>YOUNG’s modulus of the composite material</td>
<td>N/mm²</td>
</tr>
<tr>
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<td>YOUNG’s modulus of concrete</td>
<td>N/mm²</td>
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<td>bounded strain</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\varepsilon}$</td>
<td>unbounded strain</td>
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</tr>
<tr>
<td>$f$</td>
<td>loading function</td>
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<td>$f_t$</td>
<td>tensile strength of concrete</td>
<td>N/mm²</td>
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<tr>
<td>$f_t^*$</td>
<td>tensile strength of the composite material</td>
<td>N/mm²</td>
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<td>$\Gamma^\text{ext}$</td>
<td>external force vector of structure</td>
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<td>$\Gamma$</td>
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<td>$\Gamma^h$</td>
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<td>trial stress (tensor)</td>
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<td>parameter in the interface softening law</td>
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<td>N/mm$^3$</td>
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<td>N/mm$^2$</td>
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<tr>
<td>$\hat{u}$</td>
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<tr>
<td>$\mathbf{u}_{n,r,s}$</td>
<td>normal and tangential components of displacement discontinuity</td>
<td>-</td>
</tr>
<tr>
<td>$\tilde{u}$</td>
<td>equivalent displacement discontinuity in interface</td>
<td>mm</td>
</tr>
<tr>
<td>$\tilde{u}_0$</td>
<td>elastic limit value of $\tilde{u}$</td>
<td>mm</td>
</tr>
<tr>
<td>$w_{ref}$</td>
<td>reference value of crack opening</td>
<td>mm</td>
</tr>
<tr>
<td>$w_u$</td>
<td>ultimate value of crack opening</td>
<td>mm</td>
</tr>
<tr>
<td>$\mathbf{0}$</td>
<td>zero (vector)</td>
<td>-</td>
</tr>
</tbody>
</table>
7.1 Cracking process represented via degenerated solid elements

7.1.1 Kinematics and strain approximation

Figure 7.1: Domain with (a) strong discontinuity surface \( \Gamma \) or with (b) weak discontinuity band \( \Gamma^h \).

As mentioned in Section 6.4, the displacement field \( \mathbf{u} \) in a domain \( \Omega \) with a strong discontinuity surface \( \Gamma \) [Fig. 7.1(a)] consist of a continuous part \( \mathbf{\bar{u}} \) and a discontinuous component \( \mathbf{\hat{u}} \) (OLIVER 2000):

\[
\mathbf{u} = \mathbf{\bar{u}} + \mathbf{\hat{u}} = \mathbf{\bar{u}}_{\text{continuous}} + H_{\Gamma} \left[ \mathbf{u} \right]_{\text{discontinuous}},
\]

(7.1)

The resulting strain field is written as the summation of a bounded part \( \bar{\varepsilon} \) and an unbounded term \( \hat{\varepsilon} \):

\[
\varepsilon = \nabla^{\text{sym}} \mathbf{u} = \bar{\varepsilon} + \hat{\varepsilon} = \bar{\varepsilon}_{\text{bounded}} + \delta_{\Gamma} \left[ \mathbf{u} \otimes \mathbf{n} \right]_{\text{unbounded}}.
\]

(7.2)

By introducing the concept of weak discontinuity \( \Gamma^h \) with the width \( h \), as illustrated in Fig. 7.1(b), the strain field can be regularized as:

\[
\varepsilon = \bar{\varepsilon}_{\text{bounded}} + \frac{\mu_{\Gamma}}{h} \left[ \mathbf{u} \otimes \mathbf{n} \right]_{\text{regularized}},
\]

(7.3)

with \( \mu_{\Gamma} = 1 \) in \( \Gamma^h \) and \( \mu_{\Gamma} = 0 \) in \( \Omega \setminus \Gamma^h \). If the band width is so small that \( h \to 0 \), the strong discontinuity concept is recovered.

Using degenerated solid elements to represent cracks, the strain in these interface solid elements is assumed to be exclusively related to the (regularized) unbounded term \( \hat{\varepsilon} \) (MANZOLI ET AL. 2012, SÁNCHEZ ET AL. 2014):

\[
\varepsilon \approx \hat{\varepsilon} = \frac{1}{h} \left[ \mathbf{u} \otimes \mathbf{n} \right]_{\text{\hat{}}}.
\]

(7.4)

The strain tensor in an ISE can be expressed in the local coordinate system (N-R-S) as:

\[
\varepsilon \approx \frac{1}{h} \begin{bmatrix}
\left[ \mathbf{u} \right]_n & \left[ \mathbf{u} \right]_r / 2 & \left[ \mathbf{u} \right]_s / 2 \\
\left[ \mathbf{u} \right]_r / 2 & 0 & 0 \\
\left[ \mathbf{u} \right]_s / 2 & 0 & 0
\end{bmatrix},
\]

(7.5)
7.1. CRACKING PROCESS REPRESENTED VIA DEGENERATED SOLID ELEMENTS

Figure 7.2: Degenerated 3D solid element characterized by the “base”, the “apex” node-4 and its projection on the base (point 4').

with \([u]_n\), \([u]_r\), and \([u]_s\) as the normal and tangential components of the displacement jump \([u]\), respectively, which are determined from the relative displacement of the apex node with respect to its projection on the base (Fig. 7.2). The displacement of the projection can be calculated via the interpolation of the displacements of the base-nodes.

7.1.2 Interface damage model

In the present work, all bulk elements are considered to be linear elastic. The constitutive law of the degenerated solid elements is cast in a continuum form equipped with a damage law which allows to approximate the behavior of interfacial degradation mechanisms involved during the opening of cracks. According to the principles of CSDA, bounded stresses are obtained from the (regularized) unbounded strains in ISE. Note that, since \(h\) is chosen to be very small, the elastic behavior of ISE approximates a rigid interface (\([u] \approx 0\)); in this sense, the term \(1/h\) plays the role of a penalty parameter (MANZOLI ET AL. 2012).

The inelastic constitutive behavior of the ISE is formulated as:

\[
\sigma = (1 - d)C_e : \varepsilon \approx (1 - d) \frac{1}{h} C_e : ([u] \otimes n) + \sigma_n^2 n_n + \left(\frac{\sigma_{nr}}{\beta}\right)^2 + \left(\frac{\sigma_{ns}}{\beta}\right)^2,
\]

(7.6)

with \(d\) as the scalar damage variable and \(C_e\) denoting the elastic stiffness tensor with POISSON’s ratio \(\nu = 0\). The loading criterion is defined in terms of the equivalent stress \(\tilde{\sigma}\) and the displacement-like internal parameter \(\alpha\):

\[
f(\sigma, \alpha) = \tilde{\sigma} - t(\alpha) \leq 0.
\]

(7.7)

Here, the equivalent stress is defined as

\[
\tilde{\sigma} = \sqrt{\sigma_{nn}^2 + \left(\frac{\sigma_{nr}}{\beta}\right)^2 + \left(\frac{\sigma_{ns}}{\beta}\right)^2}.
\]

(7.8)

In the formula above, \(\beta\) represents the ratio between the shear and tensile strengths. For example, \(\beta = 1\) implies a semi-spherical yield surface in the local stress space; \(\beta = 2\) indicates that the shear
strength of interface is four times of the tensile strength [see e.g. CAMACHO AND ORTIZ (1996), GRASSL AND REMPLING (2008) and CABALLERO ET AL. (2008)].

Recall that the crack bridging law developed in Chapter 5 is implemented in the format of the analytical surrogate model Eq. (5.16), which allows to define the softening law of interface as (see Fig. 7.3)

\[
t(\alpha) = (f_0^* - t_1) \exp\left(-\frac{\alpha}{w_{ref}}\right) + t_1 \frac{w_u - \alpha}{w_u} + t_2 \exp\left(c_1 - c_2 \alpha\right) \alpha,
\]

(7.9)

with the parameters \(t_1, t_2\) and \(c_1, c_2\) appropriately fitted according to the numerical solution obtained from the original model Eq. (5.11) for the specific fiber cocktail. The internal parameter \(\alpha\) is defined based on the maximum value of equivalent separation experienced during the loading history as

\[
\alpha = \max(\tilde{u}) - \tilde{u}_0.
\]

(7.10)

The equivalent crack separation is defined as

\[
\tilde{u} = \sqrt{\left[\left[\frac{u}{\kappa}\right]_x^2 + \left[\frac{u}{\kappa}\right]_r^2\right]^2 + \left[\frac{u}{\kappa}\right]_s^2}.
\]

(7.11)

and \(\tilde{u}_0\) corresponds to the limit state of the elastic interface:

\[
\tilde{u}_0 = \frac{f_0^*}{K_0} = \frac{h f_0^*}{E^*} \approx 0,
\]

(7.12)

with \(K_0\) representing the “rigid” elastic stiffness of the equivalent interface behavior (Fig. 7.3).

![Figure 7.3: Equivalent traction–separation relation for the cohesive solid interface model derived from the crack bridging model (Chapter 5).](image)

The scalar damage variable \(d(\alpha)\) is obtained by comparing the secant stiffness \(K^\text{sec}\) with the elastic stiffness of the equivalent interface behavior (Fig. 7.3) as

\[
d(\alpha) = 1 - \frac{K^\text{sec}}{K_0} = 1 - \frac{h t}{E^*(\alpha + \tilde{u}_0)}.
\]

(7.13)
The model is completed with the loading-unloading conditions
\[ f \leq 0, \quad \dot{\alpha} \geq 0, \quad \dot{\alpha} f = 0, \quad (7.14) \]
and the consistency conditions
\[ \dot{\alpha}^f = 0, \quad \text{if } f = 0. \quad (7.15) \]

Note, that in structural simulations, an interface solid element that already experienced tensile damage can be unloaded and subjected to compressive stresses, which implies that a previously open crack is closed. For a closed crack, the (rigid) elastic stiffness in the normal direction should recover, so that the overlapping of interface (consequently the “penetration” of the two neighboring bulk elements) is prevented. This is implemented by means of separating the components of local stiffness corresponding to the normal and tangential behavior, respectively, and allow the recovery of the normal component while reserving the damage for the shear term; for details, see Appendix C.

### 7.1.3 Implicit/explicit integration scheme

For the computational failure analyses of structures made of inelastic materials, the explicit integration scheme generally leads to robust (regarding the capability to provide solutions) algorithms but demands small step sizes in order to reduce the error generated during computation; in contrary, the fully implicit algorithms can provide much more accurate results with relatively large step sizes, but one may suffer from the failure of convergence when the global tangent stiffness is becoming negative-definite (OLIVER ET AL. 2008). To this end, an implicit/explicit (IMPL-EX) integration scheme is proposed by OLIVER ET AL. (2006) and OLIVER ET AL. (2008), which combines the advantages of both integration schemes and leads to the following important characteristics:

- The effective algorithmic stiffness matrix is positive definite and step-wise constant. Therefore, the structural equation system is well conditioned and can be solved with only one iteration, which dramatically improves the robustness and reduces the total computational cost.
- The integration is generally unconditionally stable and the first-order accuracy of the implicit scheme is maintained, although the absolute error can be larger than the fully implicit algorithms.

The IMPL-EX integration scheme can be applied to various nonlinear material problems (OLIVER ET AL. 2008). In the present work, the IMPL-EX scheme is implemented in the context of the interface solid element (ISE) for FRC materials according to the following algorithmic structure:

1. Explicit stage: At the beginning of a loading step \([T_n, T_{n+1}]\) an explicit step is executed, which consists of the following algorithmic procedures:

   a) For every ISE in loading state, determine the explicit internal parameter \(\alpha^{ex}_{n+1}\) by means of linear extrapolation:

   \[
   \alpha^{ex}_{n+1} = \alpha_n + \Delta\alpha^{ex}_{n+1} = \alpha_n + \frac{\Delta T_{n+1}}{\Delta T_n} \Delta \alpha_n, \tag{7.16}
   \]

   with

   \[
   \Delta T_n = T_n - T_{n-1} \quad \text{and} \quad \Delta T_{n+1} = T_{n+1} - T_n. \tag{7.17}
   \]
indicating the size of the previous and the current loading steps, respectively, and
\[ \Delta \alpha_n = \alpha_n - \alpha_{n-1} \quad (7.18) \]
calculated using the already known internal variables from the last two steps.

b) Compute the explicit damage parameter \( d_{ex}^{n+1} \)
\[ d_{ex}^{n+1} = 1 - \frac{\dot{t}_{ex}^{n+1}}{E^*(\alpha_{ex}^{n+1} + \dot{u}_0)}, \quad (7.19) \]
with \( \dot{t}_{ex}^{n+1} = t_n + H_n \Delta \alpha_{ex}^{n+1}; \quad (7.20) \)
here \( t_n \) and \( H_n = dt/d\alpha \) are respectively the equivalent traction across the crack and the softening modulus computed at the implicit stage of \( T_n \).

c) Compute the explicit (algorithmic) element stiffness matrix \( k_{ex}^{n+1} \) at time step \( T_{n+1} \)
\[ k_{ex}^{n+1} = (1 - d_{ex}^{n+1}) k_e, \quad (7.21) \]
with \( k_e \) as the elastic stiffness matrix.

2. Assembly of the algorithmic stiffness matrix \( K_{ex}^{n+1} \) and solution of the linear structural equation system to compute the displacement vector \( u_{n+1} \):
\[ K_{ex}^{n+1} \cdot u_{n+1} = f_{ext}^{n+1}, \quad (7.22) \]
with \( f_{ext}^{n+1} \) as the external force vector.

3. Implicit stage:

a) For every ISE, compute the local displacement jump \( \lbrack u \rbrack_{ex}^{n+1} (u_{n+1}) \) from the computed vector of displacements \( u_{n+1} \).

b) Compute the local strains \( \varepsilon_{n+1} (\lbrack u \rbrack_{ex}^{n+1}) \) and the “trial” stress \( \sigma_{tr}^{n+1} (\varepsilon_{n+1}, d_n) \).

c) Check the loading criterion \( f(\sigma_{tr}^{n+1}, \alpha_n) \) according to Eq. (7.7)-(7.13). Subsequently, determine the updated values of internal parameter \( \alpha_{n+1} (\lbrack u \rbrack_{ex}^{n+1}) \), equivalent crack traction \( t_{n+1} (\alpha_{n+1}) \), softening modulus \( H_{n+1} (\alpha_{n+1}) \) and the damage variable \( d_{n+1} (\alpha_{n+1}) \).

4. Proceed with the next loading step.

It is noticed that due to the explicit nature of damage models the computation does not require any iteration, neither on the structural level nor on the constitutive level. This ensures the robustness and efficiency of the computational model in failure analyses of FRC structures even in cases of complex crack configurations.

### 7.2 Finite element mesh processing: insertion of ISE

#### 7.2.1 Preprocessing on full or partial domain

Using interface solid elements to capture the cracking processes in concrete structures, it is necessary to include a code block for placing solid elements into the gaps at the interfaces prior to the initiation
of cracks in the numerical implementation. For that purpose, a pre-processing algorithm needs to be executed before applying the loading conditions (SÁNCHEZ ET AL. 2014). As illustrated in Fig. 7.4, first of all, the topology of original finite element mesh is investigated. Based on the topological information (e.g. the neighboring bulk elements, the common edges and the node connectivity), the whole original mesh is fragmented. The common edges are duplicated; every bulk element shrinks via offsetting the edges by $h/2$; a “phantom mesh” is obtained [Fig. 7.4(b)]. Afterwards, every gap between the neighboring bulk elements is filled with degenerated solid elements with very high aspect ratio [Fig. 7.4(c)]; the “actual mesh” is prepared for computation.

In analogy to the finite element method using classical zero-thickness interface elements, interface solid elements are well suited for capturing complex cracking phenomena (e.g. crack initiation and opening, crack propagation, kinking or even branching) in quasi-brittle materials without applying any crack path tracking technique (MANZOLI ET AL. 2014). However, the crack pattern represented via interface elements is inevitably mesh dependent. This drawback can be alleviated by mesh refinement, at the cost of increased computational expense, which, in turn, can be reduced by pre-defining the interface elements only in the vulnerable regions (SU ET AL. 2010) or applying an adaptive algorithm for the insertion of interface elements (PANDOLFI AND ORTIZ 2002).

### 7.2.2 Adaptive insertion during simulation

In the present work, an algorithm is implemented for the adaptive insertion of ISE. This algorithm automatically places the ISE for potential cracks during the loading procedure. Similar effort has been made for the classical zero-thickness interface elements (PANDOLFI AND ORTIZ 2002); nevertheless, the adaptive technique does not seem to be implemented for the recently proposed ISE.

As illustrated for the 2D configuration (Fig. 7.5), the adaptive insertion of ISE is performed according to the following processes:

1. Before applying the loading conditions, prepare the phantom mesh [Fig. 7.4(b)].

![Figure 7.4: Pre-processing (full fragmentation): (a) original mesh; (b) phantom mesh obtained by duplicating the edges and shrinking the bulk elements; (c) actual mesh for computation, obtained after the insertion of solid elements into all interfaces.](image-url)
2. Start computation with the original mesh. During every load step:
   a) Check the stress state in the bulk elements and the resulting tractions on the interfaces. If the magnitude of traction (with positive normal component) exceeds a given threshold, i.e. \( |t| > t_{\text{thr}} \) (\( t_{\text{thr}} \) can be set as 0.8\( f_t \)), the interface is considered to be critical.
   b) The critical interface is fully separated; the cluster of phantom nodes corresponding to each node of the interface is activated and used: For the influenced bulk elements that have been using the interfacial nodes so far, the node connectivities are updated by using the correct phantom nodes; two ISE are generated based on the newly activated phantom nodes and are placed in the interfacial gap.
   c) The adjacent interfaces are partially split and filled with one ISE.
3. The updated mesh is used for the computation of current load step.

This algorithm is implemented both in 2D and 3D configurations. In Fig. 7.6, the insertion progress at one interface in the 3D configuration is illustrated and is described as follows:

1. Initially, the interface using Node-1, 2 and 3 is shared by two bulk elements, i.e. Element-T and Element-B.
2. When Node-1 is separated, activate the corresponding pair of phantom nodes (Node-1T and Node-1B). Update the nodes of bulk element-T and element-B (replace the current node-1 by the new Node-1T and Node-1B, respectively). Create the first interface solid element (ISE-I).
3. When Node-2 is split, activate the phantom nodes (2T and 2B). Update the nodes of bulk elements in the same way as described above; meanwhile, the nodes of the existing ISE-I are updated as well (using the new node-2B instead of the current Node-2). Generate the second ISE (ISE-II).
4. The node-3 is activated. Similarly, the phantom nodes 3T and 3B are activated and used; the nodes of the bulk elements as well as the existing two ISE are updated. Finally, ISE-III is created.

Note that the scheme described above (Fig. 7.6) can be used for different situations during the
interface activation procedure. This interface can be fully separated within one load step due to the critical stress level, or partially opened due to the splitting of one or two neighboring interface. Therefore, for any interface in the structure, depending on the existing number of split nodes (equals to 0, 1 or 2) and the number of new nodes that will be separated (1, 2 or 3), there are in total six cases to be considered.

It it worth emphasizing that the major difference between the present algorithm and the exiting adaptation techniques for the classical interface elements is that conventional solid elements are placed in the interfacial gaps. In the adaptive insertion algorithm, it is crucial to update the nodes of existing bulk elements as well as interface solid elements. In addition, one must ensure that a consistent node-numbering rule (e.g. the right hand rule) is used.

7.3 Verification and validation: cracking in plain concrete

In this section, the performance of interface solid elements in representing the cracking processes is demonstrated by means of selected plain concrete structural simulation examples. The effect of fibers on the structural failure behavior will be discussed in the next chapter.
7.3.1 L-shape test

The “L-shaped” panel test is documented in Winkler et al. (2004) and has been selected by a few researchers to demonstrate the performance of a concrete cracking model (Feist 2004). The 100-mm-thick panel, made of plain concrete, is fixed at the bottom and subjected to a concentrated load which causes the failure of panel, as illustrated in Fig. 7.7. During the test, the macro crack initiates at the inner corner as a result of the concentrated tensile stresses and propagates first with a small angle to the horizontal direction and then nearly horizontally through the panel.

This test is re-analyzed via numerical simulation using interface solid elements in the 2D configuration. Different finite element settings are considered and the results are compared to verify the implemented model. The material parameters used for plain concrete are: $E_c = 25.850$ MPa, $f_t = 2.7$ MPa, $G_f = 0.09$ MPa·mm (Feist 2004). The displacement is measured at the lower corner on the right edge.

Influence of finite element discretization

In order to inspect the influence of finite element discretization on the results of simulation, four cases of the (original) mesh are considered:

- structured discretization with coarse grid (mesh size = 25 mm),
- structured discretization with fine grid (6.25 mm),
- unstructured coarse mesh (25 mm)
- and unstructured fine mesh (6.25 mm).

As shown in Fig. 7.8, the macroscopic crack pattern is generally captured in all the four cases. Nevertheless, it is clearly observed that the crack paths obtained from the FE simulation using ISE are inevitably mesh dependent, due to the fact that the tensile damages are restricted at the ISE placed along the edges of the original bulk elements; this issue leads to spurious energy dissipation.
when the numerically obtained crack path deviates from the actual one. Furthermore, from the comparison among different crack patterns, it is noticed, that finer discretization enables to better follow the realistic crack path; in addition, it appears that, in this test, the unstructured mesh allows to predict more reasonable crack path, due to the curved nature of the macro crack in this situation. The force–displacement relations obtained from these four cases are contained in Fig. 7.9. Without surprise, using coarse discretization, the computed curves exhibits remarkable “zigzag” shape and noticeable overestimation of structural response, while the finer meshes lead to smoother curves.

**Figure 7.8:** Deformation (scale factor = 100) and contour plot of crack opening magnitude obtained from simulations of the L-shape test: (a) structured discretization with grid size 25 mm; (b) structured mesh with grid size 6.25 mm; (c) unstructured discretization with mesh size 25 mm; (d) finer unstructured mesh (6.25 mm).

**Influence of interface solid element width**

It is already demonstrated in MANZOLI ET AL. (2012) that with a smaller value of $h$, i.e. the width of interface solid element, the interface degradation phenomena can be better approximated by the degenerated solid elements. In the present work, this issue is verified based on the L-shape test. As shown in Fig. 7.10, the vulnerable domain is discretized with very fine unstructured mesh (approximately 3 mm element size), which allows to obtain a crack pattern that is in very good agreement with reality (Fig. 7.7). It is observed that the force–displacement relations obtained fit the experimental results very well. Nevertheless, one may notice that a large value of $h$ can lead to the underestimation of structural reaction forces, which is understood as the consequence of volume loss during the mesh fragmentation and ISE-insertion processes.

**Remark:** In principle, smaller value of $h$ allows better recovery of the strong discontinuity kinematics; however, as in many existing discrete crack models, in order to avoid the numerical instabil-
Figure 7.9: L-shape test: force-displacement relations obtained from the simulations with different finite element discretization.

Figure 7.10: L-shape test: simulation with locally refined mesh: (a) crack pattern; (b) force–displacement relations obtained with different values of $h$.

ities associated with the too high elastic stiffness $K_0$ of interface, $h$ should not be too small. In the present work, $h$ is generally selected to be approximately $10^{-3}$ of the smallest bulk element size.

Influence of load step size

It is well known that the load step size plays an important role affecting the robustness and accuracy of the computational analysis of nonlinear problems induced by inelastic material behavior. Using the implicit/explicit integration scheme, a good combination of the robustness of explicit scheme and the accuracy of an implicit scheme can be achieved (OLIVER ET AL., 2008).
In Fig. 7.11, the influence of the size of load increment \( u_{\text{inc}} \) can be observed. The crack pattern obtained from the simulation with \( u_{\text{inc}} = 0.005 \) mm and \( u_{\text{inc}} = 0.001 \) mm are shown in Fig. 7.11(a) and (b), respectively (the crack pattern with \( u_{\text{inc}} = 0.0002 \) mm is already included in Fig. 7.10). As can be seen, the macroscopic crack path are reasonably well represented in all the three cases. However, if a large loading increment is applied (particularly in conjunction with a fine mesh), the actual speed of crack propagation can be so high that a sequential activation of the ISE one-by-one along the macro crack path can not be guaranteed; this causes the “over-stress” in the vulnerable regions and leads to discontinuous crack paths from simulation [Fig. 7.11(a)]. With smaller \( u_{\text{inc}} \), this problem can be significantly reduced [Fig. 7.11(b)]. In regard of the structural responses, it is obvious that large step sizes leads to the overestimation of reaction forces which can be effectively reduced if using small step sizes. Generally speaking, without doubt, the smaller \( u_{\text{inc}} \) is, the better the numerical results are; nevertheless, a reasonable step size should be selected based on the specific problems in order to control the total computational cost.

### 7.3.2 Notched panel with hole

This example is concerned with a discontinuous crack path. As shown in Fig. 7.12, a notched panel with a hole and subjected to tension is recently tested and reported in Ambati et al. (2015). The panel is made of cement mortar which can be considered as elastic-brittle material. The lower small hole is fixed, while the upper one is loaded upwards. The test is simulated using interface solid elements, with the parameters used for the cement mortar as follows: \( E_c = 5,983 \) MPa, \( f_t = 2.0 \) MPa, \( G_f = 0.01 \) MPa-mm. Note that this test is selected to demonstrate the capability of model to follow the cracking processes; therefore, no attempt is made to compare the structural force-displacement responses, neither in the present thesis, nor in the original article.

As can be seen from Fig. 7.13, the cracking processes are captured by the simulation showing...
very good agreement with the test: The macro crack path starts at the notch-tip and propagates first horizontally and then gradually turns towards the hole [Fig. 7.13(a)]. After the first section of crack path is fully developed, the second section of crack path is generated at the edge of the hole and continues advancing horizontally, finally penetrating the remaining part of specimen [Fig. 7.13(b)].

### 7.3.3 3D examples

**Brokenshire’s test: notched prism under torsion**

The “Brokenshire’s test”, i.e. a plain concrete prism with an inclined notch subjected to torsion, is documented in Brokenshire (1996) and sometimes selected to demonstrate the performance of a concrete cracking model (Gasser and Holzapfel 2006). The dimension and loading conditions
7.3. VERIFICATION AND VALIDATION: CRACKING IN PLAIN CONCRETE

The parameters used for plain concrete are: $E_c = 30,000$ MPa, $f_t = 3.0$ MPa, $G_f = 0.09$ MPa·mm.

As shown in Fig. 7.15, the force–CMOD relationship obtained from the FE simulation using interface solid elements replicates the experimental results satisfactorily. The major crack initiates at the notch-tip and propagates towards the bottom surface with slight twisting; this non-planar major crack surface is captured by the model as well (Fig. 7.16).

**Notched beam: example of adaptive insertion**

The test of notched beam under three-point bending which has been simulated in Chapter 6, Section 6.4 is now re-analyzed in the 3D configuration considering the plain concrete case. The beam has the dimension of 600 mm × 150 mm × 150 mm. A total displacement of 0.5 mm is applied
Figure 7.16: Notched prism under torsion: activated ISE at different loading states showing the development of slightly twisted major crack surface: (a) crack initiation; (b)-(c) crack propagation; (d) the developed major crack surface.

Figure 7.17: Results of 3D simulation of the notched beam with adaptive insertion of ISE: (a) load-displacement diagram; (b) crack pattern (side view and bottom-side view).

at the middle-span on the top surface which leads to the failure of beam, characterized by a vertical crack initiates at the notch and reaches the top surface (Fig. 7.17).

This example is concerned with the performance of the implemented adaptive insertion technique for interface solid elements. The original 3D FE-discretization contains 6,743 nodes and 34,537 linear tetrahedral elements. As a reference case, the simulation is first performed with full insertion of ISE, which leads to the completely fragmented mesh filled with ISE at every interfacial gap; consequently, the pre-processed mesh contains 138,148 nodes (approximately 20 times
Figure 7.18: Results of the 3D simulation of notched beam with adaptive insertion of ISE: (a) evolution of the relative problem size with respect to the case of full insertion, for system degree of freedom and number of elements, respectively; (b) all the ISE (both damaged and undamaged) at the end of simulation ($u = 0.5$ mm).

of the original mesh) and 235,939 elements (approximately 7 times of the original mesh), among which 201,402 are ISE. The simulation results, including the load–displacement curve and the crack pattern are illustrated in Fig. 7.17.

By employing the adaptive insertion technique, the computation starts with the original mesh. Interface solid elements are first inserted in the vicinity of notch-tip and then continuously added surrounding the front of the macroscopic crack which propagates vertically until the structure fails. At the end of simulation ($u = 0.5$ mm), only 77,538 ISE are created and distributed in the vicinity of the structural crack path [Fig. 7.18(b)]. As compared to the full insertion, the efficiency of adaptive insertion is clearly shown in [Fig. 7.18(a)]: At the beginning, only 5% of the system degrees of freedom and 15% of the number of elements are used in computation; these proportions increase with the loading procedure. When the structural crack is approaching the top surface ($u > 0.3$ mm), the insertion of ISE in every load step becomes rather negligible and the problem size maintains at the level of approximately 41% for the system DOF and 48% for the number of elements.

In this example, we observe that the proposed adaptive insertion technique considerably reduces the computational expense as compared to the situation where ISE are inserted in the whole mesh, in spite of the marginal increase in the computation time due to mesh adaptation. Note that the efficiency of this method highly depends on the characteristics of specific problems; the geometry of structure, the loading conditions, the failure pattern and the original discretization have direct impact on the performance of adaptive ISE insertion.

7.3.4 Concluding remarks

As can be observed from the numerical examples presented above, using conventional solid elements to approximate the interface degradation behavior of the discrete cracks in concrete, the fracturing
phenomena such as the crack initiation, opening and propagation can be captured in a natural manner, without the need of a crack tracking algorithm. The direct use of a discrete crack model, i.e. the traction–separation law, to control the softening behavior of ISE allows to obtain realistic structural responses. Due to the explicit nature of damage models, the implicit-explicit algorithm implemented in the present guarantees the robustness of computation. The drawback of mesh dependency can be reduced by means of fine FE-discretization of the domain. The high computational cost can be significantly reduced via a priori placing the ISE in only limited regions or the adaptive insertion during simulation. By addressing these aspects, the FEM using ISE is considered as an appropriate tool for (particularly 3D) failure analyses of structures made of fiber-reinforced concrete.
Chapter 8

Validation of Fiber-Reinforced Concrete Model

By incorporating the surrogate function form of traction–separation law obtained from the crack bridging model (Chapter 5) which is based upon the single fiber pullout model (Chapter 3 and 4), the finite element model using interface solid elements can be applied to perform the numerical analyses of structures made of fiber-reinforced concrete. Thus, the present multilevel modeling scheme for FRC is ready to follow the influence of the fundamental design parameters (such as the concrete class, single fiber properties, distribution and orientation of fibers, geometry of structure), on the cracking phenomena and the failure responses of FRC structures. In this chapter, the performance of the present multilevel model for steel-fiber–reinforced concrete (SFRC) structures is demonstrated based on selected 2D and 3D numerical examples.
8.1 Notched prism under tension

The notched prism subjected to tension, which is already investigated in Chapter 5 concerning the crack bridging effect (Fig. 5.9), is now re-analyzed by means of finite element simulation. The test can only be simulated in the 3D configuration; considering the symmetry of problem, 1/4 of the structure is modeled. The material parameters used are already listed in Table 5.1. The softening behavior of every interface solid element is controlled by the analytical surrogate function (traction-separation law) obtained from the crack bridging model, i.e. the \( t(w) \) relation [Eq. (5.16)] with the values of coefficients \( t_1 = 0.77, t_2 = 0.45, c_1 = 0.50 \) and \( c_2 = 0.64 \).

![Figure 8.1: Re-analysis of notched prism under tension: (a) comparison between the analytical input law, the finite element result and the experimental results; (b) deformation (scale factor = 5) and contour plot of the crack opening magnitude.](image)

As shown in Fig. 8.1, at the very beginning, the effective stress ascends nearly vertically approaching the tensile strength; afterwards, cracking occurs from the notches. The horizontal major crack quickly emanates from the notches at the middle-height and penetrates the specimen, accompanied with a rapid drop of stress, followed by a ductile post-cracking response containing a hardening branch. As expected, the finite element results of the effective stress vs. the crack mouth opening displacement relation (the blue solid curve in the diagram) is nearly identical to the input \( t(w) \) law (the red dotted curve).

8.2 Notched beam under three-point bending

8.2.1 Problem description

The laboratory test on a series of notched beams made of different SFRC mixtures subjected to three-point bending is reported in Putke et al. (2014), where the results of numerical simulation using embedded strong discontinuity approach, as briefly presented in Chapter 6, are also included.
This problem is now simulated employing the finite element method with interface solid elements representing cracks.

As illustrated in Fig. 8.2, every specimen is a beam with the width of 150 mm; a notch with the depth of 30 mm is located at the middle-span. The specimens are made of high-strength concrete and long steel fibers with hooked ends which are identical to the case of notched prism discussed above. In addition, short straight “micro” fibers with length 13 mm, diameter 0.19 mm and yield stress 2200 MPa are used. Three different composites are tested:

• “L60”: SFRC containing 60 kg/m$^3$ long hooked-end fibers,
• “S60”: SFRC with 60 kg/m$^3$ short straight fibers,
• “L30S30”: SFRC made of a mixture containing 30 kg/m$^3$ long hooked-end fibers and 30 kg/m$^3$ short straight fibers.

Since the specimens are cast in a “lying” mold, the fibers tend to align to the X-Y plane (Fig. 8.2). A vertical structural crack develops at the notched cross section; as will be seen in the resulting structural responses, this corresponds to a favorable situation for the residual load bearing capacity.

For the numerical simulation, the traction–separation laws used to control the softening behavior of interface solid elements are obtained from the crack bridging model, as described in Chapter 5: The macroscopic crack develops with the normal direction coinciding with X-axis; the casting process results in the fiber orientation profile $\lambda^{\text{cast}} = [0.40, 0.40, 0.20]^T$; the boundary effect is taken into account for the cross section of specimen characterized by the dimension $L_y = L_z = 150$ mm and the 30 mm saw-cut notch. As a result, the obtained analytical surrogate function for the crack bridging law are as follows: For L60, the values of coefficients for the $t - w$ relation are $t_1 = 1.40$, $t_2 = 1.10$, $c_1 = 1.05$ and $c_2 = 0.78$; for S60, $t_1 = 1.04$ and the remaining three are zero; the traction separation law for the FRC “cocktail” L30S30 is the weighted average of the previous two. For simplicity, the finite element simulation is performed in the 2D configuration.

### 8.2.2 Results of numerical simulation

As shown in Fig. 8.3 and 8.4, for all the three cases, the major crack initiates at the notch-tip and propagates vertically until reaching the top surface of specimen; this cracking process is well represented by the activated interface solid elements. One may notice the minor crack branches in...
the vicinity of major crack path; these minor cracks unload while the major crack opens; similar phenomenon is observed on the specimen during the test.

The computed force versus crack mouth opening displacement relations are found in good agreement with the experimental data (Fig. 8.5). In general, the structural responses are nearly identical for all the three cases till the first peak of reaction force \( F \approx 17 \text{ kN} \). Afterwards, the structure exhibits ductile behavior with remarkable residual loading capacity, as the major crack opens and the fibers bridging the crack are progressively pulled out from the concrete matrix. Nevertheless, the responses are quite different due to different fiber–concrete bonding mechanisms: For the case L60, after a limited decrease, the reaction force continues increasing up to a level above 20 kN corresponding to approximately 2 mm crack mouth opening displacement [Fig. 8.5(a)]. Such a remarkable hardening branch in the post-cracking load bearing capacity of structure is associated with the anchorage effect of hooked-ends and the favorable fiber orientation in this case. For the case of
8.3 Round panel test

In the previous two cases analyzed, only one macroscopic crack that leads to the failure of structure is obtained. In this section, the 3D simulation concerning multiple structural cracks that evolve simultaneously is demonstrated.

8.3.1 Problem description

The ASTM C1550 round panel test is a standardized testing method proposed for the determination of the flexural behavior of fiber-reinforced concrete in the post-cracking regime using a circular plate resting on three symmetrically placed supports (see Fig. 8.6 and 8.7) and subjected to a central point load (ASTM C1550 2012). The round panel test is investigated and reported in a few publications, for the purpose of, e.g., the inverse analysis of the crack bridging effect by means of an analytical model (NOUR ET AL. 2011). Numerical methods such as layered finite element shell model can be used to simulate this problem (GÖDDE AND MARK 2010).

The experiments documented in NOUR ET AL. (2011) is selected to be re-analyzed using the present FRC modeling framework. As shown in Fig. 8.6, the panel is characterized by the radius of 400 mm and the thickness of 80 mm. Three supports are arranged at the radius of 375 mm, with 120° angle from each other. A concentrated force is applied via a loading “piston” at the center of top surface. As a result, a “Y”-shaped crack pattern, consisting of three macro cracks that join at the center, initiate at the bottom surface and propagate vertically to the top surface.

The FRC investigated here contains 80 kg/m³ hooked-end steel fibers; hence, similar to the “dog-bone” test discussed in Section 5.3, it is assumed that 60% of the fibers are effective. The material parameters used for the simulation are listed in Table 8.1. In analogy to the previous examples, the
traction–separation law is obtained by using the crack bridging model. The fiber orientation profile \( \lambda^{\text{cast}} = [0.40, 0.40, 0.20]^T \) is assumed; the boundary effect is considered for the cross section of specimen with the width of 800 mm and the height of 80 mm; the resulting analytical crack bridging law \( t(w) \) is characterized by \( t_1 = 1.58, t_2 = 1.03, c_1 = 1.41 \) and \( c_2 = 1.20 \).
8.3. ROUND PANEL TEST

Figure 8.7: Round panel test: details of support (ASTM C1550 2012).

Figure 8.8: Numerical results of the round panel test: contour plot of the crack opening magnitude in the deformed configuration (scale factor = 5): (a) bottom view, (b) side view and (c) bottom-side view.

8.3.2 Numerical simulation

It is clear that the numerical model for this problem must be built in the 3D configuration. The present FE model contains the “transfer plate” of supports (see Fig. 8.7), for which the linear elastic steel property is assumed. As can be seen in Fig. 8.8, an ideal Y-shaped crack pattern is obtained from the simulation, obviously due to the aligned FE-discretization at the expected crack paths. The three structural cracks, with 120° from each other and 60° from the closest supports, join at the symmetric line of the circular plate. Note that a less regular crack pattern is obtained in the experiment (Fig. 8.6), probably due to the stochastic nature of material properties in reality.

The reaction force vs. the displacement relations are obtained and plotted in Fig. 8.9. As can
be seen, the structure exhibits approximately a linear elastic response at the beginning of loading. With the reaction force approaching 40 kN, the cracks initiate. The force shortly remain at the level of 40 kN; afterwards, it continues ascending to the peak value of 50 kN; this ascending branch is clearly attributed to the anchorage effect of hooked ends of fibers intercepting the cracks. After the peak, the reaction force decays gradually, as the fibers are progressively sliding out of the concrete matrix and providing residual bridging stresses across the cracks. As a reference, assuming that plain concrete is used for the panel, the structure fails rapidly after the reaction force reaches 30 kN peak force. Obviously, the flexural toughness of structure is dramatically enhanced via the addition of fibers into the concrete material.

Figure 8.9: Numerical results of the round panel test: force–displacement relations compared with the experimental range.
As mentioned in the introduction of thesis, the serviceability and durability characteristics of segmental tunnel linings can be inspected by means of laboratory tests, from the material scale to the full-ring level. In the design phase of tunnel linings, simplified engineering models are used for the estimation of the load-bearing capacity of lining segments. A more detailed investigation of particularly the undesired material responses at a local level and the consequence on the structural behavior of lining segments demands sophisticated computational tools such as finite element methods. As demonstrated in the preceding chapters, in the framework of multiscale oriented modeling of fiber-reinforced concrete materials and structures, the influence of different design parameters on the structural responses can be well followed from the single fiber level to the structural scale, by means of appropriately capturing the nonlinear behavior of individual material components and their mutual interactions at different length-levels, and efficiently transmitting the model information through the scales. The developed multilevel FRC model is now applied to the numerical simulation of the failure behavior of tunnel lining segments made of different FRC composites.
9.1 Flexural behavior of lining segment

The serviceability and durability requirement of tunnel lining demands that the segments provide sufficient resistance against various loads at different stages, including the bending moments developed while in stock or caused by the external pressure or even concentrated forces from the underground (Caratelli et al. 2011, Gettu et al. 2004, Liao et al. 2015). In order to evaluate the flexural toughness of lining segments, a full-scale laboratory test on the segment made of FRC is performed by Caratelli et al. (2011); this test is selected and re-analyzed using the present FRC model.

9.1.1 Setup of experiment

The lining segment tested is cast according to the geometry of the precast segments used in the Brenner Base Tunnel, where the lining system is characterized by the external radius of 3,000 mm and the thickness of 200 mm. The full-size flexural test on the segment, which is 3,650 mm long and 1,500 mm wide in total, is carried out in the laboratory environment by applying a three-point bending load (Fig. 9.1).

The specimen is made of normal-strength concrete and contains 40 kg/m³ high-strength hooked-end steel fibers. In the present re-analysis, the material parameters used are as follows:

- Concrete matrix: elastic modulus $E_c = 34,998$ MPa; compressive strength $f_c = 50$ MPa; tensile strength $f_t = 3.62$ MPa; tensile fracture energy $G_F = 0.09$ N/mm.
- Steel fiber: length $L_f = 30$ mm; diameter $d_f = 0.35$ mm; yield stress $\sigma_y = 2,200$ MPa. The fibers are characterized by hooked-ends with the detailed geometry as $l_1^h = 1.4$ mm, $l_2^h = 0.8$ mm, $\alpha^h = 45^\circ$ and $\rho^h = 0.6$ mm. For the fiber–matrix interactions, the interfacial bond strength $\tau_{\text{max}} = 2.01$ MPa, asymptotic frictional stress $\tau_0 = 0.52$ MPa, reference slip $s_{\text{ref}} = 0.18$ mm and friction coefficient $\mu = 0.4$ are assumed (see Chapter 3 and 4 to recall the physical meaning of these parameters).
9.1.2 Numerical simulation

For the 2D numerical simulation, only the part of lining segment between the two supports is considered in the finite element model (Fig. 9.2). The nonlinear material behavior of FRC is derived according to the crack bridging model (Chapter 5). In addition to the previously mentioned properties of concrete matrix and individual fiber, considering that the specimen is cast in such a way that the fibers tend to align to the horizontal plane, the fiber orientation profile $\lambda_{\text{cast}} = [0.40, 0.40, 0.20]^T$ is assumed (Fig. 9.2). Note that this assumption may become less accurate in the regions distant from the middle span due to the curved shape of segment [in principle, more accurate fiber dispersion characteristics in a segment can be determined experimentally (CARMONA ET AL. 2016)]. Furthermore, it is clear that the macroscopic cracks relevant to the failure of segment will initiate on the inner surface and propagate in the radial direction towards the outer surface. Therefore, the segment cross section with the width of 1500 mm and the height of 200 mm is considered as the potential crack plane; based on this plane, the boundary effect on fiber orientation is analyzed. As the result of crack bridging model, the traction–separation law governing the softening behavior of any crack in this specific SFRC composite is obtained and is characterized by the values of coefficients for the surrogate function $(t - w$ relation) as $t_1 = 1.40$, $t_2 = 1.27$, $c_1 = 1.98$ and $c_2 = 2.13$ [Eq. (5.16)]. This softening law is assigned to all the interface solid elements [see Chapter 7, Eq. (7.9)].

9.1.3 Discussion of results

Experimental observations

As observed during the experiment (see the photos contained in Fig. 9.3), after reaching the elastic-limit state, the segment experiences different cracking stages at different levels of load (CARATELLI ET AL. 2011): The first visible crack is detected at the force level of approximately 95 kN at the middle span [Fig. 9.3(a)]. With the increase of load, a few minor cracks appear at the force level of 125 kN [Fig. 9.3(b)-(c)]. The maximum value of reaction force that the segment can carry reaches 140 kN; after this peak force, a major crack is observed at the middle span [Fig. 9.3(d)]; this crack continues opening and finally leads to the failure of structure.
Results obtained from numerical simulation

The computed force–displacement ($F - u$) relation is plotted in Fig. 9.4. In general, good accordance is noticed between the curves obtained from the finite element simulation and from the laboratory test. The simulation result obtained with the assumption of plain concrete is included as a reference; obviously, without any reinforcement, the segment will fail in a rather brittle manner, after reaching the reaction force of 95 kN. With the enhancement contributed by steel fibers, the structure exhibits quite a ductile post-elastic response, which is characterized by the following states corresponding to the computed crack patterns shown in Fig. 9.5:

- At the beginning, the structure is fully elastic and the force–displacement curve ascends almost linearly until $F \approx 75$ kN, when the first crack initiates on the inner surface at the middle span. The force increases and reaches the first peak at approximately 110 kN, followed by a very limited drop, as a result of the development of the first crack that propagates vertically towards the outer surface [Fig. 9.5(a)].
- Further opening of the first crack is temporarily retarded by the fiber bridging effect, particularly the contribution of hooked ends of fibers. While the local stresses are re-distributed in the vicinity of this crack, the structural reaction force increases to the level of 130 kN, at which two secondary cracks are generated nearly simultaneously. These two cracks initiate and propagate nearly symmetrically with respect to the first crack [Fig. 9.5(b)-(c)].
- The force continues to increase, until reaching the maximum reaction force of structure
9.1. FLEXURAL BEHAVIOR OF LINING SEGMENT

Figure 9.4: Results of FRC lining segment subjected to bending: comparison between the force–displacement relations obtained from experiment and simulation, including the simulation result of plain concrete as a reference. The arrows point to different states characterized by the crack patterns shown in Fig. 9.5.

Figure 9.5: Simulation results of crack pattern at different loading states, in deformed configuration (scale factor = 20): (a) the first crack developed at mid-span \( u = 1.0 \) mm; (b) the second crack \( u = 1.6 \) mm; (c) the third crack \( u = 2.0 \) mm; (d) further opening of the first crack \( u = 6.5 \) mm.

\[ F_{\text{max}} \approx 150 \text{ kN}. \] Afterwards, the first crack continues to grow and becomes the major crack [Fig. 9.5(d)], which is finally responsible for the failure of specimen, as shown in the contour plot of crack opening magnitude at the end of simulation (Fig. 9.6).

This scenario is also investigated via 3D simulation, where half width (750 mm) of the segment is considered. As can be seen in Fig. 9.7, similar results are obtained as compared to the 2D analysis, regardless of the less regular crack pattern as a result of the unstructured finite element discretization in 3D. In general, the numerical simulations replicate the laboratory test with high accuracy,
9.2 Lining segment subjected to hydraulic jack forces

The case investigated in the previous section is focused on the ductile behavior of FRC segment subjected to external concentrated load. Now the attention is drawn to the load bearing capacity and failure behavior of a lining segment subjected to hydraulic jack forces imposed by the tunnel boring machine during the excavation of tunnel. In this section, the proposed multilevel FRC model is employed to investigate the behavior of a segment suffering imperfect support from the installed lining ring. The imperfection can be induced by the deformation, misalignment, inadequate installation of exiting ring, etc., and can lead to axial cracks in the lining segment under large jack forces. As reported in Sugimoto (2006), this type of cracking is one of the most frequently observed damage patterns according the survey of dozens of tunnel lining sites in Japan. In the following parametric study on the effect of different FRC materials, the critical lining segment is investigated individually; the support provided by the neighboring segments are accounted for by means of idealized...
boundary conditions (Fig. 9.8).

Figure 9.8: Illustration of the possible damage mechanism in lining segments subjected to the hydraulic jack forces during tunnel excavation: (a) damages due to imperfect support from the previous lining ring; (b)-(c) simplified boundary conditions for the numerical analysis of the critical segment.

9.2.1 Numerical simulation

Figure 9.9: Setup for the numerical simulation of the critical lining segment subjected to hydraulic jack forces: (a) model geometry; (b) model parts and boundary conditions.

The finite element model is built according to the geometric data obtained from the cooperation and exchange between the Collaborate Research Center (SFB) 837 and the German Railway (DB Netz AG) construction project Rastatt Tunnel. The model geometry consists of five parts as described in the following (Fig. 9.9).

Segment

The segment is assumed to be made of

- normal-strength concrete having the same properties as in the previous example (Section 9.1),
- “Fiber-L”: normal-strength long hooked-end steel fiber (Dramix® RC-80/60-BN, of which the material parameters are already listed in Table 4.1)
- and “Fiber-M”: high-strength medium hooked-end steel fiber (identical to that in Section 9.1).
Figure 9.10: Numerical simulation of lining segment under jack forces: $t - w$ relations controlling the softening of interface solid elements obtained using the crack bridging model for different FRC composites.

Table 9.1: Parameters of different crack bridging laws [Eq. (5.16)] considered in the numerical simulation of tunnel lining segments made of different fiber–concrete composites.

<table>
<thead>
<tr>
<th></th>
<th>PC</th>
<th>L60</th>
<th>L120</th>
<th>M60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength $f_t^*$ (MPa)</td>
<td>3.62</td>
<td>3.68</td>
<td>3.74</td>
<td>3.68</td>
</tr>
<tr>
<td>Reference crack opening $w_{ref}$ (mm)</td>
<td>0.0255</td>
<td>0.0255</td>
<td>0.0255</td>
<td>0.0255</td>
</tr>
<tr>
<td>Ultimate crack opening $w_u$ (mm)</td>
<td>-</td>
<td>30</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>Coefficient $t_1$ (MPa)</td>
<td>-</td>
<td>1.32</td>
<td>2.26</td>
<td>2.19</td>
</tr>
<tr>
<td>Coefficient $t_2$ (MPa/mm)</td>
<td>-</td>
<td>0.82</td>
<td>1.41</td>
<td>1.45</td>
</tr>
<tr>
<td>Coefficient $c_1$ (mm/mm)</td>
<td>-</td>
<td>1.21</td>
<td>1.21</td>
<td>2.21</td>
</tr>
<tr>
<td>Coefficient $c_2$ (1/mm)</td>
<td>-</td>
<td>1.06</td>
<td>1.06</td>
<td>2.67</td>
</tr>
</tbody>
</table>

Four different steel-fiber–reinforced concrete (SFRC) composites are assumed for the purpose of parametric study:

- “PC”: plain concrete, without any reinforcement,
- “L60”: normal-content FRC containing 60 kg/m$^3$ Fiber-L,
- “L120”: FRC with high content (120 kg/m$^3$) of Fiber-L
- and “M60”: normal-content (60 kg/m$^3$) FRC made of Fiber-M.

All these crack opening laws (i.e. the $t - w$ relations as plotted in Fig. 9.10 and characterized by the values of parameters listed in Table 9.1) are generated using the crack bridging model, considering the fiber orientation profile $\lambda_{cast}^{ast} = [0.40, 0.40, 0.20]_T$, and the cross section of segment with the dimension of 2000 mm $\times$ 500 mm as the potential crack plane (Chapter 5). It is worth pointing out that both in the cases of L120 and M60, a super-critical crack bridging law, characterized by the peak of crack bridging stress in the hardening branch exceeding the tensile strength of composite, is obtained. As introduced in Section 2.4, the high performance of FRC will lead to a significantly different post-cracking behavior (more specifically, the strain-hardening associated with multiple cracking phenomena) from the situation of plain concrete and normal-performance FRC.
9.2. LINING SEGMENT SUBJECTED TO HYDRAULIC JACK FORCES

The displacements on the right half of back surface is fixed during the simulation [Fig. 9.9(b)], as one of the boundary conditions.

Other parts
At each lateral surface of segment, a fixed rigid “wall” is attached. The segment-wall interface, corresponding to the segment joint, is modeled via pre-damaged interface solid elements (see Appendix C), so that the opening of joint is allowed but the penetration between segments is prevented.

The two plates are designed to transfer the hydraulic jack forces onto the front surface of segment (Fig. 9.9). They are assumed to be made of steel and perform linear elastically. During the simulation, the distributed jack forces are applied in a load-controlled manner; the simulation is terminated when the resulting $u_1$ on the unsupported side of segment reaches 10 mm.

9.2.2 Discussion of results

![Figure 9.11: Numerical simulation of lining segment under jack forces: results of the reaction force ($F$) vs. displacement ($u_1$) relations obtained from the segments made of different FRC composites.](image)

**Jack force vs. displacement relation**

It is clear that due to the asymmetry of boundary conditions on the back surface, the equally applied jack forces result in different displacements between the two plates ($u_1 > u_2$). It is noticed that the $F - u_2$ curves in all the cases investigated exhibit almost linear elastic behavior; on the contrary, under the same level of jack force, the resulting displacement on the unsupported side ($u_1$) increases significantly due to the elastic deformation and, furthermore, the development of cracks in the segment. According to Fig. 9.11, in all the four cases, the $F - u_1$ curves ascend almost linearly upto approximately 2 MN (which is usually the maximum hydraulic jack force in reality). Afterwards, one can distinguish the responses of segments made of different SFRC composites:

- For the segment made of plain concrete (PC), with the quick growth of the macroscopic cracks, the reaction force increases slowly; when $F$ approaches 6.5 MN, the segment can hardly carry any more thrust forces.
• In the cases that the segments are enhanced by steel fibers, thanks to the crack bridging stresses contributed by fibers, significantly higher load-bearing capacity is obtained. More specifically, for the case L60, the jack force $F$ increases up to a level of 9.5 MN as $u_1$ approaches 10 mm; the force reaches 11.5 MN in Case L120.

• Without doubt, higher fiber content leads to higher load-carrying capacity of segment. Nevertheless, as mentioned in Section 2.4, the reinforcing index is not only dependent on the fiber content, but also related to several other parameters such as fiber strength. Therefore, it is not surprising to see that with normal content of high-strength fibers (Case M60), the level of reaction force is almost the same as that with high-content normal-strength fibers (L120).

By comparing the $t - w$ diagram in Fig. 9.10 and the $F - u_1$ curves in Fig. 9.11, it is clearly observed that the strengthening of structural load-bearing capacity is correlated with the level of crack bridging stress that a specific FRC composite can reach.

**Cracking process and failure pattern**

![Figure 9.12](image1.png)

**Figure 9.12**: Simulation results of lining segment under jack forces (Case PC): different cracking states indicated by the activated interface solid elements in the interior of segment in the undeformed configuration:
- (a) $F = 2.15$ MN;
- (b) $F = 3.05$ MN;
- (c) $F = 5.66$ MN.

![Figure 9.13](image2.png)

**Figure 9.13**: Simulation results of lining segment under jack forces (Case L120): different cracking status indicated by the activated interface solid elements in the interior of segment in the undeformed configuration:
- (a) $F = 3.05$ MN;
- (b) $F = 6.02$ MN;
- (c) $F = 8.99$ MN.

In general, similar structural deformation and cracking phenomena are observed for the cases PC and L60, and for the cases L120 and M60, respectively:
• In the segment made of plain concrete, an “axial crack” along the symmetric plane initiates at the front surface and propagates, parallel to the axial direction of tunnel, towards the back surface of segment. In the late stage, a “shear crack” develops in the left part of segment.
Figure 9.14: Simulation results of lining segment subjected to hydraulic jack forces: contour plots of crack opening magnitude (inner view and front view) at the jack force level \( F = 6.47 \) MN, corresponding to the failure of segment made of plain concrete. (For Case PC, the deformation scale factor equals to 20; for L60, scale factor = 40; for L120 and M60, scale factor = 100.)

(Fig. 9.12). These two structural cracks open rapidly, leading to the loss of load-carrying capacity of the unsupported part of segment and, finally, the failure of structure, as shown in Fig. 9.14.

- In the situation that the segment is made of high-performance FRC (Case L120 or M60), remarkably different cracking processes are noticed. Taking Case L120 as an example, due to the super-critical crack bridging stress (Fig. 9.10), the rapid opening of cracks is prevented;
Figure 9.15: Simulation results of lining segment under jack forces: contour plot of crack opening magnitude in the deformed configuration (scale factor = 40) at the end of simulation ($F = 11.51$ MN): (a) Case L120; (b) Case M60.

a series of (almost) parallel axial cracks and a similar group of shear cracks are generated (Fig. 9.13). These parallel cracks open simultaneously with a limited magnitude of crack width (Fig. 9.14), until one of them starts to grow into the major crack that is responsible for the failure of segment.

If one further compares the crack patterns of different segments at $F = 6.47$ MN, when the segment made of plain concrete is considered as failed, one can clearly discover the effect of fibers on the control of cracking in concrete structures (Fig. 9.14):

- If the segment is enhanced by normal-content normal-strength steel fibers (Case L60), similar cracking phenomena to Case PC are observed; however, due to the bridging effect of fibers, the cracking processes are postponed with respect to the applied jack force: As shown in Fig. 9.14, the maximum crack opening width is approximately 3 mm, which is significantly smaller as compared to 10 mm in Case PC; furthermore, the shear crack has not yet developed.

- The use of normal-content high-strength fibers (M60) results in rather similar structural responses and crack patterns to the situation of high-content yet normal-strength fibers (L120). Nevertheless, it is interesting to see in Fig. 9.14 that, as compared to Case L120, for M60 the distributed cracks are characterized by a smaller crack width (less than 0.4 mm). As mentioned at the very beginning of present thesis, the control of crack width constitutes one of the major advantages of employing fibers to enhance the durability of tunnel linings (Chapter 1).

From the discussion above, it is clear that to obtain an optimal performance of structure, high-performance FRC composites are preferred. In particular, the results of M60 demonstrates that, even with lower fiber content, a better crack width control can be achieved. Nevertheless, a drawback of M60 is that, as can be noticed from the crack bridging effect (Fig. 9.10), due to the relatively fast decay of residual stress after the peak of hardening branch, the localization of cracks and the opening of macro crack occur earlier than in the case of L120. The consequence can be clearly observed in Fig. 9.15: At the level of jack force $F = 11.51$ MN, the segment L120 still exhibits distributed cracks with small crack widths, although the major axial crack is growing; on the contrary, in the
segment M60, the distributed cracks have already localized to the major axial crack characterized by a large crack width.

Concerning the deformation of segment, one may notice that the unsupported part of segment is forced to bend in the axial direction of tunnel; however, due to the presence of neighboring segment, the deformation of segment in the circumferential direction is constrained; consequently, the damaged left half of segment is “squeezed out” in the radial direction of tunnel. The last issue to mention is that, in addition to the axial and shear cracks, minor cracks are found in the corners of joint region as well; this “chipping” phenomenon is also recognized as one of the main damage patterns in tunnel lining segments according to Sugimoto (2006).

Concluding remarks

The numerical results discussed above show: The imperfect support from the installed lining ring can result in damages to the lining segments when the TBM advances; using fibers to strengthen the segment allows to improve the load-bearing capacity; if the fibers contribute sufficient reinforcement, multiple cracking phenomena characterized by distributed cracks with small widths can be obtained. From the comparison among different cases, the following specific conclusions concerning the effect of different FRC composites in this scenario can be drawn:

- Load-bearing capacity: M60 \approx L120 > L60 > PC;
- Crack width control: M60 > L120 > L60 > PC;
- Structural ductility: L120 > M60 > L60 > PC.

The present examples clearly demonstrate that the multilevel modeling framework proposed in the thesis can be employed as a “virtual laboratory” for the purpose of evaluating the effect of fibers, predicting the failure behavior of lining segments in different scenarios and providing useful information for the design and optimization of lining segments and structures made of fiber-reinforced concrete.
Chapter 10

Conclusion and Outlook

10.1 Summary of work

As documented in this thesis, a series of model components have been proposed for the purpose of multiscale oriented analyses of the failure behavior of engineering structures such as segmental tunnel linings made of steel-fiber–reinforced concrete (SFRC). The proposed multilevel model mainly comprises three levels of modeling concerned with the pullout behavior of single fibers (Level 1), the crack bridging behavior of fiber cocktails (Level 2) and the discrete representation of cracking in SFRC structures in the framework of finite element method (Level 3).

Single fiber pullout model

As the fundamental block constituting the multilevel modeling framework, an analytical model has been proposed to describe the single fiber pullout response (Level 1). First of all, the pullout behavior of single straight steel fibers embedded in the concrete matrix, either with or without inclination angle with respect to the loading direction, was analyzed. Starting with the situation without inclination, the interfacial friction–slip mechanism was considered by capturing the fiber–matrix interactions in three different pullout stages. This model component was used to replicate the corresponding laboratory test and to calibrate the necessary parameters for the proposed interfacial friction law. Next, for the pullout of an inclined straight fiber, the additional complexities connected with the fiber plastic deformation, the concrete local damage and the extra frictional stress were taken into account in the respective submodels. A numerical algorithm was implemented in order to efficiently generate the pullout force–displacement relationship. Afterwards, the focus was put on the situation of hooked-end steel fibers. The anchorage effect of hooked ends was captured by a series of key states during the pullout; for every key state, the additional pullout force contributed by the hook was directly calculated by means of evaluating the force equilibrium on different sections of the fiber. This additional force was accounted for in the model for straight fiber pullout, allowing to predict the pullout load–displacement relation of a hooked-end steel fiber embedded in a concrete matrix with an arbitrary inclination angle to the loading direction. Finite element simulations were
performed to provide supporting information for the analytical formulation. The model was successfully validated with representative experimental results. Accordingly, it is concluded that the proposed analytical model is able to capture the major mechanisms involved in various situations of the pullout of single steel fibers.

**Crack bridging model**

Based on the single fiber pullout model, the bridging effect of an opening crack in a specific SFRC composite can be computed by the integration of the pullout resistance of all fibers intercepting the crack (Level 2). As the main parameter concerned, the anisotropic fiber orientation in the SFRC members was taken into account: An ellipsoidal profile was assumed to represent the directional preference of fibers as a consequence of the casting process; furthermore, the influence of the boundaries of mold on the fiber orientation was considered by means of dividing the corresponding cross section of specimen into different areas based on the distance from the surface of mold and analyzing the boundary effect in each area; as a result, the average probability density as a function of the fiber inclination with respect to the potential crack plane was obtained and used in the integration of crack bridging stresses. It was demonstrated by selected validation analyses, that this approach is able to reflect the directional preference of fibers in SFRC members and the influence on the post-cracking ductility; the crack bridging response can be predicted more realistically as compared to models where an isotropic fiber orientation is assumed and the boundary effect is neglected. For the purpose of implementing the model in a finite element program, an analytical parameterized function form was generated based on the numerically obtained traction–separation relations for the specific fiber cocktail.

**Numerical model for cracking in SFRC structures**

At the level of structural finite element analyses (Level 3), the recently proposed mesh fragmentation technique in conjunction with the use of degenerated solid elements to approximate the interface degradation mechanisms in concrete structures was selected and implemented. These interface solid elements (ISE) were supplied with the crack bridging model as traction–separation law to represent the discrete cracks in SFRC structures. The major advantage of this method is, that it allows to capture complex cracking phenomena such as the initiation and opening, localization and propagation, branching and merging of cracks, without the requirement of specific crack tracking techniques. The implicit/explicit scheme was applied for the integration of the highly nonlinear traction–separation model. This strategy has proven as very efficient as in each load increment only a linear system needed to be solved without iterations. For the mesh processing involved during the simulation, an adaptive insertion algorithm was implemented so that the ISE could be progressively placed in the vicinity of potential cracks, which reduced the total computational cost significantly. Selected validation examples showed, that the multilevel model is able to replicate the characteristics of the post-cracking behavior of SFRC structures for different fiber cocktails in very good agreement with the experimental results.
10.2. Future perspectives

The main feature of the proposed multilevel model is, that it allows to follow the influence of various design parameters of FRC materials and structures (such as the concrete quality, the single fiber property, the fiber content and orientation) from the single fiber scale up to the structural level. It is worth pointing out, that in the proposed modeling framework, only a few basic low-scale parameters are required as input; among these parameters, only the interfacial friction–slip law (at Level 1) needs to be calibrated from the experimental data of single straight fiber pullout (see Chapter 3), and the fiber orientation profile as a consequence of the casting process and the portion of effective fibers (at Level 2) are assumed based on experimental observations.

Being aware of the assumptions made while developing the models as well as the limitations of present work, the following aspects are considered as the future perspectives of direct relevance and special interest:

- The multilevel modeling framework can be further extended to a lower length scale, particularly to the level of friction–slip mechanisms on the fiber–matrix interface, so that the bond–slip law can be derived based on a substantial understanding and the appropriate modeling of the micro-structural damage processes in the interface transition zone, instead of calibrated with experimental data.
- Concerning the modeling of pullout of single fiber embedded in concrete matrix, which is currently limited to the situation of straight and hooked-end steel fibers, new types of fiber characterized by different material, size and geometry (such as micro or macro synthetic fibers, or steel fibers with other shapes) can be included in order to expand the scope of model applicability.
- For the post-elastic behavior of specific FRC composite material, the crack bridging model can be enriched by an appropriate model capable to predict the final status of fiber distribution and orientation characteristics in the structure, by means of quantitatively following the influence of the complete casting procedure of FRC members. Besides, a systematic experimental investigation of the group effect (including the fiber–fiber interaction) would be useful to evaluate the loss of fiber efficiency in different FRC composites.
• Instead of the present assumption of homogeneous and deterministic material properties, the influence of heterogeneous and stochastic nature of concrete and fibers should be considered.

• In the case of high-performance FRC experiencing multiple cracking, an adequate submodel capturing the stress transfer mechanisms between the fiber and matrix phases will allow the correct prediction of crack spacing and, consequently, the objective macroscopic constitutive behavior of the representative volume element of high-performance FRC.

• The finite element model using interface solid elements to capture the tensile fracture is being combined with a suitable plasticity-based model describing the compressive crushing of concrete materials so that the behavior of FRC under complex stress conditions will be well captured.

• For the purpose of model-based design and optimization with respect to the materials and substructures for segmental tunnel linings made of concrete strengthened with hybrid reinforcement (including fibers, conventional steel bars, steel mesh, etc.), additional model components describing the nonlinear behavior of individual material components and, more importantly, their mutual interactions should be developed. Furthermore, the computational efficiency of the complete modeling framework can be improved via appropriate up-scaling techniques such as using surrogate models in parameterized function forms at different length scales.

The developed multilevel model for the segmental tunnel linings made of FRC is being incorporated into a sophisticated modeling platform addressing the critical aspects (such as the prediction of ground conditions ahead of the cutting wheel, the design of segmental linings with enhanced robustness and the modeling of tunnel excavation process) involved in the tunnel construction projects (Collaborative Research Center 837 2016).


Breitenbücher, R., G. Meschke, F. Song, and Y. Zhan (2014). Experimental, analytical and numerical analysis of the pullout behaviour of steel fibres considering different fibre types, inclinations and concrete strengths. Structural Concrete 15, 126–135. [cited at p. 4, 8, 19, 57]


SHAH, S. P. AND A. E. NAAMAN (1976). Mechanical properties of glass and steel fiber reinforced mortar. Journal of the American Concrete Institute 73, 50–53. [cited at p. 3]


Appendix A

Elastic-Plastic Behavior of Fiber Segment as Cantilever

Consider the fiber Section A-B with the radius $r_f$ and length $L_{AB}$, made of ideal elastoplastic material with the elasticity modulus $E_f$ and yield stress $\sigma_y$. As a typical situation of cantilever, Point B is fixed and Point A is subjected to a transverse force $V_A$ (Fig. 3.8).

At the elastic state, applying the Timoshenko beam theory, the deflections at Point A due to bending ($w_{A,M}$) and shear ($w_{A,V}$), as well as the ratio between them are expressed as follows:

\[
\begin{align*}
  w_{A,M} &= \frac{V_AL_{AB}^3}{3E_fI_f}, \\
  w_{A,V} &= \frac{2V_AL_{AB}(1+\nu_f)}{E_fA_f f_s}, \\
  \gamma &= \frac{w_{A,M}}{w_{A,V}} = \frac{9}{4(1+\nu_f)} \left(\frac{L_{AB}}{d_f}\right)^2, \\
\end{align*}
\]

with $I_f = \pi r_f^4/4$ as the moment of inertia and $f_s = 27/32$ as the shear correction factor for circular cross sections. At the elastic limit state of the fiber cross section at Point B, the bending moment is $M_B = M_{el} = \pi \sigma_y r_f^3/4$; making use of Eq. (A-1), the total free end deflection $w_{el}^A$ at this state is written as

\[
\begin{align*}
  w_{el}^A &= w_{el}^{A,M} + w_{el}^{A,V} = M_{el} L_{AB}^2 \left(1 + \frac{1}{\gamma}\right),
\end{align*}
\]

(A-2)

with $w_{el}^{A,M}$ denoting the contribution of bending and $w_{el}^{A,V}$ due to shear.

Now, given the rotation angle $\psi$ of Point A with respect to Point B, the transverse force $V_A$ is calculated as follows:

- If $\psi \leq w_{el}^A / L_{AB}$, the complete cantilever is still elastic, and the transverse force $V_A$ is calculated as, from the equations above:

\[
\begin{align*}
  w_{A,M} &= \frac{w_A}{1+1/\gamma} = \frac{\psi L_{AB}}{1+1/\gamma} \quad \Rightarrow \quad V_A(\psi) = w_{A,M} \frac{3E_f I_f}{L_{AB}^3} = \frac{\psi}{1+1/\gamma} \frac{3E_f I_f}{L_{AB}^2},
\end{align*}
\]

(A-3)

- When $\psi > w_{el}^A / L_{AB}$, the fiber material starts to yield at Point B. The relation between the force $V_A$ and the rotation angle $\psi$ becomes nonlinear due to the elastic-plastic “hinge”. We
introduce an empirical equation as follows:

$$V_A(\psi) = V_A^{pl} - (V_A^{pl} - V_A^{el}) \exp\left(\frac{w_{A,M}^{el} - \psi L_{AB}}{w_{A,M}^{pl} - w_{A,M}^{el}}\right),$$  \hspace{1cm} (A-4)$$

with

$$V_A^{pl} = \frac{M_{pl}}{L_{AB}}, \quad V_A^{el} = \frac{M_{el}}{L_{AB}}, \quad w_{A,M}^{pl}, \quad w_{A,M}^{el} \quad \frac{w_{A,M}^{el} M_{pl}}{M_{el}}.$$  \hspace{1cm} (A-5)$$

Here $M_{pl} = 4 \sigma_y r_f^3 / 3$ is the fully plastic bending moment. The validity of this formula is checked by comparing its results with finite element simulation of cantilever (Fig. A.1).

**Figure A.1:** Elastic-plastic response of cantilever: finite element simulation compared with analytical approximation.
The fiber segment B-C remaining embedded in the matrix is idealized as an elastic beam resting on an elastic foundation (Fig. 3.9). According to the beam on elastic foundation theory [see e.g. BOWLES (1996)], the deflection $w$ as a function of the local coordinate $\xi$ along the fiber satisfy the following governing equation:

$$E_f I_f \frac{d^4 w}{d\xi^4} + kw = 0.$$  (B-1)

**Foundation stiffness**

In Eq. (B-1), $k$ denotes the foundation stiffness; its value is derived in analogy to FANTILLI AND VALLINI (2007): As shown in the cross section diagram (Fig. B.1), we assume that the reaction stress in the foundation $\sigma_{22}(r_m)$ at the distance of $r_m$, induced by a distributed force $q$ on the beam, is uniformly distributed within a range of $\pi/2$:

$$\sigma_{22}(r_m) = \frac{q}{2r_m}.$$  (B-2)

Considering the plain strain state in the matrix and neglecting the horizontal strain $\varepsilon_{11}$, we have

$$\sigma_{22} = C_{22} \varepsilon_{22}, \quad \text{with } C_{22} = \frac{E_m(1 - \nu_m)}{(1 + \nu_m)(1 - 2\nu_m)}.$$  (B-3)

By integrating the vertical strain $\varepsilon_{22}$ w.r.t. $r_m$ in the interval from the fiber radius $r_f$ to the boundary (radius) of the specimen $R_m$, we obtain the vertical displacement of fiber:

$$w = \int_{r_f}^{R_m} \varepsilon_{22}(r_m) \, dr_m = \frac{q}{2C_{22}} \ln \left( \frac{R_m}{r_f} \right),$$  (B-4)
from which the elastic foundation stiffness is derived:

\[
k = \frac{q}{w} = \frac{2E_m(1-\nu_m)}{(1+\nu_m)(1-2\nu_m)\ln(R_m/\gamma)}.
\]  

(B-5)

Figure B.1: Beam on elastic foundation: idealized distribution of foundation reaction stress due to the distributed force \( q \) acting on the beam.

Lateral behavior along the beam

The general solution of the differential equation (B-1) is

\[
w(\xi) = e^{\beta \xi} \left[ C_1 \sin \beta \xi + C_2 \cos \beta \xi \right] + e^{-\beta \xi} \left[ C_3 \sin \beta \xi + C_4 \cos \beta \xi \right], \quad \text{with} \quad \beta = \left( \frac{k}{4E_f I_f} \right)^{1/4}.
\]  

(B-6)

Considering the boundary conditions at both ends of fiber Section B-C:

\[
M(0) = 0, \quad M(L_{BC}) = M_B, \quad V(0) = 0, \quad V(L_{BC}) = V_B,
\]  

with \( L_{BC} \) the length of this section, the constants \( C_1 - C_4 \) are solved from a set of linear equations regarding \( w', w'', w''', \text{ and } w'''' \), i.e. the 1st to the 4th derivatives. With the constants obtained, Eq. (B-6) gives the deflection along the fiber. The resulting distributed reaction force of the foundation is obtained as (Fig. 3.9)

\[
p(\xi) = -kw(\xi).
\]  

(B-8)

It is noticed that when the remaining fiber embedment length \( L_{BC} \) approaches zero, the solution above gives very large values of deflections and distributed forces, which is unrealistic. In reality, the matrix material is not always linearly elastic: It has been observed in experiments that the very short and stiff fiber end cuts into the concrete (LEUNG AND SHAPIRO 1999), i.e. the concrete suffers
localized damage due to high local stresses. Therefore, we consider a reduction of the foundation reaction, by means of an empirical formula as follows:

\[
p' = \begin{cases} 
|p|, & \text{if } \eta \leq 1, \\
\frac{d_f f_c}{\ln^2 \eta + 1}, & \text{if } \eta > 1,
\end{cases}
\]  

(B-9)

where \( \eta = \frac{|p|}{d_f f_c} \) indicates the degree of utilization of the foundation compressive strength.

With the modified distributed transverse force \( p' \) along the beam obtained, the distributed frictional force \( t \) is calculated as:

\[
t = t_i + t_p = \begin{cases} 
(1 - \eta/2) \pi d_f \tau + \mu p', & \text{if } \eta \leq 1, \\
\pi d_f \tau / 2 + \mu p', & \text{if } \eta > 1,
\end{cases}
\]  

(B-10)

where the first component \( t_i \) indicates the remaining contribution of the pure interface frictional stress \( \tau(s) \) as described in Section 3.1, which is dependent on the axial slip \( s \) calculated as

\[
s = 2L_{AB} - \frac{\delta}{\cos \theta}.
\]  

(B-11)

The coefficient \( 1 - \eta/2 \) represents the degree of remaining interfacial contact: If \( \eta = 0 \), the fiber is fully in contact with the matrix; if \( \eta > 1 \), only half of the circumference is still in contact. The second term \( t_p \) is the additional Coulomb’s friction resulting from the deflection of the fiber; \( \mu \) is the friction coefficient.
Appendix C

Interface Closure

C.1 Coordinate transformation

Transformation of vector and 2nd-order tensor

The transformation of coordinate system (COS) is required, from time to time, to describe the material properties in different configurations [see e.g. Richter (2005)].

A 1st-order tensor (i.e., a vector) \( \mathbf{x} \) in the 3D Cartesian space is expressed in two different COS as follows (the Einstein summation is used in the following):

\[
x_i \mathbf{e}_i = \mathbf{x} = x_j' \mathbf{e}'_j, \quad \text{with} \quad x_i = \mathbf{x} \cdot \mathbf{e}_i \quad \text{and} \quad x_j' = \mathbf{x} \cdot \mathbf{e}'_j.
\]  

(C-1)

Here, \( \mathbf{e}_i \) are the bases of “old” COS \((i = 1, 2, 3)\), and \( \mathbf{e}'_j \) are the bases of a “new” COS \((j = 1, 2, 3)\). Considering

\[
x_j' = \mathbf{x} \cdot \mathbf{e}'_j = x_i \underbrace{\mathbf{e}_i \cdot \mathbf{e}'_j}_{T_{ij}} = x_i T_{ij},
\]

(C-2)

the transformation matrix \( \mathbf{T} \) for the components of vector \( \mathbf{x} \) transformed from the old COS to the new COS is derived as

\[
\begin{bmatrix}
x'_1 \\
x'_2 \\
x'_3
\end{bmatrix} =
\begin{bmatrix}
\mathbf{e}'_1 \cdot \mathbf{e}_1 & \mathbf{e}'_1 \cdot \mathbf{e}_2 & \mathbf{e}'_1 \cdot \mathbf{e}_3 \\
\mathbf{e}'_2 \cdot \mathbf{e}_1 & \mathbf{e}'_2 \cdot \mathbf{e}_2 & \mathbf{e}'_2 \cdot \mathbf{e}_3 \\
\mathbf{e}'_3 \cdot \mathbf{e}_1 & \mathbf{e}'_3 \cdot \mathbf{e}_2 & \mathbf{e}'_3 \cdot \mathbf{e}_3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
\]

(C-3)

whereas the transformation from the new COS to the old COS is

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} =
\begin{bmatrix}
\mathbf{e}_1 \cdot \mathbf{e}'_1 & \mathbf{e}_1 \cdot \mathbf{e}'_2 & \mathbf{e}_1 \cdot \mathbf{e}'_3 \\
\mathbf{e}_2 \cdot \mathbf{e}'_1 & \mathbf{e}_2 \cdot \mathbf{e}'_2 & \mathbf{e}_2 \cdot \mathbf{e}'_3 \\
\mathbf{e}_3 \cdot \mathbf{e}'_1 & \mathbf{e}_3 \cdot \mathbf{e}'_2 & \mathbf{e}_3 \cdot \mathbf{e}'_3
\end{bmatrix}
\begin{bmatrix}
x'_1 \\
x'_2 \\
x'_3
\end{bmatrix}
\]

(C-4)
Obviously, $T'$ is the transpose of $T$, i.e.

$$ T' = T^T. \quad (C-5) $$

For a 2nd-order tensor $A$ expressed in different COS as

$$ A_{ij} e_i \otimes e_j = A'_{mn} e'_m \otimes e'_n, \quad (C-6) $$

the components in new COS is related to the components in old COS as

$$ A'_{mn} = T_{mi} T_{nj} A_{ij}, \quad (C-7) $$

here, $m = 1, 2, 3$ and $n = 1, 2, 3$. The Eq. (C-7) can be written as

$$ A' = TAT^T. \quad (C-8) $$

**Transformation of constitutive law**

According to Eq. (C-7), the stress tensor components in the new COS can be obtained from the components in the old COS as

$$ \begin{bmatrix} \sigma'_{11} & \sigma'_{12} \\ \sigma'_{21} & \sigma'_{22} \end{bmatrix} = T \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} T^T, \quad (C-9) $$

for the 2D configuration and

$$ \begin{bmatrix} \sigma'_{11} & \sigma'_{12} & \sigma'_{13} \\ \sigma'_{21} & \sigma'_{22} & \sigma'_{23} \\ \sigma'_{31} & \sigma'_{32} & \sigma'_{33} \end{bmatrix} = T \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} T^T, \quad (C-10) $$

for the 3D case. These transformations are valid for a strain tensor $\varepsilon$ in the same way.

Since the constitutive laws are often expressed in the *matrix notation*, and a stress tensor can be written as a vector, in different coordinate systems as:

$$ [\sigma] = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}, \quad [\sigma'] = \begin{bmatrix} \sigma'_{11} \\ \sigma'_{22} \\ \sigma'_{12} \end{bmatrix} \quad (C-11) $$

in the 2D situation and

$$ [\sigma] = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{bmatrix}, \quad [\sigma'] = \begin{bmatrix} \sigma'_{11} \\ \sigma'_{22} \\ \sigma'_{33} \\ \sigma'_{12} \\ \sigma'_{23} \\ \sigma'_{13} \end{bmatrix} \quad (C-12) $$
in 3D. Consequently, the transformation Eq. (C-9) and (C-10) are reformulated as

\[
[s'] = [Q_σ][σ],
\]

with \([Q_σ]\) derived as follows: Taking the 2D transformation Eq. (C-9) as an example,

\[
\begin{bmatrix}
σ'_{11} & σ'_{12} \\
σ'_{21} & σ'_{22}
\end{bmatrix} =
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
σ_{11} & σ_{12} \\
σ_{21} & σ_{22}
\end{bmatrix}
\begin{bmatrix}
T_{11} & T_{21} \\
T_{12} & T_{22}
\end{bmatrix}
\]

leading to the following component results (noticing that \(σ_{12} = σ_{21}\)):

\[
σ'_{11} = T_{11}(T_{11}σ_{11} + T_{12}σ_{21}) + T_{12}(T_{11}σ_{12} + T_{12}σ_{22}) \\
= T_{11}T_{11}σ_{11} + T_{11}T_{12}σ_{21} + T_{12}T_{11}σ_{12} + T_{12}T_{12}σ_{22} \\
= T_{11}^2σ_{11} + T_{12}^2σ_{22} + 2T_{11}T_{12}σ_{12}
\]

\[
σ'_{22} = T_{21}(T_{21}σ_{11} + T_{22}σ_{21}) + T_{22}(T_{21}σ_{12} + T_{22}σ_{22}) \\
= T_{21}T_{21}σ_{11} + T_{21}T_{22}σ_{21} + T_{22}T_{21}σ_{12} + T_{22}T_{22}σ_{22} \\
= T_{21}^2σ_{11} + T_{22}^2σ_{22} + 2T_{21}T_{22}σ_{12}
\]

\[
σ'_{12} = T_{21}(T_{11}σ_{11} + T_{12}σ_{21}) + T_{22}(T_{11}σ_{12} + T_{12}σ_{22}) \\
= T_{21}T_{11}σ_{11} + T_{21}T_{12}σ_{21} + T_{22}T_{11}σ_{12} + T_{22}T_{12}σ_{22} \\
= T_{11}T_{21}σ_{11} + T_{12}T_{22}σ_{21} + (T_{12}T_{21} + T_{11}T_{22})σ_{12}
\]

\[
σ'_{21} = T_{11}(T_{21}σ_{11} + T_{22}σ_{21}) + T_{12}(T_{21}σ_{12} + T_{22}σ_{22}) \\
= T_{11}T_{21}σ_{11} + T_{11}T_{22}σ_{21} + T_{12}T_{21}σ_{12} + T_{12}T_{22}σ_{22} \\
= T_{11}T_{21}σ_{11} + T_{12}T_{22}σ_{21} + (T_{11}T_{22} + T_{12}T_{21})σ_{12} = σ'_{12}.
\]

Therefore,

\[
[Q_σ] = \begin{bmatrix}
T_{11}^2 & T_{12}^2 & 2T_{11}T_{12} \\
T_{21}^2 & T_{22}^2 & 2T_{21}T_{22} \\
T_{11}T_{21} & T_{12}T_{22} & T_{12}T_{21} + T_{11}T_{22}
\end{bmatrix}
\]

allows to transform the stress vector \(σ\) into \(σ'\). Similarly, in 3D

\[
[Q_σ] =
\]

\[
\begin{bmatrix}
T_{11}^2 & T_{12}^2 & T_{13}^2 & 2T_{12}T_{13} & 2T_{11}T_{13} & 2T_{11}T_{12} \\
T_{21}^2 & T_{22}^2 & T_{23}^2 & 2T_{22}T_{23} & 2T_{21}T_{23} & 2T_{21}T_{22} \\
T_{31}^2 & T_{32}^2 & T_{33}^2 & 2T_{32}T_{33} & 2T_{31}T_{33} & 2T_{31}T_{32} \\
T_{12}T_{31} & T_{22}T_{32} & T_{33} & T_{22}T_{33} + T_{23}T_{32} & T_{21}T_{33} + T_{23}T_{31} & T_{21}T_{32} + T_{22}T_{31} \\
T_{31}T_{12} & T_{32}T_{12} & T_{13}T_{33} & T_{13}T_{33} + T_{12}T_{32} & T_{11}T_{33} + T_{12}T_{31} & T_{11}T_{32} + T_{12}T_{31} \\
T_{11}T_{31} & T_{12}T_{32} & T_{13}T_{23} & T_{13}T_{23} + T_{12}T_{22} & T_{11}T_{23} + T_{12}T_{21} & T_{11}T_{22} + T_{12}T_{21}
\end{bmatrix}
\]
For a strain vector
\[
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2\varepsilon_{12} \\
2\varepsilon_{23} \\
2\varepsilon_{13}
\end{bmatrix}
\], or
\[
\begin{bmatrix}
\varepsilon'_{11} \\
\varepsilon'_{22} \\
\varepsilon'_{33} \\
2\varepsilon'_{12} \\
2\varepsilon'_{23} \\
2\varepsilon'_{13}
\end{bmatrix}
\].
\text{(C-18)}

Noticing the factor “2 ×” for the shear components, the transformation reads
\[
\begin{bmatrix}
\varepsilon'_{11} \\
\varepsilon'_{22} \\
\varepsilon'_{33} \\
2\varepsilon'_{12} \\
2\varepsilon'_{23} \\
2\varepsilon'_{13}
\end{bmatrix} = [Q] \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2\varepsilon_{12} \\
2\varepsilon_{23} \\
2\varepsilon_{13}
\end{bmatrix},
\text{(C-19)}
\]

with
\[
[Q] =
\begin{bmatrix}
T_{11}^2 & T_{12}^2 & T_{13}^2 & T_{12}T_{13} & T_{11}T_{13} & T_{11}T_{12} \\
T_{21}^2 & T_{22}^2 & T_{23}^2 & T_{22}T_{23} & T_{21}T_{23} & T_{21}T_{22} \\
T_{31}^2 & T_{32}^2 & T_{33}^2 & T_{32}T_{33} & T_{31}T_{33} & T_{31}T_{32} \\
2T_{12}T_{31} & 2T_{22}T_{32} & 2T_{23}T_{33} & T_{22}T_{33} + T_{23}T_{32} & T_{21}T_{33} + T_{23}T_{31} & T_{21}T_{32} + T_{22}T_{31} \\
2T_{11}T_{31} & 2T_{12}T_{32} & 2T_{13}T_{33} & T_{12}T_{33} + T_{13}T_{32} & T_{11}T_{33} + T_{13}T_{31} & T_{11}T_{32} + T_{12}T_{31} \\
2T_{11}T_{21} & 2T_{12}T_{22} & 2T_{13}T_{23} & T_{12}T_{23} + T_{13}T_{22} & T_{11}T_{23} + T_{13}T_{21} & T_{11}T_{22} + T_{12}T_{21}
\end{bmatrix}.
\text{(C-20)}
\]

Considering the constitutive law in different COS as
\[
[\sigma] = [C][\varepsilon], \quad \text{and} \quad [\sigma'] = [C'][\varepsilon'].
\text{(C-21)}
\]

After simple matrix operation, the transformation of stiffness matrix is obtained as
\[
[C'] = [Q_\sigma][C][Q_\varepsilon]^{-1}.
\text{(C-22)}
\]

One can also prove that
\[
[Q_\varepsilon]^{-1} = [Q_\sigma]^T, \quad \text{and} \quad [Q_\sigma]^{-1} = [Q_\varepsilon]^T.
\text{(C-23)}
\]

Therefore, one may use
\[
[C'] = [Q_\sigma][C][Q_\sigma]^T
\text{(C-24)}
\]

to compute the components of stiffness matrix in the new COS (RICHTER 2005).

### C.2 Interface stiffness under compression

#### Transformation matrix

Now, we consider the standard global (X-Y-Z) coordinate system characterized by the bases
\[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}, \quad \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}, \quad \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}.
\text{(C-25)}
\]
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For a specific interface (crack plane), the bases of the local (N-R-S) COS are denoted as

\[
\begin{align*}
\mathbf{e}_1' &= \mathbf{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}, & \mathbf{e}_2' &= \mathbf{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}, & \mathbf{e}_3' &= \mathbf{s} = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}.
\end{align*}
\] (C-26)

As a result, the transformation matrix from the global COS to the local COS is obtained as

\[
\mathbf{T}^{G \rightarrow L} = \begin{bmatrix} \mathbf{e}_1' \cdot \mathbf{e}_1 & \mathbf{e}_1' \cdot \mathbf{e}_2 & \mathbf{e}_1' \cdot \mathbf{e}_3 \\ \mathbf{e}_2' \cdot \mathbf{e}_1 & \mathbf{e}_2' \cdot \mathbf{e}_2 & \mathbf{e}_2' \cdot \mathbf{e}_3 \\ \mathbf{e}_3' \cdot \mathbf{e}_1 & \mathbf{e}_3' \cdot \mathbf{e}_2 & \mathbf{e}_3' \cdot \mathbf{e}_3 \end{bmatrix} = \begin{bmatrix} n_x & n_y & n_z \\ r_x & r_y & r_z \\ s_x & s_y & s_z \end{bmatrix}.
\] (C-27)

The inverse process, i.e. the transformation from the local COS to the global COS can be performed using

\[
\mathbf{T}^{L \rightarrow G} = (\mathbf{T}^{G \rightarrow L})^T = \begin{bmatrix} n_x & r_x & s_x \\ n_y & r_y & s_y \\ n_z & r_z & s_z \end{bmatrix}.
\] (C-28)

Resolution and transformation

For any closing crack that has experienced the crack opening accompanied with interface damage, the material is under compressive stresses. In the present work, it is assumed that the interface regains the elastic stiffness in the normal direction (N), so that the two surfaces of a crack plane do not “penetrate” each other. The stiffness degradation in shear directions (R and S) remains. To take these into account, the elastic stiffness matrix is additively split into two:

\[
\mathbf{C} = \mathbf{C}_E + \mathbf{C}_G
\] (C-29)

where, in 2D,

\[
\mathbf{C}_E = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{C}_G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_{55} \end{bmatrix},
\] (C-30)

or, in 3D,

\[
\mathbf{C}_E = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{C}_G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}.
\] (C-31)

Here \( \mathbf{C}_E \) and \( \mathbf{C}_G \) contain the components governing the bulk- and shear deformations, respectively.
Considering the existing damage value $d$ obtained when the crack is open, the material stiffness matrix under compression becomes

$$\mathbf{C}_n^L = \mathbf{C}_E + (1 - d)\mathbf{C}_G.$$  
(C-32)

Note that the current material stiffness matrix $\mathbf{C}_n'$ becomes anisotropic and it is defined in the local coordinate system of crack (N-R-S). In order to obtain the stiffness matrix $\mathbf{C}_n$ (in the global COS) for the structural solution, the following coordinate transformation has to be carried out:

$$\mathbf{C}_n = \mathbf{Q}_{\sigma}^{L\to G} \mathbf{C}_n' (\mathbf{Q}_{\sigma}^{L\to G})^T.$$  
(C-33)

Here $\mathbf{Q}_{\sigma}^{L\to G}$ is the stress transformation matrix from the local COS to the global COS. The components of $\mathbf{Q}_{\sigma}^{L\to G}$ can be determined based on $\mathbf{T}^{L\to G}$ [Eq. (C-28)], and in the same form as Eq. (C-17).

**Examples**

The first example shows the performance of the present interface model for the behavior of an existing crack under compression-shear stresses (Fig. C.1): Due to the presence of the inclined crack (for which we assume an existing value of the maximum equivalent opening $\alpha = 0.1$ mm) penetrating the panel, the upper part of specimen slides on the crack surface when the panel is subjected to compression. During the sliding process, the interface recovers the elastic stiffness in the normal direction due to the compressive stress in the normal direction; meanwhile, the damage value corresponding to $\alpha = 0.1$ mm remains for the tangential stiffness. After the sliding induced interface (trial) stresses exceed the (degraded) elastic limit, further damage of the interface is introduced. This example also shows that in the present work, the interface softening can also be caused by the mixed-mode crack opening.

![Figure C.1: Numerical example of sliding along predefined crack path.](image)

The second example concerns with the central straight through crack Brazilian disk made of brittle material (e.g. rock). The disk is characterized by an inclined pre-crack in the center and is subjected to compressive load, leading to a mixed Mode-II fracture (sliding) of the existing crack and
a so-called “wing-crack” failure pattern [Fig. C.2(left)]. As indicated by the activated (red-colored) interface solid elements in Fig. C.2(right), the numerical simulation predicts the crack pattern in good agreement with the experiment.

**Figure C.2:** Example of “wing-crack” in rock: the crack pattern obtained from experiment (left) (HAERI ET AL. 2014) and from the present numerical model using interface solid elements (right).
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<td>142</td>
</tr>
</tbody>
</table>
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