5 Conclusion

In this chapter, we summarize the contribution and provide some new problems as extension of this thesis.

5.1 Summary of Contributions

In this thesis, the optimality of employing treating interference as noise (TIN) at receivers together with Gaussian encoding at transmitters was studied for elemental networks; namely a point-to-point channel interfering with a multiple access channel (PIMAC) and $M \times 2$ X-channel. The main focus was on characterizing the optimal regime of TIN from the generalized degrees of freedom (GDoF) point of view. Our approach towards finding the GDoF optimal regime of TIN for the PIMAC has been initiated with the capacity analysis of the linear deterministic model of PIMAC (LD-PIMAC). The capacity optimal regime of TIN for the LD-PIMAC has been completely characterized. Furthermore, the capacity results obtained for the LD-PIMAC have been translated to the Gaussian counterpart. The capacity optimal regime of TIN for the LD-PIMAC corresponded completely to the GDoF optimal regime of TIN for the Gaussian PIMAC.

Interestingly, it turned out that in some regimes TIN is suboptimal although all the undesired links are very weak. The main reason of this fact is that in PIMAC, in contrast to the interference channel (IC), more than one transmitter can serve a single receiver. However by using TIN, no GDoF gain is attained by dedicating multiple transmitters to a receiver. Therefore, as it is also shown in Chapter 3 that the desired links which correspond to the inactive transmitters need to be considered as interference links in the optimality condition of TIN. On the other hand, it is shown that the intuitive condition which restricts the optimality of TIN to the regimes with very weak interference links compared to the desired links is generally not necessary. These insights obtained on the optimality of TIN in the Gaussian PIMAC have been further extended to the $M \times 2$ X-channel. In order to obtain a complete characterization of the GDoF optimal regime of TIN, the following issues were considered individually for PIMAC and $M \times 2$ X-channel.

**Different variants of TIN:** The achievable performance of using TIN at the receiver side has been studied while the transmitters were allowed to use Gaussian encoding with arbitrary power control. Depending on the transmit power, different sum-rates were achievable. Interestingly, it has been shown that in PIMAC and $M \times 2$ X-channel as long as the receivers use TIN, the GDoF optimal power control is a binary power control in which the transmitters either send with full power or are completely inactive. Moreover, it turned out that no additional GDoF gain can
be attained by allowing more than one desired transmitter for a single receiver or multiple desired receivers for a single transmitter, as long as the receivers employ TIN. These facts led us to consider different variants of TIN for each setup. While in all those variants, the decoding strategy was restricted to simple TIN, their difference was mainly originated from different message flows in those variants in a sense that the original channel was reduced to different interference channels (IC) or point-to-point (P2P) channels. Obviously, by increasing the variants of TIN the achievable sum-rate increases and subsequently the regime where TIN is GDoF-optimal.

**Tighter upper bounds:** Different upper bounds on the GDoF performance of each channel have been established. All of the bounds were genie-aided in which additional information were provided to the receivers. Some of them were inspired from the upper bound presented in [ETW08] for the 2-user IC in which the noisy version of the interference signal caused by each transmitter is provided to the desired receiver. In this thesis, a more general type of upper bound has been also established in which a linear combinations of signals was given as side information to a receiver. Additionally, in order to tighten the obtained upper bounds, we proposed a novel lemma on the maximum difference between differential entropies of noisy linear combinations of signals under power constraints. This difference is bounded by a constant independent of power and hence does not appear in the GDoF expression. Due to our new upper bound, the obtained GDoF optimal regime of TIN does not only subsume the known regime in the literature but also extends them significantly.

**Suboptimality of TIN:** It was shown for all variants of TIN in PIMAC and $M \times 2$ X-channel (excluding P2P-TIN in which the $M \times 2$ X-channel is reduced to a P2P channel) that the characterized GDoF optimal regime of TIN is complete. In other words, the suboptimality of TIN has been shown for the cases that the channel operates outside the GDoF optimal regime of TIN. To do this, transmission schemes based on interference alignment combined with common and private signaling which required interference decoding have been introduced. It turned out that TIN can be outperformed by such schemes from the GDoF perspective as long as the channel performs outside of the proposed GDoF optimal regime of TIN.

### 5.2 Future Work

There are still some further interesting open problems on the optimality of TIN and different directions to extend the result of this thesis. For instance, the complete GDoF optimal regime of P2P-TIN in the $M \times 2$ X-channel remained still open. It is interesting to know whether there exists a tighter GDoF upper bound than the bounds introduced in this work for showing the optimality of P2P-TIN. If this is the case, then one can relax the proposed conditions on optimality of P2P-TIN and extend the GDoF optimal regime of TIN for the $M \times 2$ X-channel. On the other hand, to show that the proposed conditions on GDoF optimality of TIN are not only sufficient but also necessary, the suboptimality of TIN needs to be shown from the GDoF point of view.
Another interesting direction to extend the results of the thesis is to know whether TIN performs GDoF optimally in the X-channel if the role of transmitters and receivers will be swapped. Notice that based on the proposed GDoF optimal regime of TIN in [GSJ15], if TIN is GDoF optimal in an $M \times 2$ X-channel, it performs also optimally in the $2 \times M$ X-channel. It is interesting to know whether our extended GDoF optimal regime of TIN for the $M \times 2$ X-channel is also valid for the backward channel. To get some insight knowledge on this problem, one need to study the GDoF optimality of TIN in a $2 \times M$ X-channel. We do believe that this study provides interesting insights on the duality of the GDoF optimality conditions of TIN in the X-channel.

The GDoF result is an intermediate step towards characterizing the capacity of a channel. For instance, the optimality of TIN for the 2-user IC operating in very weak interference regime has been initially shown in [ETW08] with respect to the GDoF. Knowing this approximated result, a capacity optimal regime of TIN has been found in [MK09, AV09, SKC09]. Similarly, we know a capacity optimal regime of TIN for the $2 \times 2$ X-channel from [HCJ12] which is also found through studying the GDoF optimality of TIN. It is interesting to extend the results on the GDoF optimal regime of TIN for the $M \times 2$ X-channel to the capacity optimal regime of TIN. This extension becomes more interesting by focusing on the new characterized GDoF optimal regime of TIN. The question is whether any sub-regime of our extended GDoF optimal regime might appear in the capacity optimal regime of TIN. To answer this question, our proposed lemma (Lemma 21) needs to be refined. This lemma bounds the difference between differential entropies of noisy linear combinations of random variables under some constraints by a constant. Obviously, since this constant does not scale with transmit power, it does not have any impact on the GDoF analysis. However, for the capacity analysis, we need to be able to bound the difference of the entropy terms by zero to obtain a tight upper bound.