

RUHR



# Thermodynamic Property Models for Synthetic Lubricants and their Mixtures with Working Fluids in ORCs

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### Abstract

The potential of (Organic) Rankine Cycles for Exhaust Heat Recovery (EHR) in heavyduty diesel engines for mobile applications was investigated in a joint research project. The EHR process design features ethanol as working fluid. Based on a parameter study, the potential benefit of ethanol-water mixtures was to be investigated. A new Helmholtz mixture model for ethanol-water was developed in this work, which is based on the most current pure fluid equations of state (EOS). The uncertainty of the new model with respect to the speed of sound was significantly improved compared to a preliminary model of Lemmon. This improvement is connected to the improved description of densities at elevated pressures. The uncertainties of other thermodynamic properties are in a comparable range with respect to the model of Lemmon.

A poly-alpha-olefin (PAO)-based synthetic lubricant containing several PAO base oils and an emulsifier was specially designed for the lubrication of the expansion machine used in this EHR process. The thermodynamic properties of the pseudo-pure lubricant and of its mixture with ethanol were required. Due to an insufficient data base, a generalized EOS (genEOS) based on the model of Alexandrov *et al.* was developed for the PAO, which requires less experimental data to develop than a fully optimized EOS. With the new genEOS, the available experimental data can be reproduced reasonably well. A new Helmholtz model describing ethanol-PAO mixtures was subsequently developed, which is based on the new genEOS for the PAO and the current ethanol EOS of Schroeder *et al.*. General application of this method for the estimation of fluid properties was shown.

To validate the use of a genEOS in a mixture model with asymmetric components, the systems CO<sub>2</sub>-PEC5 and CO<sub>2</sub>-PEC7 were investigated. Therefore, new genEOS for the POE base lubricants PEC5 and PEC7 were fitted and mixture models for these lubricants with CO<sub>2</sub> were developed. It was shown that the genEOS reproduce the available experimental data within the AAD range claimed by Alexandrov *et al.*. The new mixture model for CO<sub>2</sub>-PEC5 represents the available phase-equilibrium and density data reasonably. Nevertheless, bubble lines calculated from the new model exhibit an unphysical oscillation that is not caused by the genEOS but by the mixture model itself. The new mixture model for CO<sub>2</sub>-PEC5 was transferred to the CO<sub>2</sub>-PEC7 system by a simple empirical conversion developed in this work, which is based on the critical pure-fluid parameters and the molar masses only. General applicability was shown with the restriction of  $x_{CO_2} > 0.4$ . For smaller CO<sub>2</sub> fractions, the bubble lines calculated from this model show an unphysical negative slope.

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## Table of Symbols

## Latin Symbols

Molar Helmholtz energy
Cubic equation of state parameter
Parameter of the Beattie and Bridgeman equation
BWR parameter
Parameter of NA terms
Reduced Helmholtz energy in PC-SAFT
Parameter of the PC-SAFT equation defined by equation (3.47)
1 <sup>st</sup> virial coefficient
Helmholtz energy
Parameter of the Beattie and Bridgeman equation
BWR parameter
Association site of component <i>i</i>
2 <sup>nd</sup> virial coefficient
Cubic equation of state parameter
Parameter of the Beattie and Bridgeman equation
BWR parameter
Parameter of NA terms
Parameter of the Beattie and Bridgeman equation
BWR parameter
Parameter of the PC-SAFT equation defined by equation (3.48)
Association site of component <i>i</i>
3 <sup>rd</sup> virial coefficient
Heat capacity
Parameter of the Beattie and Bridgeman equation
BWR parameter
Coefficient
Integration constant of the equation for the ideal Helmholtz energy
Parameter of NA terms
BWR parameter

<i>C</i> <sub>1</sub>	Abbreviation defined by equation (3.42)
D	4 <sup>th</sup> virial coefficient
d	Temperature-dependent segment diameter
	Density exponent
	Parameter of NA terms
е	Parameter of NA terms
Ε	5 <sup>th</sup> virial coefficient
f	Parameter of NA terms
	Fugacity
F	6 <sup>th</sup> virial coefficient
	Deviation between data point and value calculated from the equation of state
$F_{ij}$	Weighting factor for generalized departure function
g	Molar Gibbs enthalpy
	Radial distribution function
h	Molar enthalpy
$I_1$	Power series in packing fraction
$I_2$	Power series in packing fraction
k <sub>ij</sub>	Binary adjustable interaction parameter
k <sub>B</sub>	Boltzman's constant
М	Molar mass
т	Number of spherical segments forming one chain
	Coefficient
n	Coefficient
Ν	Number of experimental data points
	Generalized coefficient
N <sub>C</sub>	Number of components in a mixture
N <sub>s</sub>	Total number of segments
Р	Power
р	Pressure
	Density exponent in the exponential function
Ż	Heat flow
R	Universal gas constant
r	Radial distance between two segments

S	Molar entropy
t	Temperature exponent
Т	Temperature
u	Molar internal energy
	Pair potential function
v	Molar volume
w	Speed of sound
	Adjustable parameter
W	Weight assigned to a data point or constraint in the fitting process
$X^{A_i}$	Fraction of molecules of component <i>i</i> not bonded at site <i>A</i>
x	Molar composition vector
$x_i$	Mole fraction of component <i>i</i> in a mixture
Ζ	Arbitrary thermodynamic property
z <sub>i</sub>	Overall mole fraction of component $i$ in a mixture
Ζ	Compressibility factor

## **Greek Symbols**

α	Reduced Helmholtz energy
	Anisotropy parameter of hard body part in BACKONE
	BWR Parameter
	Temperature-dependent function in the SRK equation
β	Binary reducing function parameter
	Gaussian bell-shaped parameter
	Parameter of NA terms
γ	Binary reducing function parameter
	Gaussian bell-shaped parameter
	BWR Parameter
	Coefficient in the exponential function
δ	Reduced density
Δ	Variable of the NA terms
$\Delta^{A_i B_j}$	Association strength
$\Delta_A$	Parameter of the ideal gas isobaric heat capacity equation of Joback and Reid

$\Delta_B$	Parameter of the ideal gas isobaric heat capacity equation of Joback and Reid
$\Delta_C$	Parameter of the ideal gas isobaric heat capacity equation of Joback and Reid
$\Delta_D$	Parameter of the ideal gas isobaric heat capacity equation of Joback and Reid
$\epsilon^{A_i B_j}$	Association energy between two sites $A$ and $B$ of two molecules of components $i$ and $j$
$\zeta_n$	Abbreviation defined by equation (3.33) with $n \in \{0; 1; 2; 3\}$
η	Anisotropy parameter of attractive part in BACKONE models
	Packing fraction of segments
	Gaussian bell-shaped parameter
θ	Parameter in the equation for the isobaric heat capacity of the ideal gas
	Parameter of the NA terms
θ	Parameter in the equation for the isochoric heat capacity of the ideal gas
$\kappa^{A_i B_j}$	Effective volume of an association interaction between two sites $A$ and $B$ of two molecules of components $i$ and $j$
λ	Reduced well width of the square potential
σ	Temperature-independent segment diameter
ρ	Molar density
$\hat{ ho}$	Total number density of molecules
τ	Inverse reduced temperature
arphi	Fugacity coefficient
χ	Polar factor
ω	Acentric factor

## Subscripts

0	Ideal gas state
b	Boiling point
с	Critical parameter
DATA	Experimental data point
EOS	Calculated from the equation of state
ЕТО	Ethanol
Exp	Exponential
GBS	Gaussian bell-shaped
i	Control variable
j	Control variable

k	Control variable
NA	Non-analytic
р	At constant pressure
PE	Planck-Einstein
Pol	Polynomial
R	Reducing parameter
r	Reduced variable
	Reducing parameter
Т	At constant temperature
v	Vapor state
v	At constant volume
δ	Partial derivative with respect to $\delta$
ρ	At constant density
τ	Partial derivative with respect to $ au$

## Superscripts

(0)	Simple fluid
(1)	Real fluid
а	Attractive force contribution
assoc	Association contribution
chain	Contribution of chain formation
disp	Dispersion contribution
Е	Excess property
h	Hard-body contribution
hc	Hard chain contribution
hs	Hard-sphere contribution
Ι	1 <sup>st</sup> integration
II	2 <sup>nd</sup> integration
0	Ideal gas contribution
r	Residual contribution
rep	Repulsive force contribution
seg	Contribution of monomeric segments

Saturated	liquid
	Saturated

" Saturated vapor

## Abbreviations

AAD	Average absolute relative deviation
AB	Alkyl benzene
API	American Petroleum Institute
BWR	Benedict-Webb-Rubin equation
CCSA	Classical corresponding states approach
CO2	Carbon dioxide
eCSA	Extended corresponding states approach
ЕОМ	Evolutionary optimization method
EOS	Equation of state
GBS	Gaussian bell-shaped
LLE	Liquid-liquid equilibrium
mBWR	Modified Benedict-Webb-Rubin equation
MCSA	Molecular corresponding states approach
ORC	Organic Rankine cycle
PAG	Polyalkylene glycol
PAO	Poly-alpha-olefin
PC-SAFT	Perturbed-chain statistical associating fluid theory
PE	Pentaerythritol esters
PFPE	Perfluorpolyether
POE	Polyol ester
PVE	Polyvinylether
RUB	Ruhr-Universität Bochum
SAFT	Statistical associating fluid theory
VLE	Vapor-liquid equilibrium
VLLE	Vapor-liquid-liquid equilibrium

#### 1 Introduction

With increasing awareness of the global change of climatic conditions due to increasing concentrations of atmospheric greenhouse gases, the efforts of minimizing relevant emissions have increased in the last decades. It is common understanding that the most relevant anthropogenic contribution to the increase of atmospheric greenhouse gases is related to carbon dioxide (CO<sub>2</sub>) emissions. About 18% of the anthropogenic CO<sub>2</sub> emissions of Germany are generated by the transport sector, which is a major contributor next to the energy sector (38%) and industry (21%) [1]. The emissions of the transport sector are dominated by passenger cars, which account for 61%, followed by utility vehicles which account for 35% [2].

The significant power requirements of utility vehicles feature an immense requirement of expansive fuel. Consequently, the reduction of fuel consumption serves two interests: the reduction of CO<sub>2</sub> emissions and the reduction of cost. In the past, the engines of such vehicles have been optimized thoroughly. Therefore, the major potential of fuel savings and the respective reduction of emissions lies in the surrounding periphery. Lightweight bodies to reduce the weight and the driving resistance or the reduction of the energy demand of auxiliary units are possibilities to further reduce the fuel consumption. However, almost two thirds of the fuel-bound energy are rejected by the engine cooler or are purged with the exhaust gas on a rather high temperature level.

In a joint research project, the potential of (Organic) Rankine Cycles (ORC) for Exhaust Heat Recovery (EHR) in heavy-duty diesel engines for mobile applications was investigated. The industrial partners DAIMLER AG (on-road application) and MTU (off-road applications) contributed as well as the Chair of Fluidics of the TU Dortmund University. The knowledge of thermophysical properties of the involved fluids or fluid mixtures is essential for a safe design and optimization of the ORC process. The aim of this work is to provide the required thermodynamic properties of the relevant working fluids

The proposed process design includes ethanol as working fluid with the option of using ethanol-water mixtures for potentially following designs. The possible benefit of these mixtures should be investigated based on a parameter study. The required thermody-namic fluid properties are provided by equations of state. For ethanol, an equation of state was published by Schroeder *et al.* [3]. For the ethanol-water system, a preliminary

equation is available in REFPROP [4]<sup>1</sup>, which is based on an outdated equation of state for ethanol. Therefore, a new mixture model for the ethanol-water system was developed in this work. Results are presented in chapter 4.

It was decided by all project partners to implement a screw expander with direct lubricant injection as expansion device in the ORC. Consequently, the working fluid ethanol will contain or entrain some of the lubricant. Therefore, a thermodynamic property model describing the lubricant and an additional model describing its mixture with ethanol are required. The lubricant was specially designed for this application and is commercially available. No information was given on the composition of this poly-alpha-olefin (PAO)-based fluid, which is treated as a pure fluid although it contains several compounds including an emulsifier, nor are any experimental data available in the literature. Today, it is state-of-the-art to develop a substance-specific Helmholtz equation of state based on a broad set of experimental data. However, for the adjustment of the new equation of state, only a few data points were provided by the producer. Consequently, no fluid-specific Helmholtz equation of state could be developed, but a generalized approach was employed. Results are shown in section 5.1. Subsequently, a mixture model for the binary system ethanol-PAO was adjusted to some experimental data points, which were measured by a project partner in the course of this project. Results are presented in section 5.2.

The description of such a complex mixture with a Helmholtz mixture model based on a generalized equation of state for the pseudo-pure lubricant is an innovative approach. Therefore, additional systems werechosen to prove the validity of this method. Due to the available experimental data in the literature, the systems CO<sub>2</sub>-PEC5 and CO<sub>2</sub>-PEC7 were selected. These pentaerythritol esters (PE) of different molecular length (C5 and C7) belong to the group of polyol esters (POE). New generalized equations of state for the base lubricants PEC5 and PEC7 were developed in this work, and a mixture model for CO<sub>2</sub>-PEC5 was adjusted. Subsequently, this mixture model was scaled to the system CO<sub>2</sub>-PEC7 without any adjustment. Results are shown in sections 6.1 to 6.4. The quality of the proposed pure fluid equations of state is evaluated with the help of fluidspecific Helmholtz equations of state. These equations, which are not part of this thesis,

<sup>&</sup>lt;sup>1</sup> The preliminary mixture model for the ethanol-water system developed by Lemmon was never published in the literature but is available in REFPROP [4].

were collaboratively developed with the National Institute of Standards and Technologies (NIST) and are based on a significantly broader data set. For this purpose, new measurements were conducted at NIST to support the development of the EOS. These new data were not considered in the adjustment of this work in order to validate the predictive capabilities of the new generalized equations of state.

Finally, the results of this work are summarized in chapter 7 and recommendations for future investigations are given.

## 2 Exhaust Heat Recovery for an Automotive Application

In this chapter, general aspects of exhaust heat recovery (EHR) processes are discussed. Several technical applications and implementations are briefly introduced, and the basic thermodynamics behind organic Rankine cycles are presented. Requirements and restrictions concerning the choice of working fluid and lubricant are also discussed.

## 2.1 Fundamentals of Exhaust Heat Recovery

Aiming at an optimal process regarding the consumption of energy, two aspects need to be considered. On the one hand, the energy demand of the core process itself needs to be reduced. On the other hand, the utilization of the provided primary energy needs to be optimized, which means that exergetic losses in the entire process are to be minimized [5].

For an automotive application, the core process is represented by the engine. Over the last years, the engine efficiency has been thoroughly optimized. Ringler *et al.* [6] claim that it is reaching its technical limits and further investigations do not promise significant further energy savings. Consequently, the utilization of the provided primary energy regarding the surrounding process is gaining more attention.

Figure 2.1 shows a characteristic Sankey diagram for a typical modern heavy-duty diesel engine (OM457) at nominal load [7].



Figure 2.1: Sankey diagram of a modern heavy-duty diesel engine (OM457) at nominal load [7].

Only approximately 40% of the fuel-bound energy is converted into motive power. 23% is discharged with the cooling water, and another 7% is lost by radiation. These heat flows cannot be recycled by technical means. However, almost a third of the provided primary energy is lost with the exhaust gas, which has a rather high temperature level. Hence, there is an enormous potential to reduce the energy loss of this process. That is what exhaust heat recovery aims at.

In principle, there are several ways to use the exergy of the flue gas based on different physical effects. For example, turbomachines could be used to exploit the pressure gradient of the exhaust gas. However, the main potential lies in the high temperature and would remain largely unused. Nevertheless, a turbocharger could be implemented in combination with a process which profits from the high exhaust gas temperature.

Chemical reactions can be used to benefit from the available heat but are unfavorable. The heat recovery potential of thermo-acoustic and Stirling applications does not compensate for the high complexity of the required processes. A thermo-electric device or Joule process might profit from the provided heat, but they cannot reach the potential of a Rankine cycle.

So far, (organic) Rankine Cycles working with water or an organic fluid are the preferred technology to use the exhaust heat [8]. These cycles are explained in more detail in the following section.

### 2.2 Organic Rankine Cycles and Working Fluids

An Organic Rankine Cycle (ORC) is basically the same as a common steam process used in large scale power plants. The simplest process is based on four main components, as illustrated in Figure 2.2. First, the liquid working fluid (1) is compressed in a pump (2). Subsequently, it passes the first heat exchanger in which it is isobarically preheated (3) and then evaporated (4), changing its phase from liquid to gas. Depending on the working fluid and process design, it is beneficial to additionally superheat the fluid (5). The required heat input  $\dot{Q}_{\rm EHR}$  is provided by the exhaust gas. In the following, the fluid pressure is reduced again driving the expansion machine (6). Finally, the working fluid is condensed (7) and subcooled (1) by heat removal in the second heat exchanger.



Figure 2.2: Schema of an exemplary basic ORC process.

In Figure 2.3, the exemplary process presented in Figure 2.2 is shown in a characteristic *T*,*s*-diagram for water as the working fluid.



Figure 2.3: *T,s*-diagram for water with the state points of an exemplary Rankine Cycle.

As already mentioned, Rankine processes are state of the art for large scale power generation in steam power plants, nuclear power plants, and block-type thermal power stations. Despite many years of experience and despite of the successful transfer to smaller power outputs in block-type plants, there are a lot of challenges designing such a process for a mobile application. All components need to be optimized regarding volume and mass restrictions without degrading their efficiency significantly. Friction and leakage have a much bigger impact when the size of a component is reduced drastically. Additionally, in contrast to stationary power plants, all components have to be designed to work optimally in transient states. For a further discussion of the general requirements of a mobile application see Jung [7].

In principle, there are several types of expanders available for usage in low power processes. There are micro-turbines, screw expanders, scroll expanders, and reciprocal piston expanders. Regarding the automotive ORC process, the volume-type expanders, screw and scroll, are more suitable due to the characteristically lower flow rates, higher pressure ratios, and much lower rotational speed compared with the other two expander types. [9]

While in large stationary Rankine processes water is commonly used as working fluid, there are many different fluids utilized in smaller applications on lower temperature level. Since these working fluids are usually organic compounds, the corresponding processes are called Organic Rankine Cycles (ORCs). Actually, the choice of the working fluid is part of the design and optimization process and depends on the expansion machine, heat input, and others. There are many publications dealing with the choice of an adequate working fluid for different applications, for example [10–12]. Working fluids can be categorized according to the slope of their dew line in a *T,s*-diagram, see Table 2.1.

$\left(\frac{\partial T}{\partial s}\right)_{\text{dew line}}$	category
> 0	wet
= 0	isentropic
< 0	dry

 Table 2.1: Categories of working fluids.

When the slope of the dew line is known, it is also known how the fluid expands in an isentropic ideal case. Starting from the dew line, the isentropic expansion of a wet fluid ends in the two-phase region, whereas it ends in the homogeneous vapor region for a dry fluid. An isentropic fluid would expand along the dew line. The knowledge of this characteristic feature of the working fluid is crucial for the choice and design of the expansion machine and for the modification of the ORC. In common steam processes for example, water is superheated after the evaporation to limit the amount of liquid

droplets formed during the expansion (see Figure 2.3). Otherwise, there would be damage in the turbine from impingement corrosion. For a dry fluid, superheating would not be necessary, and the heat exchanger could be designed in a simpler way. However, a screw expansion machine would profit from the formation of droplets, since they reduce the gap loss. Depending on the expander and the working fluid, even a power flash cycle might be favorable, where the fluid is only heated to the bubble line and evaporates during the pressure drop in the expansion machine. For a detailed discussion of potential ORC variants see Wiedemann [13].

As an alternative to water, organic working fluids usually have a larger vapor pressure close to ambient temperature and consequently higher vapor densities at the outlet of the expansion machine, which allow for lower volume flows in the process. In principle, there are numerous candidates for each ORC application, that fulfill the basic needs like a suitable vapor pressure in the relevant temperature range. But additionally, there are restrictions for the choice of working fluids besides availability and cost, which concern [5]

- flammability
- toxicity
- environmental compatibility
- chemical and thermal stability
- material compatibility
- heat transfer properties.

An earlier investigation by the Chair of Thermodynamics and the Chair of Combustion Engines of Ruhr University Bochum (RUB) [8] dealt with the choice of an optimal working fluid for a mobile ORC application. The study focused on the in advance selected fluids water, ethanol, methanol, hexamethyldisiloxan (MM), R245fa, and iso-pentane. Out of these six fluids, methanol, ethanol, and water were found to be most promising regarding the power output of the process. Water is the classical working fluid in commercial Rankine applications since it has the great advantage over most working fluids that it is neither flammable nor toxic. However, it is highly corrosive and has a rather high freezing point of  $t = 0^{\circ}C$  at atmospheric pressure. The latter disqualifies pure water the working fluid in a mobile application. In comparison, ethanol has a very low freezing point of  $t = -114^{\circ}C$  at atmospheric pressure while showing thermal stability up to temperatures of  $t = 400^{\circ}C$ . However, ethanol is easily flammable and has a low ignition limit resulting in the need of absolute tightness of the system. Methanol shows qualities comparable to ethanol but is discarded due to its high toxicity. As a result, Span *et al.* [8] recommend ethanol, or ethanol-water mixtures as working fluid.

As already pointed out, a screw or scroll expander is the preferred expansion device for a mobile ORC. In order to determine which of these two is the best option for this application, the Chair of Fluidics of the TU Dortmund investigated the energetic and application-specific aspects of these expander types, based on computer simulations. According to earlier findings [8], ethanol was chosen as the working fluid. It was shown that due to comparatively small inlet openings, scroll expanders cause high throttling losses that lead to power losses at high rotational speeds. Given that screw expanders can be operated at higher rotational speeds, and higher pressure ratios, this type of expansion machine was chosen for further development. [14,15]

Subsequently, it was important to decide if the screw expander should be synchronized or not. Usually, there is no liquid found in the working chambers of unsynchronized screw expanders, which leads to an increased gap mass flow causing mass and energy losses. To minimize this effect, synchronized screw expanders are operated with significantly increased circumferential speeds at the crown circle. Apart from the energetic advantages of a smaller gap mass flow, unsynchronized screw expanders profit from a lighter and smaller constructional size, and a simpler assembly. Whether an unsynchronized design can be chosen depends on the lubricity of the fluid in the working chambers. There are several options to deal with this. Given that the lubricity is sufficient, liquid working fluid could be injected. Alternatively, a lubricant can be chosen and injected into the machine. Theoretically, it can be sufficient to have condensing working fluid in the working chambers. But since there is no secure way to predict how fast the working fluid nucleates and finally condensates, direct liquid injection is preferred. [15]

Within the framework of this project, it was decided to design an unsynchronized screw expander that is flooded with lubricant. Subsequently, the best machine geometry was investigated with regard to the maximum power output of the overall process [14].

#### 2.3 Lubricants in Refrigeration and Organic Rankine Cycles

With the decision to rely on direct oil injection into the screw expander, it is clear that there is lubricant in the entire ORC. Even if an oil separator is installed, the separation rates are limited. This effect is known from screw compressors in compressed-air or refrigeration applications. Installing an oil separator is still favored since it reduces the amount of oil that is heated and cooled cyclically with the working fluid. Hence, the provided exhaust heat would not be wasted on the lubricant that does not contribute to the expansion energy. Furthermore, the oil separator makes the lubricant directly available for lubrication of bearings and for injection into the working chambers. [15] Depending on the miscibility of the present lubricant and working fluid over the relevant temperature range, the oil that escaped from the separator will return to the screw compressor or expander. However, if phase separation occurs, the lubricant can accumulate on the inside of heat exchanger tubes, which results in reduced heat transfer and reduced overall performance. This can even lead to compressor/expander oil starvation and potential breakdown. [16] Considering the accumulation of oil in the evaporator, thermal stability becomes even more an issue. Compared to the wellknown refrigeration cycles, the maximum temperature of the working fluid is considerably higher in EHR processes. The solubility of the working fluid in the lubricant influences the viscosity and lubricity of the fluid mixture in the expansion machine. [17] Consequently, knowledge of the phase behavior of the complex working fluid-lubricant mixture is key for an effective and safe process design.

For the choice of a suitable lubricant, there are the same requirements concerning toxicity, flammability, environmental compatibility, and others, just as discussed for working fluids in section 2.2. The American Petroleum Institute (API) has categorized base oils into five categories (API 1509, Appendix E) from which the first three groups are exclusively refined from petroleum crude oil. Group IV base oils are fully synthetic Poly-alpha-olefins. All other base oils not included in Groups I through IV belong to Group V. So far, mineral-oil-based lubricating oils are widely used even though they are not readily biodegradable and are frequently toxic. However, environmental aspects gain more and more importance which leads to advances in green chemical technology. [18] Different synthetic lubricants have been developed, which are shortly presented here. They differ in their chemical structure, which allows a categorization in families:



- b) Polyol ester (POE)
- c) Alkyl benzene (AB)
- d) Poly-alpha-olefin (PAO)
- e) Polyvinylether (PVE).



CIEVE

Figure 2.4: Chemical structure of the common synthetic lubricant families [16,19].

Figure 2.4 shows the different chemical structures of the synthetic lubricants used so far in ORCs. PAG lubricants are compatible with most elastomers, yet they are very hygroscopic. POEs have environmental benefits since they can be synthesized from renewable sources and are biodegradable. They are less hygroscopic than PAG lubricants, but if they adsorbed a considerable amount of moisture, the ester can break down at high temperatures. POEs are not compatible with common elastomers. Since

they are polar fluids and have high densities, they tend to dissolve many impurities. AB lubricants are synthetic aromatic hydrocarbon oils with seal compatibility and electrical insulting properties. PAOs are unsaturated hydrocarbon synthetic oils showing superior performance and stability. They are non-hygroscopic and show a high fluidity at low temperatures. Recently, PVE lubricants have been developed that are less dense and less polar than ester oils. Hence, they show a superior performance over ester oils with respect to blockage. Additionally, PVEs are less effected by moisture than POEs. [16]

A commercially available lubricant usually contains several base oils of the same or different API groups plus special additives to adapt its properties. Consequently, commercial lubricants are by no means pure fluids but mixtures of numerous compounds. In the framework of this joint project, the German company Fuchs Schmierstoffe GmbH was assigned to find the most suitable lubricant for an ORC with a screw expansion machine and ethanol as working fluid. They investigated a PAO, two different PAG formulations, and a Perfluorpolyether (PFPE). The latter describes a fully synthetic lubricant consisting exclusively of the elements fluorine, carbon, and oxygen. It was decided by the project partners to use a specially designed PAO, which builds a dispersive emulsion with ethanol. This oil guaranteed good lubricity and viscosity in the mixture with ethanol, which is crucial for an effective and safe operation of the proposed process.

#### 2.4 Need for Thermophysical Properties

Prestudies showed that for an ORC in a mobile application, ethanol and ethanol-water mixtures are promising working fluids [8] and that a screw expander is the most promising expansion device [14]. Therefore, an ORC with ethanol as working fluid and a screw expander was assessed as the framework of this joint research project. Furthermore, it was agreed to use a flooded screw expander, assuring sufficient lubrication at all times.

The knowledge of thermophysical properties is crucial for a successful process design. Based on the agreed configuration, fluid properties of ethanol and ethanol-PAO mixtures are required. Furthermore, it was agreed to investigate ethanol-water mixtures as working fluids. Therefore, a mixture model for these components is required as well. For ethanol, an empirical multiparameter equation of state (EOS) explicit in the Helmholtz energy is available in the literature [3] as is for water [20]. For mixtures of these two components, there is a preliminary model by Lemmon available in REFPROP [4], which is explicit in the Helmholtz energy. However, this model is based on the outdated pure fluid equation for ethanol by Dillon and Penoncello [21], which has been replaced by the new equation of Schroeder *et al.* [3], which was fitted to an updated, broader set of experimental data. Therefore, a new mixture model based on the most accurate pure fluid equations is required. Table 2.2 summarizes the models needed and shows which of them are developed in the course of this work. The models highlighted in grey are needed for the description of the desired EHR process. All mixture models are developed in this work.

**Table 2.2:** Matrix of the source of the thermodynamic property models for the discussed fluids and fluid mixtures. All mixture models are developed in this work. The models required for the description of the EHR process are highlighted in grey.

Mixture (fluid 1 – fluid 2)	fluid 1	fluid 2		
ethanol-water	Schroeder et al. [3]	Wagner and Pruß [20]		
ethanol-PAO	Schroeder et al. [3]	this work		
CO <sub>2</sub> -PEC5	Span and Wagner [22]	this work		
CO <sub>2</sub> -PEC7	Span and Wagner [22]	this work		

As already pointed out, the PAO employed here was specifically designed for this process. Consequently, there is no model available predicting its thermodynamic properties. The company Fuchs Schmierstoffe GmbH, which developed this product, insisted on keeping the oil's composition secret. However, they notified that it is a mixture of several different PAOs and further additives, resulting in an average molar mass of M = 1773 g/mol. Additionally, they provided some density, vapor pressure, and isobaric heat capacity data for the oil, which could be used to adjust a model. Despite the different compounds, the incorporated oil is treated as a pure fluid, and for simplicity it is referred to as PAO. Due to the very limited data base, no fluid-specific empirical Helmholtz energy model was developed but a generalized approach was applied. Further information on the corresponding theory is provided in chapter 3, and the results are shown in section 5.1.

For mixtures of ethanol and PAO, there is neither a model nor experimental data available in the literature. Therefore, the Chair of Process Technology of the RUB measured phase equilibria and some densities for ethanol-PAO mixtures, which were used for the development of a new mixture model. This mixture model is based on the ethanol equation by Schroeder *et al.* [3] and the newly developed generalized equation for the PAO. Developing a mixture property model based on the Helmholtz energy for such different components, one even being described just by a generalized equation of state, is a challenging new path. Therefore, an additional model describing carbon dioxide (CO<sub>2</sub>)-PEC5 mixtures, based on a newly developed generalized equation of state for PEC5, has been developed in this work. Additionally, a generalization of this mixture model to the similar system CO<sub>2</sub>-PEC7 was investigated. Hence, an additional generalized equation of state describing PEC7 was developed. In contrast to PAO, PEC5 and PEC7 actually are pure fluids that might be mixed for commercial use. There are some data available in the literature for these lubricants that were used for the adjustment in this work. However, there has been a major investigation of the POEs PEC5, PEC7, and PEC9 at the National Institute for Standards and Technologies (NIST), including new measurements of the pure component properties, which all are to be published. Additionally, fluid-specific equations of state for these fluids [23] have been developed based on this new data, which are to be published as well. Despite the collaboration on the development of the new Helmholtz equations, the data set used for the adjustment of the generalized equations of state presented in this work was intentionally restricted to the data published in the literature. In this manner, the pure fluid equations of Lemmon and Eckermann [23] cannot only be used for a general validation of the generalized pure fluid equations of state developed in this work but rather for an evaluation of the prediction of caloric properties which the models have not been adjusted to.

CO<sub>2</sub> is one of the most promising alternative refrigerants. Due to rising concerns about the environmental impact of working fluids and lubricants, traditional chlorinated refrigerants are more and more replaced by hydrofluorocarbons (HFCs) and CO<sub>2</sub>. [17] There are several publications dealing with density and solubility properties of CO<sub>2</sub> with POEs, e.g. [24–26].

Summing up, there are two main tasks to be addressed in the course of this work. A new mixture model for the experimentally well investigated ethanol-water system is required. Accurate pure fluid equations for these components have already been published. The theory is discussed in section 3.6, and results are shown in chapter 4.

Additionally, new property models for a new lubricant and its mixtures with the working fluid ethanol are required. There is very limited data available, so that a generalized equation of state is used for the new pure fluid equation of state. This is explained in chapter 3 and results are presented in chapter 5. Since it is an innovative approach to model such an asymmetric mixture with multiparameter Helmholtz equations of state, an additional refrigerant-lubricant system, CO<sub>2</sub>-PEC5, has been accordingly developed in order to verify this approach. Furthermore, the developed model for CO<sub>2</sub>-PEC5 is transferred to mixtures of CO<sub>2</sub>-PEC7. The results of this development are described in chapter 6.

### 3 Equations of State

The need for knowledge of thermophysical properties for design and optimization of technical processes is universal. If available, equations of state have replaced property tables as the source of information. Nevertheless, fitting a highly accurate multiparameter equation of state to a specific fluid or fluid mixture requires a considerable amount of reliable experimental data. Consequently, accurate empirical multiparameter property models are available in the literature for rather well measured fluids only. Water and CO<sub>2</sub> are very well and highly accurately measured fluids for which reference equations of state have been developed [20,22]. For ethanol, a multiparameter equation of state is available as well [3]. But since the experimental data set used for the adjustment of this equation of state is not as comprehensive and accurate as the data sets for CO<sub>2</sub> and water, the uncertainty of the equation is higher than the uncertainty of the reference equations of state. For the lubricants, there is little to no data available. This is a widespread problem throughout research and industry. For this reason, generalized equations of state, which are based on a physical background in a varying degree, are in wide use. As a matter of course, these models aren't as accurate as the multiparameter equations, but they allow for the prediction of properties depending on only a few fluid-specific parameters. Therefore, the usage of such models is somewhat independent of the experimental situation.

In the course of this chapter, the historical development of pure fluid and mixture property models is presented. Subsequently, the models that are used in this work are presented in more detail.

### 3.1 Historical Background

#### 3.1.1 Empirical and Semi-Empirical Pure Fluid Equations

Generally, in equations of state, two independent state variables are used to describe another state variable. Most commonly, the pressure p or the Helmholtz energy a are formulated as a function of temperature T and molar volume v (or molar density  $\rho$ ).

In 1834, Clapeyron [27] formulated the first equation of state describing the ideal gas behavior with

$$pv = RT, (3.1)$$

*R* being the universal gas constant. The underlying assumptions of this model are molecules that are point particles without any extension, which exclusively interact with perfectly elastic collisions. For gases in the zero-density limit, particle volumes and interactions become negligible and the model is exact. At low pressures, this model is still quite accurate.

A few years later in 1873, van der Waals [28] proposed the first cubic equation that could describe not only the gaseous region but the liquid and supercritical regions as well as vapor-liquid equilibria. Van der Waals improved the ideal gas law by introducing two parameters, the attraction parameter a and the covolume of the molecules b. Thereby, he acknowledged that real molecules have an extension (b) and interact in a more complex way (a). Consequently, there is a contribution of attractive (a) as well as repulsive forces (rep):

$$p = \frac{RT}{v} \cdot \frac{v}{\underbrace{v-b}_{\text{rep}}} - \frac{a}{\underbrace{v^2}_{p^a}}.$$
(3.2)

The two model parameters can be determined by evaluating the critical conditions  $\left(\frac{\partial p}{\partial v}\right)_{T_{\rm c}} = 0$  and  $\left(\frac{\partial^2 p}{\partial v^2}\right)_{T_{\rm c}} = 0$ , which leads to

$$a = \frac{27R^2T_c^2}{64p_c}$$
, and (3.3)

$$b = \frac{RT_{\rm c}}{8p_{\rm c}}.\tag{3.4}$$

The results show that based on two critical parameters, the complete phase behavior is determined. Applying the reduced variables  $p_r = p/p_c$ ,  $v_r = v/v_c$ , and  $T_r = T/T_c$  leads to the following reduced form of the van der Waals equation:

$$\left(p_{\rm r} + \frac{3}{v_{\rm r}^2}\right)(3v_{\rm r} - 1) = 8T_{\rm r}.$$
 (3.5)

This equation yields the same results for every substance evaluated for a given set of reduced variables. Therefore, the van der Waals equation of state is the first corresponding states approach with two fluid-specific parameters. In the following, this is described by the classical corresponding states approach (CCSA) with two parameters. In 1913, van der Waals discussed limitations of this approach [29]. He found the resulting relation  $b = 1/3 v_c$  not suitable for many fluids. Experimental data proved this value to be too high. Van der Waals was aware that the two-parameter corresponding states approach reached its limitations.

In the 1930/1940s, Pitzer [30] and Guggenheim [31] discussed the physical assumptions that are necessary to apply this corresponding state principle correctly. Pitzer derived the theory of corresponding states for so called perfect liquids.

Redlich and Kwong [32] published another cubic formulation with a new description of the attractive pressure in 1948:

$$p = \frac{RT}{v-b} - \frac{a}{\sqrt{T}v(v+b)}.$$
(3.6)

The authors proposed a temperature-independent limiting value of  $0.26 \cdot v_c$  for the volume of all gases at high pressures. Consequently, the equation has been constructed to satisfy the condition  $b = 0.26V_c$ . This model is based on the two-parameter CCSA as is the van der Waals equation.

In 1955, Pitzer *et al.* [33] proposed a third parameter that should help overcoming the shortcomings of the two-parameter CCSA. They introduced the so-called acentric factor  $\omega$ . This new parameter accounts for intermolecular forces resulting from interactions between acentric parts of complex molecules with

$$\omega = -\log p_{\rm r} - 1 \text{ and } p_{\rm r} = \frac{p_{\rm v}(0.7 \cdot T_{\rm c})}{p_{\rm c}},$$
(3.7)

where  $p_v$  is the vapor pressure at a corresponding temperature. The theory postulates that any substances with the same acentric factor should behave equally according to the principle of corresponding states. However, the acentric factor is simply a measure for the slope of the vapor pressure curve with respect to reduced variables improving the description of phase equilibria.

In 1972, Soave [34] adapted the equation by Redlich and Kwong and incorporated the acentric factor in the description of the attractive pressure. This equation is discussed in detail in section 3.3. Another variation of the van der Waals equation was presented by Peng and Robinson in 1976 [35], who also incorporated the acentric factor in their new formulation of the attractive pressure. All cubic equations of state can be classified as

generalized equations since they only need the fluid-specific critical properties and potentially the acentric factor.

A completely different approach was used by Kamerlingh Onnes [36]. In 1902, he published an entirely new type of equation of state after having tried to empirically fit the two parameters of the van der Waals equation of state, *a* and *b*, to improve the representation of real fluids. He proposed a series formulation in molar volume with temperature-dependent coefficients, which he called virial coefficients. The first virial equation of state reads

$$pv = A(T) + \frac{B(T)}{v} + \frac{C(T)}{v^2} + \frac{D(T)}{v^4} + \frac{E(T)}{v^6} + \frac{F(T)}{v^8},$$
(3.8)

where each virial coefficient *A*, *B*, *C*, *D*, *E*, *F* is a polynomial function of temperature. The coefficients of these equations were fitted to experimental pvT data of hydrogen, oxygen, nitrogen, and carbon dioxide. This is the first multiparameter equation of state developed and it is based on the corresponding states principle as well. The critical temperature and pressure are used as corresponding state parameters, so this is a two-parameter CCSA. In 1928, Beattie and Bridgeman [37] proposed a different equation of state, that can be seen as an improvement of the van der Waals equation with

$$p = \frac{RT(1 - c/VT^3)}{V^2} [V + B_0(1 - b/V)] - \frac{A_0}{V^2} (1 - a/V).$$
(3.9)

where  $A_0$ ,  $B_0$ , a, b, and c are adjustable parameters. Beattie and Bridgeman [37] aimed at a better physical description of real gases. Therefore, they analyzed limiting values as well as general relations and dependencies of temperature and density. Finally, they adjusted the equation parameters individually to several substances. This fact makes this equation the first not generalized empirical equation of state. Although this model is an improvement of the van der Waals equation based on physical considerations, it is no longer cubic with respect to the volume. Moreover, the authors converted the equation into the virial form following Kamerlingh Onnes [36] path.

In 1940, Benedict *et al.* [38] published the so called Benedict-Webb-Rubin equation (BWR) based on the work of Beattie and Bridgeman [37]. Among other things, they introduced an exponential correction term depending on the density into the virial expansion, which significantly improved the description of properties at high densities and low
temperatures. They fitted the empirical parameters to experimental data for methane, ethane, propane, and n-butane specifically for each substance. The BWR equation is given by

$$p = RT\rho + \left(B_0RT - A_0 - \frac{C_0}{T^2}\right)\rho^2 + (bRT - a)\rho^3 + a\alpha\rho^6 + \frac{c\rho^3(1 + \gamma\rho^2)\exp(-\gamma\rho^2)}{T^2},$$
(3.10)

with the adjustable parameters  $A_0$ ,  $B_0$ ,  $C_0$ , a, b, c,  $\alpha$ , and  $\gamma$ .

The BWR equation is more commonly expressed in the compressibility factor

$$\frac{p(T,\rho)}{\rho RT} = Z(T,\rho) = 1 + \sum_{i=1}^{6} n_i T^{t_i} \rho^{d_i} + \sum_{i=7}^{8} n_i T^{t_i} \rho^{d_i} \exp\left(-\left(\frac{\rho}{\rho_r}\right)^2\right), \quad (3.11)$$

with the transformed coefficients  $n_i$ , and temperature and density exponents  $t_i$  and  $d_i$ , respectively.

In 1969, Carnahan and Starling [39] proposed an equation for non-attracting rigid spheres based on a virial series, that is still in use today, and has been introduced into other models that are presented later in this chapter. In 1973, Starling [40] published a variation of the BWR equation with the same density dependence. He fitted the parameters of the equation specifically for methane but proposed an additional generalized set of parameters that had been simultaneously fitted to multiple hydrocarbons. For this generalization, he also incorporated the three-parameter CCSA based on the acentric factor, the critical temperature  $T_c$ , and the critical density  $\rho_c$ .

In 1970, both Bender [41] and Wagner [42] developed algorithms to include vapor-liquid equilibrium (VLE) and *pvT* data simultaneously in the fitting process. Using this new multiproperty method, Bender [41] published virial-based equations for argon, nitrogen, oxygen, carbon dioxide, and methane with a new accuracy level concerning the description of saturated and caloric properties.

Jacobsen and Stewart [43] developed a virial-based equation of state for nitrogen with the help of a stepwise multiple regression analysis. As Bender, they included *pvT* data simultaneously with VLE data. They were able to reproduce the available experimental data within the uncertainty of the measurements. This type of equation (mBWR) has been widely used for other fluids.

In 1975, Lee and Kesler [44] developed a three-parameter CCSA model using two reference fluids. They applied the idea of Pitzer *et al.* [33] from 1955 of using a linear function in the acentric factor  $\omega$  to calculate the compressibility factor of the desired fluid with

$$Z = Z^{(0)} + \omega Z^{(1)}, \tag{3.12}$$

where  $Z^{(0)}$  is the compressibility factor of a simple fluid and  $\omega Z^{(1)}$  is the deviation of the compressibility factor of the real fluid from  $Z^{(0)}$ . Lee and Kesler reformulated this expression for better convenience to the following form

$$Z = Z^{(0)} + \frac{\omega}{\omega^{(r)}} \left( Z^{(r)} - Z^{(0)} \right), \tag{3.13}$$

with  $Z^{(1)} = (Z^{(r)} - Z^{(0)})/\omega^{(r)}$ , and  $Z^{(r)}$  being the compressibility factor of a reference fluid. This kind of generalization is referred to as extended corresponding states approach (eCSA) in the following. The simple and reference fluids were modeled with the BWR equation [38].

Platzer and Maurer [45] developed a four-parameter eCSA model especially designed for polar fluids. They additionally applied the polar factor  $\chi$ , which was introduced by Halm and Stiel [46] with

$$\chi = \ln \left(\frac{p_{v}}{p_{c}}\right)_{\frac{T}{T_{c}}=0.6} + 1.7\omega + 1.552.$$
(3.14)

This builds the so called extended-Lee-Kesler-Bender method (eLKB). The interpolation is based on the acentric and polar factor using three reference fluids. The extension of the method by Pitzer *et al.* [33] by introducing a third reference fluid was developed by Stipp *et al.* [47]. In contrast to the model by Lee and Kesler [44], which incorporated BWR equations, the reference fluids are described by Bender-type equations.

So far, all introduced equations of state were explicit in pressure. These equations alone allow for the calculation of thermal properties. If caloric properties are to be calculated as well, thermal equations have to be transformed to a residual fundamental equation by integration and then be combined with an equation describing the ideal gas behavior. Fundamental equations of state formulated in the internal energy u(v, s), enthalpy h(p, s), Gibbs enthalpy g(T, p), or Helmholtz energy a(T, v) allow for the calculation of all thermodynamic properties. Of these four, the Helmholtz energy is the common standard in the development of equations of state today, since it is not depending on the not measurable entropy, and in contrast to the Gibbs enthalpy, it is continuous through the two-phase region. The molar Helmholtz energy *a* is defined by

$$a(T,v) = u - Ts.$$
 (3.15)

The Helmholtz energy is divided into two contributions, the ideal part *a*<sup>o</sup> and the residual part *a*<sup>r</sup>. For a pure fluid, the Helmholtz energy can be written as

$$a(T,\rho) = a^{o}(T,\rho) + a^{r}(T,\rho).$$
(3.16)

The idea of simple corresponding states is applied to the Helmholtz energy as well. For that reason, a reduced formulation is utilized:

$$\alpha(\tau,\delta) = \frac{\alpha(T,\rho)}{RT},\tag{3.17}$$

with the reduced variables

$$\tau = \frac{T_{\rm r}}{T} \text{ and } \delta = \frac{\rho}{\rho_{\rm r}},$$
 (3.18)

where usually  $T_r = T_c$  and  $\rho_r = \rho_c$  are used for pure fluids.

Pressure explicit equations of state are often expressed in the compression factor  $Z = p/\rho RT$  or can easily be transferred to this form. This way, they can then be integrated to the reduced residual Helmholtz energy as follows:

$$\alpha^{\mathrm{r}}(\tau,\delta) = \int_{0}^{\delta} \frac{Z(\tau,\delta) - 1}{\delta} \mathrm{d}\delta \,. \tag{3.19}$$

In combination with an equation describing the ideal gas behavior of the fluid, all thermophysical properties, thermal and caloric, can be calculated from combinations of derivatives of these functions as well, see Table 3.3 in section 3.2.

Consequently, for equations of state explicit in pressure, terms that can be integrated to the reduced residual Helmholtz energy have exclusively been used. The required integration restricts the flexibility of chosen terms extremely. In 1969, Keenan *et al.* [48] published the first equation of state that was exclusively formulated in the free Helmholtz energy. However, it was Pollak in 1974 [49] who was the first to propose a functional form originally based on the Helmholtz energy including terms that could not have been used in a pressure explicit equation.

In 1985, Schmidt and Wagner [50] introduced exponential terms with density exponents higher than two, which are especially convenient for the description of the critical region. Even more important, the use of a powerful optimization algorithm led to an accurate equation with as few terms as possible. In 1974, Wagner [51] had proposed a stepwise regression analysis, which has the capability to select terms from a bank of terms, finding an optimized functional form based on mathematical analysis instead of the correlator's experience. This approach was improved in form of an evolutionary optimization method (EOM) by Ewers and Wagner in 1982 [52,53]. In 1989, Setzmann and Wagner [54] published a new optimization algorithm (OPTIM), which combines a modified stepwise regression analysis with elements of the evolutionary optimization method. Many powerful equations of state, that are still in use today, have been developed by means of this algorithm as are the reference equations for water [20] and CO<sub>2</sub> [22]. Recent equations of state have been developed with a different fitting technique, proposed and applied by Lemmon and Jacobsen [55] to develop an equation for R125 in 2005. This approach is also used in this work and is discussed further in section 4.2.

In 2003, Span and Wagner [56] published two technical equations of state, which are accurate but rather simple in structure, one for polar and another for non-polar fluids, that had been developed using an adapted optimization algorithm (SIMOPT). The difference to the classical approach is the simultaneous use of experimental data from multiple substances to determine the functional form of the equation. Like Starling [40], they published substance-specific coefficients [57,58] as well as a generalized version [59]. For the generalization, they applied the three-parameter CCSA but used the corresponding states parameter as adjustable parameters in order to reach the desired accuracy [59]. This model is further discussed in section 3.5. In 2004, Sun and Ely [60] published a universal equation of state explicit in the Helmholtz energy that had been adjusted to polar and nonpolar fluids simultaneously using a simulated annealing method for the optimization. Lustig *et al.* [61] observed a high similarity between the universal equation of Sun and Ely [60] and the two original technical equations of Span and Wagner [56]. They showed that, except for one term, the Sun and Ely equation [60] is a linear combination of terms of the equations of Span and Wagner [56]. In 2005, Sun and Ely [60] proposed a generalization for their equation based on a four-parameter eCSA [62]. In 2013, Alexandrov et al. [63] used the universal equation of Sun and Ely and generalized it by means of the three-



Figure 3.1: Schematic overview of the historical development of empirical pure fluid models.

parameter CCSA in order to describe *n*-alkanes from *n*-pentane (C5) to *n*-pentacontane (C50). This equation is presented in detail in section 3.5. Figure 3.1 summarizes the most important pure fluid models presented so far and their interrelations.

### 3.1.2 Empirical and Semi-empirical Mixture Models

Evidently, the historical development of thermodynamic mixture models is strongly linked to the evolution of pure fluid equations of state. So far, the history of pure fluid equations of state explicit in pressure or reduced Helmholtz energy has been presented.

In 1873, van der Waals [64] proposed mixing rules for his two equation parameters *a* and *b* for a mixture consisting of *N* components with

$$a = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j a_{ij},$$
(3.20)

and

$$b = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j b_{ij}, \qquad (3.21)$$

 $x_i$  and  $x_j$  being the mole fraction of component *i* and *j*, respectively.

The unlike-interaction parameters can be determined by

$$a_{ij} = \sqrt{a_i a_j},\tag{3.22}$$

and

$$b_{ij} = \frac{b_i + b_j}{2},\tag{3.23}$$

leading to a simpler formulation of the parameter *b* with

$$b = \sum_{i}^{N} x_i b_i. \tag{3.24}$$

Since all mixture components are combined to one pseudo-pure fluid with the help of these mixing rules, this theory is widely referred to as the one-fluid theory.

Redlich and Kwong [32] incorporated the van der Waals mixing rules for their newly proposed cubic equation in 1949. In the 1960s, Leland and co-workers [65–67] rederived these mixing rules based on statistical-mechanical theory of radial distribution functions. They were able to link van der Waals' equation parameters a and b to the molecular parameters  $\epsilon$  and  $\sigma$ , which will be introduced in the context of SAFT theories in section 3.2. Based on this work, Kwak and Mansoori [68] stated that the mixing rules were used in a wrong way by Redlich and Kwong, since they changed the parameter dependency in a way that they no longer fulfilled the statistical mechanics behind the theory.

In 1970, Zudkevitch and David [69] introduced the adjustable binary interaction parameter  $k_{ij}$  representing the deviation of parameter  $a_{ij}$  from the classical geometric mean assumption leading to

$$a_{ij} = (1 - k_{ij})\sqrt{a_i a_j}.$$
 (3.25)

Soave [34] as well as Peng and Robinson [35] introduced this binary interaction parameter in their description of mixtures using van der Waals' one-fluid theory.

In 1972, Bishnoi and Robinson [70] published mixing rules for the BWR equation parameters based on the one-fluid theory as proposed by van der Waals as well. They also introduced a binary interaction parameter into some of the mixing rules. Since his equation is a slight variance of the BWR, Starling [40] published analogous mixing rules in 1973, referring to Bishnoi and Robinson [70].

In 1978, Plöcker *et al.* [71] extended the generalized equation of state by Lee and Kesler [44] to mixtures. They proposed mixing rules to determine the pseudocritical properties of the mixture and introduced a binary interaction parameter in the formulation of the pseudocritical temperature. Plöcker *et al.* [71] introduced an additional constant exponent in the formulation of the pseudo-critical temperature accounting for asymmetric mixtures consisting of components of significantly different molecular size. If the value of this exponent is set to one, the mixing rules transfer to the van der Waals mixing rules. In 1993, Platzer and Maurer [72] extended their previously published generalized Bender equation [45] to predict properties of multicomponent mixtures. Therefore, they proposed three additional binary parameters for their mixing rules, two of them influencing the pseudo-critical temperature and one influencing the pseudo-critical density. In 1994, Huber and Ely [73] proposed an extended corresponding states model based on a wide-ranging modified BWR equation of state (mBWR). They chose density and temperature



Figure 3.2: Schematic overview of the historical development of empirical mixture models.

dependent shape factors to transform results for a well-known reference fluid to the desired substance. To account for the composition dependence, they chose the van der Waals one-fluid mixing rules. In this model, there are two binary interaction parameters that can be adjusted.

The concept of pure fluid multiparameter Helmholtz energy equations was first extended to mixtures by Tillner-Roth [74] in 1993 and Lemmon [75] in 1996. They independently developed models, that describe a mixture's behavior in a very similar manner. In 1999, Lemmon and Tillner-Roth [76] revised and generalized the description of these mixture models based on highly accurate equations of state for the pure components, which are combined at the reduced temperature and density of the mixture. In 2000, Lemmon et al. [77] published a mixture model for air-like mixtures that was based on this work. In the same year, Klimeck [78] developed a preliminary multi-fluid mixture model for natural gas mixtures. This model was developed further by Kunz et al. [79]. It is known as the GERG-2004 equation of state. Considering up to 18 components, it sets new standards for the highly accurate description of multi-component natural gas mixtures. In 2012, this model was extended to 21 components and published as the GERG-2008 by Kunz and Wagner [80]. Recently, Gernert and Span [81] proposed a multi-fluid mixture model for combustion gases and combustion gas-like mixtures called EOS-CG. This model is designed in a similar way and can be seen as an extension of the GERG-2008 since it incorporates the already developed mixture models. This type of mixture models is presented in detail in section 3.6, since it is used for the development of the new model for ethanolwater mixtures. Figure 3.2 summarizes the so far presented mixture models.

### 3.1.3 Physical Property Models

A completely different approach of property modelling is the Statistical Associating Fluid Theory (SAFT) [82]. In contrast to the macroscopic point of view of the so far presented models, SAFT relates non-ideality to intermolecular forces. Therefore, it can be classified as a physical model even though its practical implementation contains empirical elements.

In 1989, Chapman *et al.* [82,83] published the original SAFT theory based on Wertheim's disturbance theory [84–87], which provides the basis for the description of a thermodynamic reference system. Wertheim expanded the Helmholtz energy in a series of integrals of molecular distribution functions and the association potential. Based on this approach, Chapman *et al.* [82] divided the residual Helmholtz energy in three physical contributions: repulsion-dispersion contributions of segment-segment interactions, contributions resulting from the formation of chains, and site-site specific association interactions, for example hydrogen bonding. Consequently, the residual Helmholtz energy is given by

$$A^{\rm r} = A^{\rm seg} + A^{\rm chain} + A^{\rm assoc}, \tag{3.26}$$

where the superscripts seg, chain, and assoc refer to the contributions from the "monomeric" segments, from the formation of chains, and from the existence of association sites, respectively. [82] The original SAFT theory already accounts for mixtures using van der Waals' one-fluid theory.

SAFT has been refined and adapted by many authors. One of the important changes was introduced by Huang and Radosz [88,89], who applied the dispersion term by Chen and Kreglewski [90] to the original SAFT theory [82,83]. This term was derived by adjusting a perturbation expansion to experimental data of argon [90].

In section 3.2, the perturbed-chain theory (PC-SAFT), which was published by Gross and Sadowski in 2001 [91], is presented more thoroughly since it will be used for comparison with the models developed in this work.

Another important semi-empirical generalized model is the Backone equation [92,93]. The authors adopt van der Waals' approach and divide the residual behavior in a hardbody (h) and an attractive contribution (a)

$$A^{\rm r} = A^{\rm h} + A^{\rm a}.\tag{3.27}$$

The ideal gas behavior, which is known quite well from statistical thermodynamics, describes a fluid's behavior in the limit of zero-density. Recent perturbation theories allow for a good description of hard bodies, which is a good approximation of fluids in the limit of very high densities. Saager *et al.* [92] introduced the fluid-specific anisotropy parameter  $\alpha$  to the hard-body contribution, which describes the shape of the molecule. For  $\alpha = 1$ , which means that the molecule is spherical, the contribution reduces to the Carnahan and Starling equation [39]. For the attractive contribution, a double polynomial equation dependent on a scaled temperature and a packing fraction with 24 universal parameters was fitted to experimental data of ethane. The anisotropy is described separately for both contributions with  $\eta$  being the anisotropy parameter of the attractive part. Additionally, a characteristic density  $\rho_0$  and temperature  $T_0$  are required as corresponding states

parameters. This makes this model a four-parameter CSA. For polar substances, the description of the Helmholtz energy is extended [93]. An additional parameter for the reduced dipole or quadrupole moment is required. These extended equations are called DI-BACK and QUABACK, respectively.

Müller *et al.* [94] changed the description of the attractive part by adjusting it to experimental data of methane, oxygen, and ethane using the optimization algorithm of Setzmann and Wagner [54]. In that manner, they found an optimized functional form that could use the anisotropy parameter  $\alpha$  instead of the separate  $\eta$ , which was originally used in the hard-body contribution, as well as a scaled density instead of the packing fraction. The resulting equation for non-polar substances is called SIMBACKONE. For polar substances, they use the same additions published by Saager and Fischer [93]. It is important to notice that the additional equations for dipolar or quadrupolar substances cannot be used simultaneously, since they do not account for cross combinations. Weingerl *et al.* [95] published mixing rules for all equation parts of the BACKONE theory.

In the following, the models that are used in this work are presented in more detail.

## 3.2 PC-SAFT

The Perturbed-Chain Statistical Associating Fluid Theory (PC-SAFT) developed by Gross and Sadowski [91] is a modification of the SAFT version of Huang and Radosz [88,89], which is based on the original SAFT model of Chapman *et al.* [82]. Gross and Sadowski developed a new dispersion term by applying the perturbation theory to a hard-chain of spherical hard-sphere segments as reference instead of the hard-sphere molecules themselves. The resulting new model constants of the dispersion expression were adjusted to *pvT* and vapor pressure data of *n*-alkanes. Since the hard-chain is build-up of uniform hard-spheres, it knows only repulsive forces. All attractive chain interactions are modeled as perturbations in the dispersion term (disp). For associating fluids, the residual Helmholtz energy consists of the following contributions:

$$A^{\rm r} = A^{\rm hc} + A^{\rm disp} + A^{\rm assoc}.$$
(3.28)

The repulsive forces between the individual hard-chain molecules (hc) are modeled using the first order perturbation theory according to Wertheim [84–87]. The PC-SAFT model can be extended to describe the molecular behavior of the required substance more precisely. The extensions, for example extensions for dipolar or quadrupolar substances, can simply be added to the existing Helmholtz equation as additional perturbations. Figure 3.3 illustrates the proposed molecular setup behind the PC-SAFT theory.



**Figure 3.3:** The molecular setup of PC-SAFT [91]. (a) An initial hard-sphere reference system of equally sized spherical segments. (b) Chain molecules are formed by bonding of *m* segments. (c) The hard-chains experience dispersive forces. (d) Association sites allow molecules to form hydrogen bonds. [96]

## **Molecular model**

The molecular model underlying the PC-SAFT theory is a modified square well potential suggested by Chen and Kreglewski [90]:

$$u(r) = \begin{cases} \infty, \ r < (\sigma - s_1) \\ 3\epsilon, \ (\sigma - s_1) \le r < \sigma \\ -\epsilon, \ \sigma \le r < \lambda\sigma \\ 0, \ r \ge \lambda\sigma \end{cases}$$
(3.29)

where u(r) is the pair potential, r is the radial distance between two segments,  $\sigma$  is the temperature-independent segment diameter,  $\epsilon$  is the depth of the potential well, and  $\lambda$  is the reduced well width. As suggested by Chen and Kreglewski [90], a ratio of  $s_1/\sigma = 0.12$  is set. However, no additional temperature dependence of the potential depth is proposed.



Figure 3.4: Modified square well potential by Chen and Kreglewski [90].

As a result, there are three substance specific parameters describing the molecule:  $\sigma$ ,  $\varepsilon$ , m; m being the number of spherical segments forming one chain. Figure 3.4 presents the applied potential model graphically.

Despite the simplicity of the molecular model, the step function accounts for the soft repulsion of molecules which is an essential feature of a real fluid's behavior. The validity of the temperature-independent segment diameter  $\sigma$  as the collision diameter is restricted to the zero-temperature limit. For higher temperatures, the collision diameter becomes smaller. Therefore, a temperature dependence of the collision diameter is introduced with

$$d(T) = \int_0^\sigma \left[ 1 - \exp\left(\frac{u(r)}{k_{\rm B}T}\right) \right] \mathrm{d}r.$$
(3.30)

The temperature-dependent segment diameter d for component i described by the proposed molecular model yields

$$d_i(T) = \sigma_i \left[ 1 - 0.12 \exp\left(-\frac{3\epsilon_i}{k_{\rm B}T}\right) \right].$$
(3.31)

### Hard chain

As already pointed out, the innovation of the PC-SAFT model is the new hard-chain reference fluid consisting of hard-spheres. The reduced Helmholtz energy of the hard-sphere is defined in relation to the total number of segments  $N_s$  and is given by the expression of Mansoori *et al.* [97] reading

$$\tilde{a}^{\rm hs} = \frac{A^{\rm hs}}{N_{\rm s}k_{\rm B}T} = \frac{1}{\zeta_0} \left[ \frac{3\zeta_1\zeta_2}{(1-\zeta_3)} + \frac{\zeta_2^3}{\zeta_3(1-\zeta_3)^2} + \left(\frac{\zeta_2^3}{\zeta_3^3} - \zeta_0\right) \ln(1-\zeta_3) \right], \tag{3.32}$$

with the abbreviation

$$\zeta_n = \frac{\pi}{6} \hat{\rho} \sum_{i=0}^{N_c} x_i m_i d_i^n \text{ and } n \in \{0, 1, 2, 3\},$$
(3.33)

with  $x_i$  being the mole fraction of component *i* in the mixture with  $N_c$  components. The packing fraction or reduced segment density is defined as

$$\eta = \zeta_3 = \frac{\pi \hat{\rho}}{6} \sum_{i=1}^{N_c} x_i m_i d_i^3, \qquad (3.34)$$

with  $\hat{\rho}$  being the total number density of molecules

$$\hat{\rho} = \rho \cdot N_{\rm A} \cdot 10^{-30} \frac{\rm m^3}{\rm \AA^3}.$$
(3.35)

The radial distribution function of the hard-sphere fluid according to Boublik [98] is given by the following relation:

$$g_{ii}^{\rm hs} = \frac{1}{(1-\zeta_3)} + \frac{1}{2}d_i \frac{3\zeta_2}{(1-\zeta_3)^2} + \frac{1}{4}d_i^2 \frac{2\zeta_2^2}{(1-\zeta_3)^3} .$$
(3.36)

As already pointed out, the repulsive forces between the individual chain molecules are modeled using the first order perturbation theory according to Wertheim [84–87]. The reduced Helmholtz energy of the hard-chain fluid is composed of the radial distribution function and the reduced Helmholtz energy of the hard-sphere fluid and can be determined from

$$\tilde{a}^{\rm hc} = \frac{A^{\rm hc}}{Nk_{\rm B}T} = \bar{m}\tilde{a}^{\rm hs} - \sum_{i=1}^{N_{\rm c}} x_i(m_i - 1)\ln g_{ii}^{\rm hs}(\sigma_{ii}), \qquad (3.37)$$

with the average segment number

$$\overline{m} = \sum_{i=1}^{N_c} x_i m_i \tag{3.38}$$

to account for mixtures.

## Dispersion

The dispersive interactions are modeled as second order perturbations based on the theory of Barker and Henderson [99,100] with

$$\widetilde{a}^{\text{disp}} = \frac{A_1}{Nk_BT} + \frac{A_2}{Nk_BT} = -2\pi\widehat{\rho}I_1\overline{m^2\epsilon\sigma^3} - \pi\widehat{\rho}\overline{m}C_1I_2\overline{m^2\epsilon^2\sigma^3}.$$
(3.39)

For the utilization in mixtures, van der Waals' one-fluid theory is applied. Conventional Lorentz-Berthelot combining rules are proposed to obtain the mixture parameters

$$\sigma_{ij} = \frac{\sigma_{ii} + \sigma_{jj}}{2},\tag{3.40}$$

and

$$\epsilon_{ij} = \sqrt{\epsilon_i \epsilon_j} (1 - k_{ij}), \qquad (3.41)$$

with  $k_{ij}$  being the only binary interaction parameter.

The following abbreviations are introduced in equation (3.39):

$$C_1 = \left(1 + \overline{m} \,\frac{8\eta - 2\eta^2}{(1 - \eta)^4} + (1 - \overline{m}) \,\frac{20\eta - 27\eta^2 + 12\eta^3 - 2\eta^4}{[(1 - \eta)(2 - \eta)]^2}\right)^{-1},\tag{3.42}$$

$$\overline{m^2 \epsilon \sigma^3} = \sum_{i=1}^{N_c} \sum_{j=1}^{N_c} x_i x_j \, m_i m_j \left(\frac{\epsilon_{ij}}{kT}\right) \sigma_{ij}^3 \,, \tag{3.43}$$

and

$$\overline{m^2 \epsilon^2 \sigma^3} = \sum_{i=1}^{N_c} \sum_{j=1}^{N_c} x_i x_j \, m_i m_j \left(\frac{\epsilon_{ij}}{kT}\right)^2 \sigma_{ij}^3 \,. \tag{3.44}$$

To shorten the volume integrals of Barker and Henderson [99,100], the following power series in the packing fraction are introduced:

$$I_1 = \sum_{j=0}^6 a_j(\bar{m})\eta^j,$$
 (3.45)

and

$$I_2 = \sum_{j=0}^{6} b_j(\bar{m})\eta^j,$$
(3.46)

with

$$a_{j}(\overline{m}) = a_{0j} + \frac{\overline{m} - 1}{\overline{m}} a_{1j} + \frac{\overline{m} - 1}{\overline{m}} \frac{\overline{m} - 2}{\overline{m}} a_{2j}, \qquad (3.47)$$

and

$$b_j(\bar{m}) = b_{0j} + \frac{\bar{m} - 1}{\bar{m}} b_{1j} + \frac{\bar{m} - 1}{\bar{m}} \frac{\bar{m} - 2}{\bar{m}} b_{2j}.$$
(3.48)

Equations (3.47) and (3.48) account for the bonding of one segment to a nearest-neighbor segment and for the possible bonding of the neighbor segment to the next-nearest neighbor segment.

## Association

If associating fluids are considered, there are two additional parameters that characterize the interactions between the association site  $A_i$  and  $B_i$  of a pure component *i*: the association energy  $\varepsilon^{A_iB_i}/k$  and the effective association volume  $\kappa^{A_iB_i}$ . For mixtures, simple combining rules for the cross-association of substances by Wolbach and Sandler [101] are applied:

$$\kappa^{A_i B_j} = \sqrt{\kappa^{A_i B_i} \kappa^{A_j B_j}} \left( \frac{\sqrt{\sigma_{ii} \sigma_{jj}}}{1/2 \left( \sigma_{ii} + \sigma_{jj} \right)} \right)^3, \tag{3.49}$$

and

$$\varepsilon^{A_i B_j} = \frac{1}{2} (\varepsilon^{A_i B_i} + \varepsilon^{A_j B_j}). \tag{3.50}$$

There is no additional binary interaction parameter proposed. Therefore, only the dispersive interactions are corrected.

The contribution of association to the reduced Helmholtz energy according to Huang and Radosz [89] is defined as

$$\frac{a^{\text{assoc}}}{RT} = \sum_{i=1}^{N} x_i \left[ \sum_{A_i=1}^{M_i} \left( \ln X^{A_i} - \frac{X^{A_i}}{2} \right) + \frac{1}{2} M_i \right],$$
(3.51)

with  $A_i$  being the associating site A on component i,  $M_i$  the total number of associating sites of component i, and  $X^{A_i}$  the fraction of molecules of component i not bonded at site A. This fraction is determined from

$$X^{A_{i}} = \left(1 + \sum_{j=1}^{N} \sum_{B_{j}=1}^{M_{j}} \rho_{j} X^{B_{j}} \Delta^{A_{i}B_{j}}\right)^{-1}$$
(3.52)

with the associating strength

$$\Delta^{A_i B_j} = \sigma_{ij}^3 g_{ij}^{\text{seg}}(d_{ij}) \kappa^{A_i B_j} \left[ \exp\left(\frac{\varepsilon^{A_i B_j}}{kT}\right) - 1 \right].$$
(3.53)

The radial distribution function of the segment is approximated with that of the hardsphere:

$$g_{ij}^{\text{seg}}(d_{ij}) \approx g_{ij}^{\text{hs}}(d_i, d_j)$$
  
=  $\frac{1}{(1 - \zeta_3)} + \left(\frac{d_i d_j}{d_i + d_j}\right) \frac{3\zeta_2}{(1 - \zeta_3)^2} + \left(\frac{d_i d_j}{d_i + d_j}\right)^2 \frac{2\zeta_2^2}{(1 - \zeta_3)^3}.$  (3.54)

Equation (3.54) resembles the Carnahan-Starling EOS [39] for a mixture of hard-spheres [102].

Huang and Radosz [88] published approximations for the fraction of sites not bonded to site *A* of a molecule. Table 3.1 presents an overview over the relevant bonding types for pure ethanol, and water. For any alcohol, the rigorous type is 3C since there are two lone pairs per oxygen atom (O), and one electron acceptor (H), which add to a total of three associating sites. In case of type 2B, both lone pairs are combined to one associating site. Consequently, the rigorous type of water is 4C, although it is often modeled as type 2B.

Formula -		$X^A$	$X^A$
Туре	$\Delta$ approximations	approximations	site fractions
Alcohol (2B)	$\Delta^{AA} = \Delta^{BB} = 0;$	$X^A = X^B$	$\frac{-1+\sqrt{1+4\rho\Delta}}{2\rho\Lambda}$
н В	$\Delta^{nn} \neq 0$		2pt
Alcohol (3B) A $\vdots$ B H C	$\Delta^{AA} = \Delta^{AB}$ $= \Delta^{BB} = \Delta^{CC} = 0;$ $\Delta^{AC} = \Delta^{BC} \neq 0$	$X^A = X^B;$ $X^C = 2X^A - 1$	$\frac{-(1-\rho\Delta)+\sqrt{(1+\rho\Delta)^2+4\rho}}{4\rho\Delta}$
Water (2B)	$\Delta^{AA} = \Delta^{BB} = 0;$ $\Delta^{AB} \neq 0$	$X^A = X^B$	$\frac{-1+\sqrt{1+4\rho\Delta}}{2\rho\Delta}$
Water (4C) B A : 0 : H C H D	$\Delta^{AA} = \Delta^{AB} = \Delta^{BB} = \Delta^{CC}$ $= \Delta^{CD} = \Delta^{DD} = 0;$ $\Delta^{AC} = \Delta^{AD} = \Delta^{BC} = \Delta^{BD} \neq 0$	$X^A = X^B = X^C = X^D$	$\frac{-1+\sqrt{1+8\rho\Delta}}{4\rho\Delta}$

Table 3.1: Bonding types in alcohols and water, illustrating the basic associating schemes 2B, 3B, and 4C.

Table 3.2 presents the association strength for the cross association of the components alcohol and water with the association schemes 2B, and 2B and 4C, respectively.

Formula - Type	$\Delta$ approximations
Alcohol (2B) - Water (2B) $ A_1$ $A_2$ $A_2$ $H$ $H$ $B_2$ $B_1$	$\Delta^{A_1A_2} = \Delta^{B_1B_2} = 0$
Alcohol (2B) - Water (4C) $- \begin{array}{c} & & & \\ & & \\ & & \\ & & \\ & & \\ & H \\ & & \\ B_1 \\ \end{array} \begin{array}{c} B_2 \\ & & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\Delta^{A_1A_2} = \Delta^{A_1B_2} = \Delta^{B_1C_2} = \Delta^{B_1D_2} = 0;$

Table 3.2: Cross association of the basic associating schemes 2B - 2B, and 2B - 4C.

# 3.3 SRK

In this section, Soave's [34] adaption of the model by Redlich and Kwong [32], the SRK, is presented. In the sense of the van der Waals equation, there are two contributions to the overall pressure, an attractive part as well as a repulsive part resulting from hard-body interactions. Soave changed the temperature dependence of the attractive part by simply turning the attraction parameter *a* into a function of temperature *T*. This new dependency leads to the following formulation:

$$p = \frac{RT}{v-b} - \frac{a(T)}{v(v+b)}.$$
(3.55)

For a pure component *i*, the attraction parameter *a* is now given by

$$a_i(T) = \alpha_i(T) \cdot a_{c,i},\tag{3.56}$$

with  $\alpha_i(T_{c,i}) = 1$ , where subscript c denotes the critical condition. As introduced in section 3.1, both parameters  $a_{c,i}$  and  $b_i$  are determined by evaluating the equation at the critical point of component *i*, which yields

$$a_{c,i} = 0.42747 \frac{R^2 T_{c,i}^2}{p_{c,i}}$$
, and (3.57)

$$b_i = 0.08664 \frac{RT_{c,i}}{p_{c,i}}.$$
(3.58)

Soave investigated in the description of the vapor pressure. From plotting calculated vapor pressures of hydrocarbons, he found the following relation:

$$\alpha_{i}(T) = \left(1 + m_{i}\left(1 - \sqrt{T/T_{c,i}}\right)\right)^{2}.$$
(3.59)

As an improvement, Soave introduced the acentric factor as an additional corresponding states parameter. Therefore, he correlated the empirical parameter  $m_i$  against  $\omega_i$ , to find

$$m_i = 0.48 + 1.57\omega_i - 0.176\omega_i^2. \tag{3.60}$$

The SRK can be used predictively if the critical properties  $T_{c,i}$  and  $p_{c,i}$  as well as the acentric factor  $\omega_i$  are known. In section 5.1.1, this model is used to adjust the corresponding states parameters to a synthetic PAO lubricant based on a very limited data base since the critical parameters are not known.

As already presented, Soave incorporates the one-fluid mixing rules for the calculations of a mixture consistent of *N* components:

$$a = \sum_{i}^{N} \sum_{j}^{N} x_{i} x_{j} a_{ij},$$
 (3.20)

with

$$a_{ij} = (1 - k_{ij})\sqrt{a_i a_j},$$
 (3.22)

 $k_{ij}$  being a binary adjustable parameter, and

$$b = \sum_{i}^{N} x_i b_i. \tag{3.24}$$

### 3.4 Helmholtz Equations of State for Pure Fluids

In this section, Helmholtz equations of state for pure fluids are presented. The corresponding mixture approach is shown separately in section 3.6. As already introduced in equation (3.16), the contribution of the ideal gas behavior to the reduced Helmholtz energy is modeled separately from the residual part.

#### **Ideal part**

For an ideal gas, the Helmholtz energy is defined by

$$a^{o}(T,\rho) = u^{o}(T) + Ts^{o}(T,\rho).$$
(3.61)

The internal energy of the ideal gas  $u^{0}$  can be obtained by integrating the isochoric heat capacity of the ideal gas  $c_{v}^{0}$  over the temperature as

$$u^{o}(T) = u_{0}^{o} + \int_{T_{0}}^{T} c_{\nu}^{o} \, \mathrm{d}T \,.$$
(3.62)

The ideal gas entropy can be determined by

$$s^{o}(T,\rho) = s_{0}^{o} + \int_{T_{0}}^{T} \frac{c_{v}^{o}}{T} dT - R \ln\left(\frac{\rho}{\rho_{0}}\right).$$
(3.63)

The density at the reference point of the ideal gas is determined by the thermal equation of state for the ideal gas given in equation (3.1) and is calculated from the reference pressure  $p_0$  and the reference temperature  $T_0$ :

$$\rho_0 = \frac{p_0}{RT_0}.$$
 (3.64)

The combination of equations (3.62) and (3.63) results in the following description of the ideal Helmholtz energy:

$$a^{o}(T,\rho) = u_{0}^{o} - Ts_{0}^{o} + \int_{T_{0}}^{T} c_{v}^{o} dT - RT - T \int_{T_{0}}^{T} \frac{c_{v}^{o}}{T} dT + RT \ln\left(\frac{\rho}{\rho_{0}}\right).$$
(3.65)

In contrast to the residual part, the ideal gas contribution can be modeled by physically meaningful equations based on statistical thermodynamics. Usually, an equation for the ideal gas isobaric heat capacity  $c_p^o$  is formulated in the following form:

$$\frac{c_p^o}{R} = n_0^* + \sum_{i=1}^{I_{\text{Pol}}} n_i^* T^{t_i^*} + \sum_{i=I_{\text{Pol}}+1}^{I_{\text{Pol}}+I_{\text{PE}}} m_i \left(\frac{\theta_i}{T}\right)^2 \frac{\exp(\theta_i/T)}{(\exp(\theta_i/T) - 1)^2}.$$
(3.66)

Apart from the constant  $n_0^*$ , the common formulation may contain polynomial (Pol) and Planck-Einstein terms (PE). The constant  $n_0^*$  describes contributions from translation as well as external rotation of the molecules. The Planck-Einstein terms characterize the internal molecular vibrations and enable a description of the ideal gas isobaric heat capacity over a wide temperature range with good extrapolation behavior with the help of relatively few terms. Despite the underlying physical idea, the  $\theta_i$  and  $m_i$  in these terms are used as adjustable parameters in this context. For many fluids, Planck-Einstein terms in combination with the constant  $n_0^*$  suffice for a description of the ideal gas behavior. In case the individual modes interact, polynomial terms can be used additionally. Equation (3.66) can be fitted to either experimental or theoretical data. For further details on the ideal gas behavior see Span [59].

The ideal reduced Helmholtz energy  $\alpha^{0}$  is linked to the ideal isochoric heat capacity  $c_{\nu}^{0}$  as follows:

$$\left(\frac{\partial \alpha^{\rm o}}{\partial \tau^2}\right)_{\delta} = -\frac{c_{\nu}^{\rm o}}{R\tau^2}.$$
(3.67)

Consequently, the ideal isobaric heat capacity  $c_p^0$  is transformed to the ideal isochoric heat capacity  $c_v^0$  according to  $c_p^0/R = c_v^0/R + 1$ , which finally leads to

$$\frac{c_{\nu}^{o}}{R} = n_0 + \sum_{i=1}^{I_{\text{Pol}}} n_i \tau^{t_i} + \sum_{i=I_{\text{Pol}}+1}^{I_{\text{Pol}}+I_{\text{PE}}} m_i (\vartheta_i \tau)^2 \frac{\exp(\vartheta_i \tau)}{(\exp(\vartheta_i \tau) - 1)^2},$$
(3.68)

where  $n_0 = n_0^* - 1$ ,  $n_i = n_i^* \cdot T_r^{t_i^*}$ ,  $t_i = -t_i^*$ , and  $\vartheta_i = \theta_i / T_r$ .

This equation can then be integrated twice with respect to the inverse reduced temperature  $\tau$ :

$$\alpha^{o}(\tau,\delta) = c^{II} + c^{I}\tau + c_{0}\ln(\tau) + \sum_{i=1}^{I_{Pol}} c_{i}\tau^{t_{i}} + \sum_{i=I_{Pol}+1}^{I_{Pol}+I_{PE}} m_{i}\ln[1 - \exp(-\vartheta_{i}\tau)] + \ln(\delta), \quad (3.69)$$

with  $c_0 = n_0 = n_0^* - 1$ , and  $c_i = -n_i^* / (t_i^*(t_i^* + 1)) \cdot T_r^{t_i^*}$ . The integration constants  $c^I$  and  $c^{II}$  are defined by an arbitrary reference state.

# **Residual part**

In contrast to the ideal part, the residual fluid behavior is modeled by a completely empirical formulation. Generally, four different types of terms can be used to describe the residual reduced Helmholtz energy: polynomial (Pol), exponential (Exp), Gaussian bellshaped (GBS), and non-analytic (NA) terms:

$$\alpha^{\rm r}(\tau,\delta) = \alpha^{\rm r}_{\rm Pol}(\tau,\delta) + \alpha^{\rm r}_{\rm Exp}(\tau,\delta) + \alpha^{\rm r}_{\rm GBS}(\tau,\delta) + \alpha^{\rm r}_{\rm NA}(\tau,\delta).$$
(3.70)

These types of terms differ in complexity and number of parameters, as can be seen below

$$\alpha^{\mathrm{r}}(\tau,\delta) = \sum_{i=1}^{I_{\mathrm{Pol}}} n_{i}\tau^{t_{i}}\delta^{d_{i}} + \sum_{i=I_{\mathrm{Pol}}+1}^{I_{\mathrm{Pol}}+I_{\mathrm{Exp}}} n_{i}\tau^{t_{i}}\delta^{d_{i}}\exp(-\delta^{p_{i}}) 
+ \sum_{\substack{I_{\mathrm{Pol}}+I_{\mathrm{Exp}}+I_{\mathrm{GBS}}\\I_{\mathrm{Pol}}+I_{\mathrm{Exp}}+I_{\mathrm{GBS}}+I_{\mathrm{NA}}} n_{i}\tau^{t_{i}}\delta^{d_{i}}\exp(-\eta_{i}(\delta-\epsilon_{i})^{2} - \beta_{i}(\tau-\gamma_{i})^{2}) 
+ \sum_{\substack{i=I_{\mathrm{Pol}}+I_{\mathrm{Exp}}+I_{\mathrm{GBS}}+I_{\mathrm{NA}}\\I_{\mathrm{Pol}}+I_{\mathrm{Exp}}+I_{\mathrm{GBS}}+I_{\mathrm{NA}}} n_{i}\delta\Delta^{b_{i}}\exp(-e_{i}(\delta-1)^{2} - f_{i}(\tau-1)^{2}),$$
(3.71)

with 
$$\Delta = \{(1-\tau) + c_i [(\delta-1)^2]^{1/(2\beta_i)}\}^2 + d_i [(\delta-1)^2]^{a_i}.$$
 (3.72)

Amongst others, the polynomial and exponential terms have already been used by Schmidt and Wagner in 1975 [50]. In 1991, Setzmann and Wagner [103] applied for the first time Gaussian bell-shaped terms to a fundamental equation of state which were introduced by Haar *et al.* [104]. They are especially useful for the description of the critical region. However, depending on the parameters, they can improve the fluid's description in the entire fluid region [105]. Non-analytic terms are only in use in two reference equations of state, in the reference equation for water by Wagner and Pruß [20], and in the reference equation for CO<sub>2</sub> by Span and Wagner [22]. The non-analytic terms were introduced for the equations to show infinite heat capacities as well as zero speed of sound at the critical point, as proposed by theory. However, these terms are very sensitive and lead to numerical problems if used in mixtures. This is the reason why non-analytic terms are no longer found in current equations of state.

To calculate thermophysical properties, the derivatives of the reduced Helmholtz energy  $\alpha$  with respect to the independent variables  $\tau$  and  $\delta$  are needed. The relations of the different properties to the reduced Helmholtz energy are given in Table 3.3. The following notations are applied:

$$\alpha_{\tau} = \left(\frac{\partial \alpha}{\partial \tau}\right)_{\delta} \quad (3.73), \qquad \alpha_{\tau\tau} = \left(\frac{\partial^2 \alpha}{\partial \tau^2}\right)_{\delta} \quad (3.74), \qquad \alpha_{\delta\tau} = \left(\frac{\partial^2 \alpha}{\partial \delta \partial \tau}\right) \quad (3.75),$$
$$\alpha_{\delta} = \left(\frac{\partial \alpha}{\partial \delta}\right)_{\tau} \quad (3.76), \qquad \alpha_{\delta\delta} = \left(\frac{\partial^2 \alpha}{\partial \delta^2}\right)_{\tau} \quad (3.77).$$

Table 3.3: Definitions of common	thermodynamic properties and t	heir relation to the reduced	Helmholtz energy.

Property		Relation to reduced Helmholtz energ	gy
Pressure			
$p(T,\rho) = -\left(\frac{\partial a}{\partial v}\right)_T$	(3.78)	$\frac{p}{\rho RT} = 1 + \delta \alpha_{\delta}^{\rm r}$	(3.79)
Fugacity coefficient of component <i>i</i>			
$\ln \varphi_i(T, p, \boldsymbol{n}) = \int_0^p \left[\frac{v_i}{RT} - \frac{1}{p}\right] \mathrm{d}p_{T, \boldsymbol{n}}$	(3.80)	$\ln \varphi_i = \left(\frac{\partial n\alpha^r}{\partial n_i}\right)_{T,V,n_{j\neq i}} - \ln Z$	(3.81)
Fugacity of component i			
$f_i(T, p, \boldsymbol{x}) = x_i p \varphi_i(T, p, \boldsymbol{x})$	(3.82)	$f_i = x_i \rho RT \exp\left(\frac{\partial n\alpha^r}{\partial n_i}\right)_{T,V,n_{j\neq i}}$	(3.83)
Entropy			
$s(T,\rho) = -\left(\frac{\partial a}{\partial T}\right)_{\nu}$	(3.84)	$\frac{s}{R} = \tau(\alpha_{\tau}^{\rm o} + \alpha_{\tau}^{\rm r}) - \alpha^{\rm o} - \alpha^{\rm r}$	(3.85)
Internal energy			
$u(T,\rho)=a+Ts$	(3.86)	$\frac{u}{RT} = \tau(\alpha_{\tau}^{\rm o} + \alpha_{\tau}^{\rm r})$	(3.87)
Enthalpy			
$h(T,\rho)=u+pv$	(3.88)	$\frac{h}{RT} = 1 + \tau(\alpha_{\tau}^{\rm o} + \alpha_{\tau}^{\rm r}) + \delta \alpha_{\delta}^{\rm r}$	(3.89)
Isobaric heat capacity			
$c_p(T,\rho) = -\left(\frac{\partial h}{\partial T}\right)_p$	(3.90)	$\frac{c_p}{R} = -\tau^2 (\alpha_{\tau\tau}^0 + \alpha_{\tau\tau}^r) + \frac{(1 + \delta \alpha_{\delta}^r - \delta \tau \alpha_{\delta\tau}^r)^2}{1 + 2\delta \alpha_{\delta}^r + \delta^2 \alpha_{\delta\delta}^r}$	(3.91)
Gibbs energy			
g(T,p)=h-Ts	(3.92)	$\frac{g}{RT} = 1 + \alpha^{\rm o} + \alpha^{\rm r} + \delta \alpha^{\rm r}_{\delta}$	(3.93)
Speed of sound			
$w(p,T) = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$	(3.94)	$\frac{w^2}{RT} = 1 + 2\delta\alpha^{\rm r}_{\delta} + \delta^2\alpha^{\rm r}_{\delta\delta}$ $-\frac{(1 + \delta\alpha^{\rm r}_{\delta} - \delta\tau\alpha^{\rm r}_{\delta\tau})}{\tau^2(\alpha^{\rm o}_{\tau\tau} + \alpha^{\rm r}_{\tau\tau})}$	(3.95)
2 <sup>nd</sup> thermal virial coefficient			
$B(T) = \lim_{\rho \to 0} \left( \frac{\partial \left( \frac{p}{\rho RT} \right)}{\partial \rho} \right)_{T}$	(3.96)	$B\rho_{\rm r} = \lim_{\delta \to 0} \alpha^{\rm r}_{\delta}$	(3.97)

#### Table 3.3 continued.

$$3^{\text{nd}} \text{ thermal virial coefficient}$$

$$C(T) = \frac{1}{2} \lim_{\rho \to 0} \left( \frac{\partial^2 \left( \frac{p}{\rho RT} \right)}{\partial \rho^2} \right)_T \qquad (3.98) \qquad C\rho_r^2 = \lim_{\delta \to 0} \alpha_{\delta\delta}^r \qquad (3.99)$$

The relations presented in Table 3.3 are of a general kind. They do not depend on the type of model, which is used to describe  $\alpha^{o}$  or  $\alpha^{r}$ , nor on whether the equation was originally formulated in the Helmholtz energy or has been integrated from  $c_{p}^{o}$  or pressure. The derivatives of the Helmholtz equations presented in this section and relations for further properties can be found in Span [59].

## 3.5 Generalized Empirical Equations of State for Pure Fluids

As already introduced at the beginning of this chapter, there are different publications dealing with the generalization of an empirical equation of state. In this section, two such models are presented in more detail, including the model by Alexandrov *et al.* [63] that has been used in this work.

In 2003, Span and Wagner [56] published two technical equations of state, that had been optimized for multiple fluids simultaneously, one for polar and one for non-polar fluids. Subsequently, they proposed substance-specific coefficients for numerous fluids that had been adjusted to experimental data of the respective fluids [57,58].

Additionally, they were looking for a way to generalize the coefficients  $N_i$  using the acentric factor as corresponding states parameter. Their generalized equation finally reads [59]<sup>2</sup>

$$\alpha^{\rm r}(\tau,\delta) = \sum_{i=1}^{5} N_i \delta^{d_i} \tau^{t_i} + \sum_{i=6}^{10} N_i \delta^{d_i} \tau^{t_i} \exp(-\gamma_i \delta^{p_i}), \qquad (3.100)$$

with

$$N_i = c_{1,i} + c_{2,i}w + c_{3,i}w^4. aga{3.101}$$

<sup>&</sup>lt;sup>2</sup> The generalized equation of Span and Wagner was only published in chapter 7.2.2 of Ref. [59] and never in a journal article.

The parameter *w* is introduced in order to distinguish between the adjusted value and the true acentric factor  $\omega$  of the respective substance. The reducing temperature and density are used as adjustable parameters as well, which only resemble the critical properties. The values for  $d_i$ ,  $t_i$ , and  $p_i$  of the generalized equation of state given in equation (3.100) and for  $c_{1,i}$ ,  $c_{2,i}$ , and  $c_{3,i}$  given in equation (3.101) can be taken from Ref. [59].

The numerical stability of such a generalization is crucial for a beneficial predictive use. In comparison to the model of Platzer and Maurer [45], which was shortly introduced in section 3.1.1 and which is based on a four-parameter eCSA with three reference fluids, the calculated coefficients in dependence of the acentric factor of the model by Span and Wagner [59] are better behaved as can be seen in Figure 3.5.



Platzer and Maurer

**Figure 3.5:** Values of the coefficients  $N_i$  which result from equation (3.101) and the generalization by Platzer and Maurer [45] for 11 nonpolar fluids. [59]

The model of Span and Wagner [59] presents a constant linearity over the acentric factor. In contrast, the coefficients of the model by Platzer and Maurer [45] show discontinuities, which can lead to numerical instabilities in the usage of this model. Figure 3.5 also illustrates the fluids relevant to the development of the model by Span and Wagner. It can be seen, that the highest acentric factor of a considered fluid was  $\omega_{n-octane} \approx 0.4$ . Theoretically, for a spherical fluid the acentric factor would be zero, as can be seen for argon with  $\omega_{argon} \approx -0.00219$ . With rising complexity of the molecule's form, the acentric factor deviates further from this ideal.

Alexandrov *et al.* [63] published a generalized empirical equation of state that was adjusted to long-chained *n*-alkanes and is valid for  $\omega = 0.25$  to 1.8. The purpose of this model is to describe *n*-alkanes from *n*-pentane (C<sub>5</sub>) to *n*-pentacontane (C<sub>50</sub>). It is applicable over a temperature range from the triple point to 700K with pressures up to 100 MPa.

As a basis, Alexandrov *et al.* [63] used the universal equation developed by Sun and Ely [60], but chose a different generalization method than the authors in their following publication [62]. Sun and Ely proposed a four-parameter eCSA with three reference fluids, as used by Platzer and Maurer [45]. In contrast, Alexandrov *et al.* [63] presented a three-parameter CCSA based on the critical temperature and density as well as the acentric factor, as used by Span and Wagner [59]. This led to the following formulation:

$$\alpha^{\rm r}(\tau,\delta) = \sum_{i=1}^{6} N_i \delta^{d_i} \tau^{t_i} + \sum_{i=7}^{14} N_i \delta^{d_i} \tau^{t_i} \exp(-\gamma_i \delta^{p_i}), \qquad (3.102)$$

with

$$N_i = c_{1,i} + c_{2,i}w + c_{3,i}w^{c_{4,i}}.$$
(3.103)

The coefficients of equation (3.103) and the temperature exponents  $t_i$  were simultaneously fitted. The values for  $d_i$ ,  $t_i$ , and  $p_i$  of the generalized equation of state given in equation (3.102) and for  $c_{1,i}$ ,  $c_{2,i}$ ,  $c_{3,i}$ , and  $c_{4,i}$  given in equation (3.103) can be taken from Appendix A.

Figure 3.6 shows the values of the generalized coefficients  $N_i$  of the model of Alexandrov *et al.* [63] for the full range of validity from  $\omega = 0$  to 1.8 in comparison to the model of Span and Wagner [59].



**Figure 3.6:** Values of the coefficients  $N_i$  which result from equation (3.101) and equation (3.103) for  $\omega = 0$  to 1.8. The individual coefficients of the model of Alexandrov *et al.* [63] show a linear course in the complete region of validity, whereas the coefficients of the model of Span and Wagner [59] do not extrapolate well for acentric factors higher than  $\omega = 0.6$ . Since the model has not been adjusted in this region, the values of the coefficients rapidly change, one coefficient even reaching a value over 100. Such high coefficients can have a strongly negative influence on the behavior of an equation. However, it can be seen from Figure 3.6 that the values of the coefficients of Alexandrov et al. [63] have a wider spread compared to the coefficients of Span and Wagner in their range of validity, and this spread expands with increasing acentric factor significantly. This leads to numerical problems of the model of Alexandrov *et al.* [63] as well. In Figure 3.7, *p*,  $\rho$ -diagrams for several fluids with varying acentric factors are presented, which were calculated with the model of Alexandrov et al. [63]. For pentane, decane, and pentadecane, the saturation lines show a steady course. However, already for decane, the curvature of the saturation lines descending from the critical point looks odd, whereas for pentadecane, a convexity of the saturation lines in the near-critical region becomes visible. The saturation lines of icosane, triacontane, and pentacontane show a discontinuity at elevated temperatures. This is caused by the presented course of the near-critical isotherms of these fluids. For icosane ( $T_c = 768$  K), the



**Figure 3.7:** p,  $\rho$ -diagram for six exemplary fluids of different acentric factors containing the respective phase boundaries and a maximum of two isotherms, calculated with the model of Alexandrov *et al.* [63].

isotherm in the discontinuity (T = 712.3 K) as well as a higher isotherm of T = 720 K are illustrated. The outer extrema of the isotherm T = 720 K no longer overlap with respect to pressure. In order to find the coexisting densities as well as the vapor pressure of a two-phase equilibrium, the corresponding outer extrema are sought, and the pressure is varied until the areas between the individual extrema and the vapor pressure are of the same size in the vapor and liquid region. For icosane at T = 712.3 K and T = 720 K, a false solution is found, since the phase equilibrium cannot be solved with the outer extrema. Hence, Figure 3.7 shows numerical problems of the model of Alexandrov *et al.* [63] for acentric factors of  $\omega > 0.7$  approximately. However, these numerical problems only occur in an elevated temperature region, probably beyond the thermal stability of the fluids. Nevertheless, the incorporation of such an equation of state in a mixture model can lead to the evaluation of the equation of state in this region, as will be explained in the next section.

Since the lubricants which are considered in this work are complex molecules, they are expected to have higher acentric factors than are allowed for use with the model of Span and Wagner [59]. For this reason, the model of Alexandrov *et al.* [63] is chosen for their description.

Alexandrov *et al.* [63] specify an average absolute relative deviation AAD of 0.5% to 1% for the representation of the collected density data in the development of their model. For the representation of vapor pressures, an AAD of 0.5% to 2.5% is given. The authors state an AAD of 1% for the isobaric heat capacity in the liquid, and an AAD of 1% to 4% for the speed of sound.

In sections 5.1.2, 6.1 and 6.2, the results of the adjustments for the lubricants considered in this work are presented.

### 3.6 Helmholtz Equations of State for Mixtures

In the following, the extension of the multiparameter Helmholtz energy approach for pure fluids presented in the previous section to mixtures is shown in detail.

The separation of the ideal gas behavior from the residual behavior can also be applied for mixtures. However, the molar composition vector  $\boldsymbol{x}$  is an additional independent variable which leads to the following expression:

$$a(T, \rho, \mathbf{x}) = a^{o}(T, \rho, \mathbf{x}) + a^{r}(T, \rho, \mathbf{x}).$$
(3.104)

Analogously to the pure fluid equation, the Helmholtz energy is reduced with the temperature *T* and the universal gas constant *R* 

$$\alpha(\tau, \delta, \mathbf{x}) = \frac{\alpha(T, \rho, \mathbf{x})}{RT}.$$
(3.105)

The ideal gas part can be calculated simply based on the knowledge of the pure fluid equations for the ideal gas behavior and an additional contribution due to ideal mixing with

$$\alpha^{o}(T,\rho,\mathbf{x}) = \sum_{i=1}^{N} x_{i} [\alpha_{o,i}^{o}(\tau_{o,i},\delta_{o,i}) + \ln x_{i}], \qquad (3.106)$$

where *N* is the number of components in the mixture,  $\alpha_{o,i}^{o}$  is the dimensionless ideal-gas part of the Helmholtz free energy, and  $x_i$  is the mole fraction of component *i* in the mixture, respectively. The sum  $x_i \ln x_i$  accounts for the entropy of mixing in the ideal mixture. The equations for the ideal part of the pure fluids are evaluated at the reduced variables of the pure fluids:

$$\tau_{o,i} = \frac{T_{c,i}}{T} \text{ and } \delta_{o,i} = \frac{\rho}{\rho_{c,i}}.$$
 (3.107)

The residual part of the mixture can be described as a combination of the pure fluids' residual parts evaluated at the reduced temperature and density of the mixture, which is referred to as an extended corresponding states approach, and possibly an additional departure function:

$$\alpha^{\mathrm{r}}(\tau,\delta,\mathbf{x}) = \sum_{\substack{i=1\\ \mathrm{residual \, part \, pure \, fluids}}}^{N} x_i \alpha_{\mathrm{o}i}^{\mathrm{r}}(\tau,\delta) + \sum_{\substack{i=1\\ j=i+1\\ \mathrm{departure \, function}}}^{N-1} F_{ij} x_i x_j \alpha_{ij}^{\mathrm{r}}(\tau,\delta).$$
(3.108)

The weighing factor  $F_{ij}$  was introduced by Lemmon and Jacobsen [106] for the use of generalized departure functions for a group of binary systems. For a binary case it follows:

$$\alpha^{\rm r}(\tau,\delta,\mathbf{x}) = \sum_{i=1}^{2} x_i \alpha^{\rm r}_{0i}(\tau,\delta) + F_{12} x_1 x_2 \alpha^{\rm r}_{12}(\tau,\delta).$$
(3.109)

According to the corresponding states principle for mixtures, the residual pure fluid equations are evaluated at the reduced temperature and density determined with the help of the following equations. This approach leads to the evaluation of the individual pure fluid equations at different temperatures and densities. However, to determine the dimensionless variables  $\tau$  and  $\delta$ , reducing functions for the reducing temperature  $T_r$  and density  $\rho_r$ are required, which depend on the composition of the mixture with

$$\tau = \frac{T_{\rm r}(\mathbf{x})}{T} \text{ and } \delta = \frac{\rho}{\rho_{\rm r}(\mathbf{x})}.$$
(3.110)

For a multicomponent mixture the reducing functions  $T_r$  and  $\rho_r$  are given by

$$\frac{1}{\rho_{\rm r}(\mathbf{x})} = \sum_{i=1}^{N} x_i^2 \frac{1}{\rho_{\rm c,i}} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} 2x_i x_j \beta_{\nu,ij} \gamma_{\nu,ij} \frac{x_i + x_j}{\beta_{\nu,ij}^2 x_i + x_j} \frac{1}{8} \left( \frac{1}{\rho_{\rm c,i}^{1/3}} + \frac{1}{\rho_{\rm c,j}^{1/3}} \right)^3, \quad (3.111)$$

and

$$T_{\rm r}(\boldsymbol{x}) = \sum_{i=1}^{N} x_i^2 T_{{\rm c},i} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} 2x_i x_j \beta_{T,ij} \gamma_{T,ij} \frac{x_i + x_j}{\beta_{T,ij}^2 x_i + x_j} \frac{1}{8} \sqrt{T_{{\rm c},i} T_{{\rm c},j}}.$$
 (3.112)

The adjustable parameters  $\beta_{\nu}$ ,  $\gamma_{\nu}$ ,  $\beta_T$  and  $\gamma_T$  are fitted to experimental data of each binary combination of the mixture's components. For a binary mixture, these functions reduce to

$$\frac{1}{\rho_{\rm r}(\mathbf{x})} = \sum_{i=1}^{2} x_i^2 \frac{1}{\rho_{{\rm c},i}} + 2x_1 x_2 \beta_{\nu,12} \gamma_{\nu,12} \frac{x_1 + x_2}{\beta_{\nu,12}^2 x_1 + x_2} \frac{1}{8} \left( \frac{1}{\rho_{{\rm c},1}^{1/3}} + \frac{1}{\rho_{{\rm c},2}^{1/3}} \right)^3, \tag{3.113}$$

and

$$T_{\rm r}(\boldsymbol{x}) = \sum_{i=1}^{2} x_i^2 T_{{\rm c},i} + 2x_1 x_2 \beta_{T,12} \gamma_{T,12} \frac{x_1 + x_2}{\beta_{T,12}^2 x_1 + x_2} \frac{1}{8} \sqrt{T_{{\rm c},1} T_{{\rm c},2}}.$$
 (3.114)

The binary parameters  $\beta$  and  $\gamma$  allow for the description of arbitrary symmetric and asymmetric shapes of these functions which are derived from quadratic mixing rules. They form a composition-dependent surface between the critical parameters for the density and temperature of the pure components in the mixture. The binary parameters obey the following symmetry rules:

$$\beta_{T,ij} = \frac{1}{\beta_{T,ji}}, \quad \beta_{\nu,ij} = \frac{1}{\beta_{\nu,ji}}, \quad \gamma_{T,ij} = \gamma_{T,ji}, \quad \gamma_{\nu,ij} = \gamma_{\nu,ji}$$
(3.115)

If  $\beta_T$ ,  $\gamma_T$ ,  $\beta_v$ , and  $\gamma_v$  are set to unity, the equations merge into the combining rules by Lorentz and Berthelot as discussed in detail by Kunz *et al.* [79].

Very often, it is sufficient to fit the reducing functions to describe a mixture's non-ideal behavior. In case this is not enough, and more degrees of freedom are required, a departure function can be introduced. However, to fit this additional function, a good experimental data base concerning quality as well as quantity is mandatory.

A common functional form of the departure function is shown below:

$$\alpha_{ij}^{r(\tau,\delta)} = \sum_{k=1}^{K_{\text{Pol},ij}} n_{k,ij} \tau^{t_{k,ij}} \delta^{d_{k,ij}} + \sum_{k=K_{\text{Pol},ij+1}}^{K_{\text{Pol},ij}+K_{\text{Exp},ij}} n_{k,ij} \tau^{t_{k,ij}} \delta^{d_{k,ij}} \exp(-\delta^{l_{k,ij}}) + \sum_{k=K_{\text{Pol},ij+K_{\text{Exp},ij+1}}}^{K_{\text{Pol},ij+K_{\text{Exp},ij}}} n_{k,ij} \tau^{t_{k,ij}} \delta^{d_{k,ij}} \exp(-\eta_{k,ij} (\delta - \epsilon_{k,ij})^2 - \beta_{k,ij} (\tau - \gamma_{k,ij})^2).$$
(3.116)

These functions contain polynomial, exponential, and Gaussian-Bell shaped terms. The latter have just recently been introduced into these mixture models by Bell *et al.* [107]. The GERG [80] and EOS-CG [81] models both incorporate an additional type of exponential terms that has not been used in this work.

Theoretically, the mixture models based on this approach are independent of the utilized pure fluid equation, since the mixture model should exclusively describe the mixing behavior. The mixture model should not compensate shortcomings of the pure fluid equations, which in practice is impossible to ensure. Hence, if there is a new pure fluid equation available that seems superior to the original one, as is the case for ethanol and water, it is usually useful to refit an existing mixture model.

### 4 A New Model for Ethanol-Water Mixtures

In this chapter, a new model for ethanol-water mixtures is presented. It is based on the most current pure fluid equations of state, the equation of Wagner and Pruß [20] for water and the equation of Schroeder *et al.* [3] for ethanol. In 2008, Lemmon [4] developed a mixture model that utilizes the now outdated pure fluid equation of state for ethanol of Dillon *et al.* [21]. The equation of Schroeder *et al.* [3] was fitted to a broader experimental data base and has reduced uncertainties. Well-built pure fluid equations of state should be exchangeable, since a mixture model only to describes the mixing behavior. Figure 4.1 illustrates relative density deviations of the mixture model of Lemmon [4] calculated with both ethanol equations of state from selected experimental data. On top, the mixture model is evaluated with the originally utilized pure fluid equation of state. At the bottom, the pure fluid equation describing ethanol has been exchanged and the equation of Schroeder *et al.* [3] is used instead of



**Figure 4.1:** Relative density deviations of the mixture model of Lemmon [4] from selected exemplary experimental data. At the top, the original setup of the mixture model that uses the ethanol equation of state of Dillon *et al.* [21] is plotted. At the bottom, the ethanol equation of Schroeder *et al.* [3] was used instead.

the equation of Dillon *et al.* [21]. It is evident that the change of the pure fluid equation of state for this binary mixture has a strong, negative influence on the description of these mixture densities due to larger differences in the pure fluid equations than is typical with many other fluids.

A new mixture model, which incorporates the most accurate equations of state for the pure fluids involved, was developed in this work. In the following, the experimental data for ethanol-water mixtures found in the literature are presented. Subsequently, the fitting technique that was used to develop the new model is discussed. Finally, the new model is presented and evaluated based on the available experimental data. Additionally, the data and the new model are compared to the model of Lemmon [4].

## 4.1 Available Experimental Data

The development of an equation of a state depends on the availability of reliable experimental data for the thermodynamic properties of the fluid or fluid mixture. The knowledge of as many different properties as possible in a wide temperature, pressure, and composition range is desirable to describe the fluids behavior as accurately as possible. The quality of the equation strongly depends on the quality, quantity and location of the data.

There are many data sets for ethanol-water mixtures published in the literature including phase equilibria, density, speed of sound, isobaric heat capacity, and excess enthalpy. Table 4.1 gives an overview of the data published for ethanol-water mixtures.

Data type	Data sets	Data points	T <sub>min</sub> / K	T <sub>max</sub> / K	p <sub>min</sub> / MPa	p <sub>max</sub> / MPa
Phase equilibria	81	2613	273.15	623.11	<0.1	8.97
Density	56	5599	273.15	523.15	0.1	8.64
Speed of sound	7	68	293.12	606.3	0.1	0.1
Isobaric heat capacity	5	447	241.15	308.15	0.1	0.1
Excess enthalpy	24	1362	273.15	548	0.1	20

Table 4.1: Available experimental data for ethanol-water mixtures.

The phase behavior of this mixture has been investigated thoroughly throughout the last decades. There are many data sets available especially at atmospheric pressure. For elevated pressures, the quantity of data sets for phase equilibria of ethanol-water mixtures reduces drastically. At first glance, the density of ethanol-water mixtures seems very well investigated. Unfortunately, this is only true for the liquid phase. Only Bazaev *et al.* [108] and Abdurashidova *et al.* [109] measured vapor phase densities and densities at high temperatures. In this region, there is only one further experimental data set by Safarov and Shakhverdiev [110] available.

Similar to the density, there are only homogeneous liquid speed-of-sound data available in the literature. Those few data sets are limited to atmospheric pressure. This also corresponds to the isobaric heat capacity. There are only homogeneous liquid phase data at atmospheric pressure available, which only cover a very narrow temperature range. Some of the authors published excess isobaric heat capacities. For fitting and comparison purposes, those values have been transferred to the full heat capacity of the mixture with the help of the pure fluid equations [3,20].

The excess enthalpy of ethanol-water mixtures has been well investigated. In contrast to the speed of sound and the isobaric heat capacity, there are many data sets available that agree well in the description of the mixture behavior. The available data sets cover a broad temperature and pressure range.

# 4.2 Fitting Process

Although the introduced empirical model is explicit in the Helmholtz energy as a function of temperature, density, and composition, there is no information on the shape of this thermodynamic surface. Since the Helmholtz energy cannot be measured experimentally, the energy surface is exclusively shaped by fitting the Helmholtz energy derivatives to experimental data. Therefore, it is crucial to ensure that derivatives with respect to all independent variables are included; the higher the order, the better. However, the uncertainty of the composition plays a major role in the fitting process. A significant error in composition can be problematic to fit several fluid properties.

The relations between the available experimental data and the derivatives are given in Table 4.2.

In principle, there are two ways to fit the parameters of empirical multiparameter models. They can either be fit linearly or non-linearly. Numerous Helmholtz equations of reference quality have been published by the Wagner group in the 1980s and 1990s that have been developed with a combination of linear and non-linear fitting techniques. Additionally, these equations were optimized with the help of a sophisticated

Thermodynamic property	Related derivatives of $\alpha(\tau, \delta, x)$
$p, \rho, T$ relation $p(T, \rho, \mathbf{x})$	$lpha_{\delta}^{ m r}$
Fugacity $f_i(T, \rho, \mathbf{x})$ for VLE calculation	$\alpha^{\mathrm{r}}_{\delta}, \alpha^{\mathrm{r}}_{\delta\delta}, \alpha^{\mathrm{r}}_{\tau\delta}, \alpha^{\mathrm{r}}_{\tau}, \alpha^{\mathrm{r}}_{\tau\tau}, \alpha^{\mathrm{r}}_{x}, \alpha^{\mathrm{r}}_{xx}$
Speed of sound $w(T, p, x)$	$lpha_{\delta}^{ m r}, lpha_{\delta\delta}^{ m r}, lpha_{\tau\delta}^{ m r}, lpha_{\tau\tau}^{ m r}, lpha_{\tau\tau}^{ m o}$
Isobaric heat capacity $c_p(T, p, x)$	$lpha^{ m r}_{\delta}, lpha^{ m r}_{\delta\delta}, lpha^{ m r}_{\tau\delta}, lpha^{ m r}_{\tau au}, lpha^{ m o}_{ au au}$
Excess enthalpy $h^{\rm E}(T, p, x)$	$\alpha^{\mathrm{r}}_{\delta}, \alpha^{\mathrm{r}}_{\tau}, \alpha^{\mathrm{o}}_{\tau}$

**Table 4.2:**Thermodynamic properties available as experimental data and the involved partial derivatives of the reduced Helmholtz free energy  $\alpha$ .

combinatory algorithm based on the work of Setzmann and Wagner [54]. This algorithm determines the optimal mathematical structure of the equation from a bank of terms. The bank of terms contains numerous terms that differ in temperature and density exponents and potentially the form of terms itself, e.g. polynomial or exponential. Only the coefficients are directly fitted to data. However, a precorrelation for the density is required since Setzmann and Wagner [54] used a linear optimization technique. Consuming significantly more time and resources, a non-linear method was applied in the last iteration only to refit the coefficients  $n_i$  non-linearly. The mixture model for natural gases (the GERG-2008 of Kunz and Wagner [80]) was developed with the optimization algorithm of Setzmann and Wagner [54]. For more details on this fitting method, see Wagner and Pruß [20] or Span [59].

The non-linear fitting algorithm that was used for the development of the mixture models in this work has been developed and was provided by Dr. Eric W. Lemmon at the National Institute of Standards and Technology (NIST) in Boulder, Colorado [55]. This algorithm has been used successfully for the development of many pure fluid equations as well as for the extensive mixture model EOS-CG of Gernert and Span [81].

By means of the non-linear algorithm, all thermophysical properties can directly be integrated in the fit in the original configuration of the experimental setup. Thus, the independent variables of the data sets do not need to be identical to those of the model. Another advantage of non-linear algorithms is the possibility to use flexible constraints based on inequalities instead of direct comparisons. With the help of such constrains, regions on the thermodynamic surface without experimental information can be forced to exhibit physically reasonable behavior. Additionally, the extrapolation behavior can be controlled to show reasonable trends.
There are numerous adjustable parameters simultaneously included in the fit: the parameters of the reducing functions (equations (3.113) and (3.114)), the coefficients of the departure function as well as the temperature exponents, and the Gaussian bellshaped term parameters (equation (3.116)). It is important to mention that there are no density exponents included in the fit. These parameters are exclusively manipulated by hand to ensure that they always have positive integer values in order to maintain physical correct extrapolation behavior of the equation in the zero-density limit. Technically, density exponents can be considered in the fit, but the mandatory final rounding often leads to worse solutions. The flexibility of the equation is thus limited and depends on the correlator's experience. The temperature exponents are restricted to positive values of a considerate magnitude below ten to guarantee reasonable extrapolation behavior.

The development of a binary mixture model is an iterative procedure during which the deviation between the current equation and the selected experimental data as well as the physical constraints have to be evaluated repeatedly. As a measure of quality, the sum of square SSQ is introduced as

$$SSQ = \sum_{i=1}^{I} W_{\rho,i} F_{\rho,i}^{2} + \sum_{j=1}^{J} W_{p,j} F_{p,j}^{2} + \sum_{k=1}^{K} W_{w,k} F_{w,k}^{2} + \cdots,$$
(4.1)

with W being the assigned weight of the considered data point or constraint. The relative error F of an arbitrary property z is defined as

$$F_z = \frac{z_{\rm exp} - z_{\rm EOS}}{z_{\rm exp}},\tag{4.2}$$

where  $z_{exp}$  is the experimental data point and  $z_{EOS}$  is the calculated value from the equation of state. The different contributions to the SSQ of the selected and weighted experimental data are listed in Table 4.3.

The fitting algorithm minimizes the SSQ by adjusting the parameters of the mixture model. It is important to keep in mind that the absolute value of the SSQ depends on the weights assigned to the data points. Ideally, each data point would be weighted according to its experimental uncertainty. Unfortunately, many publications either do not contain a discussion of the uncertainty at all or underestimate it significantly. Therefore, the correlator has to observe not only the SSQ but also the relative change

of its value during the fit. Another important way to evaluate the fitting progress is to observe deviation plots and various property diagrams, e.g. p over  $\rho$  or p over  $x_i$ .

**Table 4.3:** Available experimental mixture data used to fit the mixture model, and their contribution to the sum of squares.

Thermodynamic property	Contribution to SSQ
Density	$F_o^2 = \left(\frac{p_{\text{exp}} - p_{\text{EOS}}(T, \rho, \boldsymbol{x})}{\left(\frac{\partial p}{2}\right)^{-1}}\right)^2$
$\rho(p,T,\mathbf{x})$	$p \left( p_{\exp} \left( \partial \rho \right)_{T,x} \right)$
Pressure	$p_{\text{E2}} - \left(p_{\text{exp}} - p_{\text{EOS}}(T, \rho, \boldsymbol{x})\right)^2$
$p(\rho, T, \boldsymbol{x})$	$F_{p} = \left(\frac{p_{exp}}{p_{exp}}\right)$
VLE data <i>p,T,<b>x</b>',<b>x</b>'' given</i>	$F_{pTx'x''}^{2} = \sum_{i=1}^{2} \left( \frac{f_{i}'(p,T,x') - f_{i}''(p,T,x'')}{1 + f_{i}'(p,T,x')} \right)^{2}$
VLE data <i>p,T,<b>x</b>'</i> or <i>p,T,<b>x</b>''</i> given	$F_{pTx'}^{2} = \sum_{i=1}^{2} (x'_{i,\text{exp}} - x'_{i,\text{EOS}})^{2} + \left(\frac{p_{\text{exp}} - p_{\text{EOS}}(\rho', T, x')}{p_{\text{exp}}}\right)^{2}$ $F_{pTx''}^{2} = \sum_{i=1}^{2} (x''_{i,\text{exp}} - x''_{i,\text{EOS}})^{2} + \left(\frac{p_{\text{exp}} - p_{\text{EOS}}(\rho'', T, x'')}{p_{\text{exp}}}\right)^{2}$
Speed of sound $w(T, p, \mathbf{x})$	$F_{w}^{2} = \left(\frac{w_{\exp} - w_{EOS}(T, p, \boldsymbol{x})}{w_{\exp}}\right)^{2}$
Isobaric heat capacity $c_p(T, p, x)$	$F_{c_p}^2 = \left(\frac{c_{p,\text{exp}} - c_{p,\text{EOS}}(T, p, \boldsymbol{x})}{c_{p,\text{exp}}}\right)^2$
Excess enthalpy $h^{\rm E}(T, p, \mathbf{x})$	$F_{h^{\rm E}}^2 = \left(\frac{h^{\rm E}_{\rm exp} - h^{\rm E}_{\rm EOS}(T, p, \boldsymbol{x})}{h^{\rm E}_{\rm exp}}\right)^2$

The used non-linear fitting algorithm has the ability to adjust both coefficients and exponents of an equation of state at the same time. All types of thermodynamic data can be fit simultaneously without any previous linearization or pre-correlations. Additionally, the software allows for the use of a wide variety of constraints and control parameters. A detailed discussion of this minimization algorithm and its application on mixture models is given in Gernert [111].

Fitting ethanol-water mixtures is considerably challenging, since both components are complex fluids. Water shows a maximum in density in the liquid phase at  $T \approx 277$  K. This phenomenon affects all kinds of properties and consequently many absolute property plots, which differ in shape from those of simpler fluids. Additionally, Wagner and Pruß [20] utilized non-analytic terms in their pure fluid equation, which are known to be challenging in mixture models. The very high temperature and density exponents of this equation can be problematic especially in the description of low temperatures and

high densities. Ethanol is strongly self-associating and cross-associates with water. As a result, some of the excess properties of the mixture change drastically with temperature and composition.

In the course of the adjustment of the newly developed mixture model, several constraints were used to enforce a specific behavior. Once the model became stable, the final fit used only one constraint. The isotherm T = 800 K was constrained to have positive curvature with respect to  $(Z - 1)/\rho$  over density from vapor-like densities to liquid-like densities, as shown in Figure 4.2 in the following section.

## 4.3 The New Mixture Model

The new mixture model was developed with non-linear multiproperty fitting techniques. The model was fitted to selected experimental density data including phase equilibria, excess enthalpies, speeds of sound, and isobaric heat capacities. Additionally, constraints were added to control the qualitative behavior of the model.

Table 4.4 presents the parameters of the new reducing functions as described in equations (3.113) and (3.114).

$\beta_T$	$\gamma_T$	$\beta_{v}$	$\gamma_{v}$
0.9866	0.9971	1.0124	0.9558

Table 4.4: Parameters of the new reducing functions according to equations (3.113) and (3.114).

All four parameters are very close to unity. Table 4.5 gives the parameters of the newly fitted departure function, which contains polynomial, exponential, and Gaussian-bell shaped terms, as described in equation (3.116). All employed parameters show reasonable values.

i	<b>n</b> i	<b>t</b> i	<b>d</b> i	li	$\eta_i$	$\boldsymbol{\beta}_i$	$\varepsilon_i$	Υi
1	-0.2726	1.68	1	0				
2	0.027	0.73	4	0				
3	-0.01483	4.55	3	1				
4	1.773	1.17	2		0.585	0.19	1.11	1.08
5	6.9	0.15	1		0.510	2.12	1.64	0.75
6	-6.42	0.43	1		0.700	1.22	1.64	1.34

Table 4.5: Parameters of the new departure function according to equation (3.116).

For comparison the parameters of the preliminary mixture model of Lemmon are given in Table 4.6 and Table 4.7.

Table 4.6: Parameters of the reducing functions of Lemmon [4].

β <sub>T</sub>	$\gamma_T$	$\beta_v$	$\gamma_v$
0.971761	0.93996768	1.0624092	0.8687762

The model of Lemmon [4] contains a generalized departure function with  $F_{ij} = 0.3285109$  (see equation (3.109)). This departure function utilizes rather high temperature and density exponents and exclusively consists of exponential terms.

ni	ti	di	li
1.09765	0.26	2	1
1.94679	7.3	3	2
-2.16809	5.3	5	2
-0.137077	2.3	5	1
0.0486690	0.7	7	1
1.04024	3.3	6	2

Table 4.7: Parameters of the departure function of Lemmon [4].

Figure 4.2 illustrates  $c_v^r$  over temperature, PIP over temperature, and  $(Z - 1)/\rho$  over density for the pure fluid equations of state for ethanol [3] (a) and water [20] (b), and for the mixture model of Lemmon [4] (c) and the mixture model developed in this work (d). These properties are chosen, since they are often used to control the qualitative and extrapolation behavior of an equation of state as introduced by Gao *et al.* [112] as well as Thol and Lemmon [113]. It is important to consider the pure fluid equations of state incorporated in the development of a mixture model in order to not "correct" a special effect of a pure fluid equation that contributes to the mixture property.

For ethanol (a), there are several differences from the classical course of these graphs. The association occurring in the liquid phase enforces a maximum of the bubble line for  $c_v^r$  and the PIP at  $T \approx 400$  K. In the vicinity of the critical point, the bubble line of the ethanol equation shows a little spike which has no physical explanation. The  $(Z - 1)/\rho$  over density plot is shaped as expected [112]. For water (b), the  $c_v^r$  over temperature plot looks as expected as well. At the critical point, the non-analytic terms



**Figure 4.2:**  $c_v^r$  over temperature including the saturation lines as well as several isobars, PIP over temperature including the saturation lines as well as several isobars, and  $(Z - 1)/\rho$  over density including the saturation lines as well as several isotherms calculated from the pure fluid equations for ethanol [3] (a) and water [20] (b), and from the mixture model of Lemmon [4] (c) and the mixture model developed in this work (d) for  $x_{\text{Ethanol}} = 0.5$ .

cause the maximum to go to infinity as proposed by theory. The PIP over temperature plot of the water equation of state reveals a pole in the low temperature region. The isotherms at a lower temperature level in the  $(Z - 1)/\rho$  over density plot show a

negative slope in the gaseous region, which is unusual. Both effects are also found for the current heavy water equation of state of Herrig *et al.* [114], which does not use as extensively high temperature and density exponents as are used in the water equation of state of Wagner and Pruß [20]. Hence, these effects seem to be caused by the properties of these fluids themselves.

For the model of Lemmon [4] (c),  $c_v^r$  shows a positive slope and negative curvature with respect to increasing temperature at low temperatures. The pressure dependence of the PIP is inverted at  $T \approx 480$  K. The mixture model of Lemmon [4] includes very high temperature exponents that contribute heavily in the liquid region and might cause these effects. Similar to the water equation of state [20], both mixture models (c and d) show isotherms with negative slopes in the lower temperature region for  $(Z - 1)/\rho$  over density. The PIP and  $c_v^r$  over temperature plots calculated from the new mixture model present a reasonable behavior.

#### 4.4 Comparison to Experimental Data

Ideally, the new equation should be able to reproduce all experimental data within their uncertainty of measurement. To assess the representation with the aid of a statistical evaluation, the percentage deviation of a measuring point z of any thermodynamic property from the equation of state is calculated according to the following equation:

$$\Delta z = 100 \cdot \left(\frac{z_{\exp} - z_{EOS}}{z_{\exp}}\right). \tag{4.3}$$

For the comparison of complete data sets, the average absolute relative deviation (AAD) is calculated for each author and property individually:

$$AAD = \frac{1}{N} \sum_{i=1}^{N} |\Delta z_i| , \qquad (4.4)$$

*N* being the amount of experimental data points of the respective author of the respective tive thermodynamic property.

However, potential systematic deviations of the model or a data set cannot be detected by an exclusively statistical evaluation. In contrast, graphical comparisons of the new model with the experimental data can reveal these effects. Therefore, both methods are used in the following section. Depending on the property of interest, the plots illustrate deviations or absolute values.

#### 4.4.1 Phase Equilibria

Numerous experimental data sets have been published that investigated the phase equilibria of ethanol-water mixtures. Unfortunately, most of them do not include a comprehensive and reasonable discussion of the experimental uncertainty. Therefore, deviations are presented either based on the saturation temperature or saturation pressure. Subsequently, the AAD of all data sets are evaluated in Table 4.8 with respect to the saturation temperature and saturation pressure.

Figure 4.3 presents a *T*, *x*-diagram with experimental data and values calculated from the new mixture model and from the model of Lemmon [4] for p = 0.101325 MPa as well as relative deviations between these data and the new mixture model. Many data sets are available that agree well with each other as well as with the new mixture model. The representation of these data by the model of Lemmon [4] is nearly identical to the representation by the new model. The experimental data on the bubble and dew line is depicted within  $\pm 0.2\%$  with respect to the saturation temperature by the new model.

Figure 4.4 illustrates a p, x-diagram with experimental data and values calculated from both mixture model for p = 0.101325 MPa as well as relative deviations between these data and the new mixture model. As for the T, x-diagram at atmospheric pressure, there is no difference to be seen in the representation of these experimental data by the new model or the model of Lemmon [4]. The deviations between the data of Pemberton and Mash [115] and Kurihara *et al.* [116] with respect to the saturation pressure on the dew and bubble lines are lower than  $\pm 0.5\%$ .

Kurihara *et al.* [116] state individual uncertainties for their temperature, pressure, and composition measurements, as their technique involves the analysis of vapor and liquid samples. The combined uncertainty is estimated to be  $\pm 0.003$  with respect to the ethanol mole fraction in the vapor phase and  $\pm 0.0045$  in the liquid phase. Exemplary calculations prove that the new model represents these data within their estimated uncertainty ( $\pm 0.0021$  and  $\pm 0.0006$  at p = 0.023 MPa). The same analysis could be pursued for the analogous data of Pemberton and Mash [115], which are also represented within their estimated uncertainty.



**Figure 4.3:** Experimental vapor–liquid equilibrium data at p = 0.101325 MPa and calculated values from the new model, and from the model of Lemmon [4] (dashed curve). Bottom: Relative dew and bubble point temperature deviations between experimental data and the new mixture model over the ethanol fraction.

In Figure 4.5, experimental data describing the phase equilibrium at T = 523.15 K as well as calculated values from the compared models are presented. The new model follows the data more accurately than the model of Lemmon [4], which overestimates the saturation pressure. The new model represents the available data within  $\pm 2\%$  with respect to the saturation pressure. Barr-David and Dodge [117] published an extensive data set with high temperature phase equilibrium data. Their data at this isotherm agree well with the data published by other authors. However, the uncertainty of their



**Figure 4.4:** Experimental vapor–liquid equilibrium data at T = 323.15 K and calculated values from the new model, and from the model of Lemmon [4] (dashed curve). Bottom: Relative dew and bubble point pressure deviations between experimental data and the new mixture model over the ethanol fraction.

experimental technique is estimated to be of considerable magnitude. The analysis of the liquid and vapor samples was conducted by a determination of the respective liquid sample densities at T = 298.15 K and they do not further discuss the transformation to composition, which was based on a theoretical property model.

Figure 4.6 shows high temperature phase equilibria in a *p*-*x*-diagram for three isotherms. The absolute values are shown as well as relative deviations in the saturation pressure between the experimental data and the new model. Unfortunately, there are only few data sets available in this region which do not agree very well. The data of Barr-David and Dodge [117] were preferred in the fitting process, since they agree well with others in lower temperature regions. The model deviates from the data with -2%to +3%, which is also assumed to be the approximate uncertainty of the data. The deviations do not increase significantly with increasing temperature but with increasing



**Figure 4.5:** Experimental vapor–liquid equilibrium data at T = 523.15 K and calculated values from the new model, and from the model of Lemmon [4] (dashed curve). Bottom: Relative dew and bubble point pressure deviations between experimental data and the new mixture model over the ethanol fraction.

composition, which is caused by the tendency to prolong the phase boundaries of the new mixture model. The model of Lemmon [4] exhibits the same behavior. Due to slightly rotated saturation lines in the p, x diagram, the new model better represents the dew lines, whereas the model of Lemmon [4] agrees slightly better with the data on the bubble line.

The uncertainty of the new model is estimated to be  $\pm 0.5\%$  in saturation pressure for temperatures  $T \le 400$  K. From T > 400 K to  $T \le 550$  K, these deviations are assessed with  $\pm 2\%$  increasing to  $\pm 3\%$  for higher temperatures.



**Figure 4.6:** Experimental vapor–liquid equilibrium data at T = 573.15 K, T = 598.15 K, and T = 623.15 K, and calculated values from the new model, and from the model of Lemmon [4] (dashed curve). Bottom: Relative dew and bubble point pressure deviations between experimental data and the new mixture model over the ethanol fraction.

	Nr-			(T <sub>min</sub> - T <sub>max</sub> )	$(p_{\min} - p_{\max})$	AAD(	T <sub>s</sub> ) / %	AAD(p	9 <sub>s</sub> ) / %
Author	Pts	Pts-x	Pts-y	/ K	/ MPa	bub.	dew	bub.	dew
Abu Al-Rub et al. [118]	11	11	9	351.27 - 373.14	0.1	0.64	0.05	6.191	0.648
Aldrich and Querfeld [119]	10	10	0	351.53 - 365.73	0.1	1.427	0	17.773	0
Arce et al. [120]	23	23	0	351.42 - 373.15	0.1	0.307	0	2.999	0
Babinets et al. [121]	4	0	4	351.53 - 351.93	0.1	0	0.195	0	2.766
Balcazar-Ortiz et al. [122]	26	26	0	323.14 - 323.14	0.01 - 0.03	0.243	0	2.558	0
Barr-David and Dodge [117]	82	79	77	423.11 - 623.11	0.56 - 18.55	0.197	0.174	1.768	1.585
Beebe et al. [123]	70	70	70	307.74 - 358.43	0.01 - 0.1	0.522	0.468	8.814	7.924
Bloom <i>et al.</i> [124]	36	18	18	351.33 - 367.43	0.1	0.061	0.932	0.828	11.284
Carey and Lewis [125]	14	14	14	293.25 - 295.59	0.1	20.95	20.965	94.971	95.111
Carroll et al. [126]	11	11	0	351.33 - 373.12	0.1	0.618	0	5.901	0
Chaudhry et al. [127]	24	24	0	323.14	0.01 - 0.03	0.268	0	2.836	0
Dutta Choudhury [128]	12	12	12	323.14	0.01 - 0.03	0.162	0.177	2.632	2.896

Table 4.8: Absolute average deviation of the new mixture model from the experimental VLE data.

#### Table 4.8 continued.

	Nr-			(T <sub>min</sub> - T <sub>max</sub> )	$(p_{\min} - p_{\max})$	AAD(7	(r <sub>s</sub> )/%	AAD(p	0 <sub>s</sub> )/%
Author	Pts	Pts-x	Pts-y	/ K	/ MPa	bub.	dew	bub.	dew
Christensen [129]	6	6	0	322.05 - 372.25	0.01 - 0.1	0.039	0	0.548	0
Cristino et al. [130]	74	74	74	363.3 - 423.7	0.13 - 0.99	0.137	0.087	1.619	1.041
Dalager [131]	27	27	27	351.33 - 373.12	0.1	0.034	0.079	0.471	1.129
Dalmolin <i>et al.</i> [132]	28	0	20	288 - 323	0 - 0.03	0	1.26	0	13.307
D'Avila and Cotrim [133]	10	10	10	303.14 - 308.14	0.01 - 0.01	0.082	0.074	1.382	1.261
D'Avila and Silva [134]	25	25	0	283.15 - 303.14	0 - 0.01	0.106	0	1.978	0
Dobson [135]	12	12	10	298.14	0 - 0.01	0.495	0.038	5.737	0.587
Dornte [136]	22	22	20	298.14	0 - 0.01	0.482	0.518	7.075	8.771
Dulitskaya [137]	8	8	0	323.15 - 323.15	0.01 - 0.03	0.768	0	8.026	0
Garbarenko and Stabnikov [138]	114	114	0	273.15 - 323.15	0 - 0.03	0.357	0	3.899	0
Ghosh and Ghosal [139]	12	11	10	273.15 - 361.33	0 - 0.07	0.066	0.106	0.948	1.497
Gonzalez and Van Ness [140]	20	20	0	323.14 - 323.14	0.02 - 0.03	0.018	0	0.29	0
Griswold et al. [141]	18	17	18	423.11 - 548.11	0.74 - 10.29	0.353	0.401	3.163	3.722
Hall et al. [142]	10	10	10	298.14	0 - 0.01	0.047	0.034	0.846	0.607
Heitz [143]	4	4	4	351.13 - 366.93	0.1	0.18	0.209	2.418	2.846
Herraiz et al. [144]	9	9	9	313.15 - 313.15	0.01 - 0.02	0.021	0.016	0.347	0.269
Hong et al. [145]	30	30	30	372.52	0.1	0.116	0.119	1.532	1.576
Hrncirik [146]	17	17	0	351.3 - 373.15	0.1	0.428	0	4.221	0
Hughes and Maloney [147]	19	19	19	351.29 - 372.05	0.1	0.093	0.083	1.319	1.191
Iwakabe and Kosuge [148]	28	28	28	351.39 - 364.23	0.1	0.059	0.031	0.802	0.432
Johnson and Furter [149]	15	15	13	351.43 - 373.12	0.1	0.494	0.061	4.951	0.867
Jones <i>et al.</i> [150]	13	13	11	351.39 - 373.12	0.1	0.545	0.078	5.313	0.961
Kauer and Bittrich [151]	9	9	8	351.13 - 368.02	0.1	0.777	0.063	7.485	0.833
Kirschbaum and Gerstner [152]	84	84	84	308.15 - 370.85	0.01 - 0.1	0.07	0.061	1.083	0.922
Kojima and Kato [153]	170	170	0	319.97 - 437.49	0.03 - 0.69	0.266	0	2.586	0
Kojima et al. [154]	21	21	19	351.35 - 373.12	0.1	0.326	0.046	3.142	0.575
Kojima et al. [155]	21	21	0	351.33 - 373.12	0.1	0.326	0	3.127	0
Kolbe and Gmehling [156]	122	122	0	363.23 - 423.61	0.07 - 1	0.273	0	2.422	0
Kurihara <i>et al.</i> [157]	107	107	107	323.15 - 333.15	0.02 - 0.05	0.022	0.028	0.341	0.426
Kurihara <i>et al.</i> [116]	18	18	18	351.33 - 362.19	0.1	0.053	0.057	0.73	0.798
Linderstroem-Lang and Vaslow [158]	20	20	0	298.16	0 - 0.01	0.381	0	4.989	0
Liu et al. [159]	15	15	13	351.2 - 373.1	0.1	0.487	0.265	4.764	3.4
Lozovoi <i>et al.</i> l[160]	8	8	8	295.74 - 309.94	0.01	0.179	0.147	3.132	2.553
Macarron [161]	34	34	33	349.55 - 370.75	0.1	0.546	0.485	7.18	6.574
Manczinger and Tettamanti [162]	7	7	7	357.33 - 371.02	0.1	0.042	0.045	0.574	0.618
Mertl [163]	39	39	39	313.14 - 343.13	0.01 - 0.07	0.061	0.091	0.97	1.435
Niesen <i>et al.</i> [164]	21	19	19	423.11 - 523.11	0.74 - 7.12	0.121	0.073	1.168	0.723
Nikolskava [165]	23	23	18	298.14 - 348.13	0 - 0.09	0.567	0.078	6.479	1.055

#### Table 4.8 continued.

	Nr-			$(T_{\min} - T_{\max})$	$(p_{\min} - p_{\max})$	AAD(7	(r <sub>s</sub> )/%	AAD(p	9 <sub>s</sub> ) / %
Author	Pts	Pts-x	Pts-y	/ K	/ MPa	bub.	dew	bub.	dew
Novella and Tarraso [166]	17	17	17	351.33 - 368.32	0.1	0.109	0.098	1.494	1.363
Novitskiy et al. [167]	36	18	18	423.1 - 598.1	0.5 - 13.7	0.344	0.499	2.953	4.983
Noyes and Warfel [168]	60	60	0	351.32 - 372.8	0.1	0.159	0	1.715	0
Othmer <i>et al.</i> [169]	43	43	43	377.92 - 431.91	0.19 - 0.86	1.641	1.623	23.36	23.588
Otsuki and Williams [170]	127	127	127	351.33 - 481.61	0.1 - 2.07	0.165	0.146	2.23	2.051
Paul [171]	13	13	13	351.25 - 369.22	0.1	0.1	0.077	1.381	1.086
Pemberton and Mash [115]	95	95	95	303.14 - 363.13	0 - 0.16	0.037	0.072	0.568	0.808
Phutela et al. [172]	14	14	12	298.13	0 - 0.01	0.417	0.038	4.79	0.613
Protsyuk and Devyatko [173]	14	14	14	351.29 - 371.52	0.1	0.206	0.163	2.849	2.289
Rarey and Gmehling [174]	60	60	0	343.15	0.03 - 0.07	0.038	0	0.559	0
Rieder and Thompson [175]	36	36	34	351.43 - 373.12	0.1	0.238	0.05	2.483	0.667
Rius Miro <i>et al.</i> [176]	12	12	12	352.73 - 370.82	0.1	0.058	0.043	0.793	0.599
Safarov and Shakhverdiev [110]	21	20	0	373.15 - 523.15	0.14 - 8.96	0.794	0	8.117	0
Stabnikov et al. [177]	126	126	126	351.31 - 372.93	0.1	0.063	0.059	0.865	0.816
Stabnikov <i>et al.</i> [178]	27	27	27	334.68 - 371.42	0.05 - 0.1	0.064	0.063	0.902	0.885
Svoboda <i>et al.</i> [179]	12	12	12	350.83 - 369.82	0.1	0.091	0.08	1.247	1.112
Takiguchi <i>et al.</i> [180]	25	25	0	340 - 420	0.05 - 0.92	0.071	0	0.901	0
Tochigi <i>et al.</i> [181]	6	6	0	338.71 - 351.32	0.06 - 0.1	0.034	0	0.488	0
Tsiklis et al. [182]	6	6	6	573.11 - 573.11	8.83 - 13.14	0.611	0.783	4.338	5.992
Udovenko and Fatkulina [183]	36	36	30	313.14 - 333.13	0.01 - 0.05	0.553	0.18	5.94	2.387
Van Zandijcke and Verhoeye [184]	13	13	13	351.23 - 365.13	0.1	0.098	0.056	1.363	0.783
Vostrikova <i>et al.</i> [185]	8	8	8	351.33 - 351.53	0.1	0.047	0.047	0.653	0.651
Voustas et al. [186]	51	51	45	307.44 - 344.57	0.01 - 0.03	0.367	0.033	3.998	0.563
Vu <i>et al.</i> [187]	28	28	26	313.15	0.01 - 0.02	0.258	0.044	3.001	0.713
Wilson <i>et al.</i> [188]	17	17	0	323.14 - 323.14	0.01 - 0.03	0.362	0	3.764	0
Wormald and Vine [189]	14	14	7	423.2 - 533.2	0.84 - 7.65	2.999	0.15	21.365	22.551
Wrewsky [190]	70	70	39	312.9 - 347.92	0.01 - 0.09	0.324	0.2	3.662	3.198
Yamamoto <i>et al.</i> [191]	11	11	11	298.15 - 298.15	0 - 0.01	0.049	0.067	0.888	1.202
Yang and Wang [192]	20	20	20	351.6 - 372.97	0.1	0.075	0.097	1.041	1.331
Zielkiewicz and Konitz [193]	23	23	23	313.15 - 313.15	0.01 - 0.02	0.021	0.022	0.356	0.373

# 4.4.2 Densities

Except for two data sets, only liquid or liquid-like densities are available in the literature for ethanol-water mixtures. For those experimental data that were published as excess volumes, the pure fluid volumes were calculated with the equation of state for water of Wagner and Pruß [20] and the equation of state for ethanol of Schroeder *et al.*  [3], weighted with their molar fraction, and added to the excess value to obtain the total volume that was subsequently converted to a density.

In Figure 4.7, comparisons of densities calculated from the new mixture model to selected experimental liquid data up to T = 450 K are presented. The mixture model of Lemmon [4] is evaluated at p = 0.101325 MPa for T < 350 K and p = 5 MPa for T > 350 K, and plotted for comparison. The data sets agree very well. The mixture model of Lemmon [4] follows the data well in this temperature range.

The new model oscillates around the data showing systematical deviations. This oscillation could be reduced in the fitting process at the cost of an inferior representation of the VLE and speed-of-sound data. Therefore, a compromise had to be found. Up to T = 350 K, the data can be represented within  $\pm 0.5\%$ . For very high pressures only, the deviations increase to  $\pm 1\%$ .

A few selected data sets, which are considered to be the most accurate, are discussed in more detail. Zarei et al. [194] (Anton Paar oscillating u-tube) measured over a temperature range from T = 283.15 K to T = 313.15 K at atmospheric pressure and over the complete composition range. The combined uncertainty of their experimental setup is estimated to be within 0.05% in molar density, which seems rather optimistic regarding the use of a flexural resonator. The new model represents these data with a maximum deviation of ±0.5% and an AAD of 0.254%. Hervello and Sánchez [195] (An-Paar vibrating-tube) conducted their experiments from ton T = 283.15 K to T = 298.15 K also at atmospheric pressure over the complete composition range. The uncertainty of their data is estimated to be within 0.025% in molar density without including the uncertainty in composition, which is again considered to be an underestimation. The new mixture model depicts these data with a maximum deviation of  $\pm 0.46\%$  and an AAD of 0.21%. An extensive data set with pressures up to 40 MPa was published by Pečar and Doleček [196] (vibrating tube densimeter). Their data cover a range from T = 298.15 K to T = 348.15 K and over the complete composition range. The combined uncertainty has been determined to be 0.04% in molar density, which is considered to be underestimated regarding the experimental method used. The data are represented with a maximum deviation of  $\pm 0.5\%$  and an AAD of 0.16%.



**Figure 4.7:** Relative deviations of selected experimental liquid densities over the molar ethanol fraction from the new mixture model. Results from the model of Lemmon [4] (dashed curve) evaluated at p = 0.101325 MPa for T < 350 K and p = 5 MPa for T > 350 K are plotted for comparison.





The measurements of Tanaka *et al.* [197] (Adam piezometer), which were conducted over the complete composition range, included pressures up to p = 350 MPa. The temperature range went from T = 298 K to T = 323 K. The authors claim a combined uncertainty of 0.05% in molar density for their experimental setup. The new model deviates by a maximum of +0.4% to -0.95% from these data with an AAD of 0.23%.

One major improvement of the new mixture model over the model of Lemmon [4] is the representation of densities at high pressures. This feature is illustrated in Figure 4.8, where selected experimental data containing measurements conducted at higher pressures are shown. At the top, deviations of the model of Lemmon [4] from the experimental data are presented. With increasing pressure, the absolute deviations



increase to over 4%. Whereas at the bottom, the deviations of the new model never exceed  $\pm 1\%$ .

**Figure 4.8:** Relative deviations of selected experimental densities over pressure with 273 K < *T*< 450 K from the model of Lemmon [4] (top) and from the new mixture model. (bottom).

The experimental data at high temperatures with  $x_{\text{Ethanol}} = 0.5$ , which also cover the vapor phase, and their representation by the new mixture model are illustrated in Figure 4.9. The representation by the model of Lemmon [4] is given in Figure 4.10. Abdurashidova *et al.* [109] and Bazaev *et al.* [108] claim an uncertainty of 0.15% of their partly identical experimental data that have been conducted with a constant-volume piezometer. Since these data were measured along isochores, they are presented accordingly. The respective plots for the isopleths  $x_{\text{Ethanol}} = 0.2$  and  $x_{\text{Ethanol}} = 0.8$  are presented in Appendix C. Safarov and Shakhverdiev [110] estimate the uncertainty of their measurements to be within 0.15%. They also used a constant-volume piezometer for their investigations.



**Figure 4.9:** Relative deviations of the high temperature experimental densities along isochores over pressure from the new mixture model with  $x_{\text{Ethanol}} = 0.5$ .



#### Figure 4.9 continued

Figure 4.9 presents good agreement of the new model with the data in the homogeneous phase at these high temperatures within  $\pm 1\%$ . However, the deviations increase significantly in the vicinity of the phase boundary. For all isochores and  $T > (T_s + 50 \text{ K})$ , the new model deviates by  $\pm 5\%$  from the experimental data by up to  $\pm 10\%$  directly at the phase boundary. For densities with  $\rho > 18 \text{ mol/dm}^3$ , the deviations are  $\pm 3\%$  including the vicinity of the phase boundary.

The model of Lemmon [4] represents these data in a similar way. Nevertheless, the deviations in the homogeneous phases apart from the phase boundary, especially at high densities, are significantly larger with down to -5%.

The representation of the densities in the high temperature region was significantly improved by the new model and can be estimated with an uncertainty of  $\pm 2\%$  away from the phase boundary.



**Figure 4.10:** Relative deviations of the high temperature experimental densities along isochores over pressure from the model of Lemmon [4] with  $x_{\text{Ethanol}} = 0.5$ .



## Figure 4.10 continued

# Table 4.9 gives the average relative deviations of all experimental density data sets.

Table 4.9: Absolute average deviation of the new mixture model from	m the experimental	density data.
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Author	NrPts	( <i>T</i> <sub>min</sub> - <i>T</i> <sub>max</sub> ) / K	(p <sub>min</sub> - p <sub>max</sub> ) / MPa	X <sub>min</sub> - X <sub>max</sub>	AAD / %
Abdulagatov <i>et al.</i> [198]	124	292.95-448.15	2.45-40.14	0.0163-0.0946	0.245
Abdurashidova <i>et al.</i> [109]	87	523.15-673.15	5.38-50.36	0.2-0.8	35.197
Arce <i>et al.</i> [199]	28	298.15-298.15	0.1	0-1	0.263
Arce <i>et al.</i> [200]	21	298.15-298.15	0.1	0-1	0.253
Arce <i>et al.</i> [201]	7	298.15-298.15	0.1	0.2553-0.8988	0.268
Arce <i>et al.</i> [202]	8	298.15-298.15	0.1	0.2018-0.8959	0.241
Arce et al. [203]	15	298.15-298.15	0.1	0.0519-0.952	0.25
Atik [204]	13	298.15-298.15	0.1	0.1104-0.9468	12.608
Bai <i>et al.</i> [205]	28	298.15-298.15	0.1	0.0063-0.9732	0.206
Bazaev et al. [108]	290	456.15-673.15	4-51.76	0-0.8	2.58
Belda Maximino [206]	13	298.15-298.15	0.1	0-1	0.24
Belda <i>et al.</i> [207]	84	283.15-343.15	0.1	0-1	1.118
Benson and Kiyohara [208]	203	288.15-308.14	0.1	0.0229-0.9992	0.429
Bruun <i>et al.</i> [209]	14	298.14-298.14	0.1	0-1	0.181
Chauhdry and Lamb [210]	120	298.14-323.14	3-220.3	0.0226-0.862	0.209

#### Table 4.9 continued.

Author	NrPts	( <i>T</i> <sub>min</sub> - <i>T</i> <sub>max</sub> ) / K	(p <sub>min</sub> - p <sub>max</sub> ) / MPa	<i>x</i> <sub>min</sub> - <i>x</i> <sub>max</sub>	AAD / %
Dizechi and Marschall [211]	80	283.15-323.14	0.1	0-1	0.2
Dunstan and Thole [212]	21	293-303	0.1	0-0.9798	0.162
Ernst <i>et al.</i> [213]	11	298.14-298.14	0.1	0-1	0.216
Friedman <i>et al.</i> [214]	20	274-323	0.1	0.002-1	0.071
Galleguillos <i>et al.</i> [215]	24	298.15-313.15	0.1	0.0454-0.4909	0.173
Götze and Schneider [216]	28	273.16-348.13	0.1-250	0.5-0.5	0.183
Gonzalez et al. [217]	37	293.15-303.15	0.1	0-1	0.233
Grolier and Wilhelm [218]	20	298.14-298.14	0.1	0.0128-0.9373	0.243
Hafiz-ur-Rehman and Shahid An- sari [219]	108	268.15-323.15	0.1	0.1-0.9	0.25
Herraez and Belda [220]	13	298.15-298.15	0.1	0-1	0.232
Hervello and Sanchez [195]	164	283.15-298.15	0.1	0-1	0.213
Hurley et al. [221]	33	283.15-303.15	0.1	0-1	0.196
Hynčica et al. [222]	189	298.15-573.15	0.39-30.3	0.0016-0.0184	0.044
Ivanov [223]	70	278.15-318.15	0.1	0.9634-0.999	0.071
Kabir <i>et al.</i> [224]	45	303.15-323.15	0.1	0-1	1.061
Khattab <i>et al.</i> [225]	77	293-323	0.1	0-1	0.334
Kikuchi and Oikawa [226]	88	288.16-323.17	0.1	0-1	0.188
Konobeev and Lyapin [227]	24	293.14-333.13	0.1	0-1	0.133
Kubota <i>et al.</i> [228]	180	298.15-323.15	0.1-348.6	0.2-0.8	0.252
Kushare et al. [229]	12	298.15-298.15	0.1	0.9552-1	0.125
Marsh and Richards [230]	306	278.15-338.13	0.1	0.0002-0.9978	0.175
Mathews and Cooke [231]	5	273.15-343.15	0.1	0.2424-0.2424	0.934
Mori <i>et al.</i> [232]	45	298.15-338.15	0.1	0.0937-0.8979	0.273
Niazi <i>et al.</i> [233]	8	298.15-298.15	0.1	0-0.4771	0.263
Nikam and Hasan [234]	20	298.15-313.15	0.1	0.0329-0.3155	0.229
Noda <i>et al.</i> [235]	11	298.15-298.15	0.1	0-1	0.206
Ott et al. [236]	239	298.15-348.15	0.4-15	0-1	0.236
Pecar and Dolecek [196]	810	298.15-348.15	0.1-40	0.003-1	0.164
Pires et al. [237]	18	298.15-318.15	0.1	0.0949-0.9002	0.293
Romero and Rojas [238]	5	298.15-298.15	0.1	0-0.1435	0.142
Safarov and Shakhverdiev [110]	217	298.15-523.15	0.1-60	0.1153-0.5398	1.577
Spells [239]	11	283.15-283.15	0.1	0-1	0.193
Stahlberger and Guyer [240]	11	298-298	0.1	0-0.8533	0.418
Subnis et al. [241]	24	303-323	0.1	0-1	2.089
Takiguchi <i>et al.</i> [180]	753	320-420	0.1-200	0.1002-0.8904	0.35
Tanaka <i>et al.</i> [197]	322	298-323	0.1-348.64	0-1	0.225
Wang <i>et al.</i> [242]	18	303.15-328.15	0.1	0.5398-0.6891	0.199
Winnick and Kong [243]	9	323.14-323.14	0.1	0.1-0.9	0.249
Yang and Wang [244]	15	293.15-313.15	0.1	0.0416-0.2811	0.183
Zarei <i>et al.</i> [194]	84	283.15-313.15	0.1	0.0395-0.9195	0.254
Zhu [245]	8	303-303	0.1	0.1361-0.7589	0.635

#### 4.4.3 Excess Enthalpies

There are many data sets available for the excess enthalpy of ethanol-water mixtures that agree very well and ca be directly used in the fitting process. Figure 4.11 illustrates the absolute experimental values and their representation by the newly developed mixture model. The mixture model of Lemmon [4] is included for comparison.

Up to T = 323 K, the curvature of the excess enthalpy changes twice with increasing ethanol composition. At T = 273.15 K, the excess enthalpy shows a drastically steep minimum at  $x_{\text{Ethanol}} \approx 0.2$ , which reduces fast with increasing temperature. A zero crossing occurs for temperatures higher than T = 333 K. For high water content, the excess enthalpy stays negative, whereas its value becomes positive for higher ethanol fractions. For temperatures T > 380 K, the mixture behavior is much simpler without any zero crossings or changes in curvature over composition.

A few representative publications are introduced briefly. Zhao and Tremaine [246] measured at T = 298.15 K from p = 0.4 MPa to p = 10 MPa with an estimated uncertainty of 2%. Ott *et al.* [247] investigated the excess enthalpy from T = 323.14 K to T = 373.12 K and from p = 0.40 MPa to p = 15 MPa. The authors claim an uncertainty of 1% for their experimental data. Wormald and Lloyd [248] measured from T = 398 K to T = 548 K at p = 15 MPa and claim an uncertainty of 3% in the excess enthalpy.

Figure 4.11 shows that both models struggle with the representation of the extreme shape of the excess enthalpy at low temperatures and low ethanol fractions. But even for the less complicated isotherms, the deviations of the models from the experimental data are of a magnitude of  $\pm 10\%$ . The model of Lemmon [4] is slightly superior in the low temperature region, whereas the new model follows the experimental data at higher temperatures more accurately. During the fitting process, the shape of the new model could be forced to the shape of the experimental data at low temperatures, but again at the cost of a significantly worse representation of other properties.

To assess the impact of the excess enthalpy and its deviation, Table 4.10 lists the AADs of all collected publications with respect to the "total" enthalpy, that was calculated as the sum of the excess enthalpy and the pure fluid enthalpies calculated from the respective pure fluid equations of state [3,20]. A constant of h = 1000 J/mol was added to all enthalpies in order to avoid zero crossings that cause excessively high relative deviations.



Figure 4.11: Experimental excess enthalpy data of the binary mixture ethanol-water over the ethanol fraction. Results from the presented new model and from the model of Lemmon [4] (dashed curve) are plotted for comparison.



Figure 4.11 continued.



Figure 4.11 continued.

**Table 4.10:** Absolute average deviation of the new mixture model from the experimental excess enthalpy data converted to total enthalpies.

Author	NrPts	( <i>T</i> <sub>min</sub> - <i>T</i> <sub>max</sub> ) / K	(p <sub>min</sub> - p <sub>max</sub> ) / MPa	<i>x</i> <sub>min</sub> - <i>x</i> <sub>max</sub>	AAD / %
Belousov and Makarova [249]	96	273.15 - 348.12	0.101325	0.0023 - 0.972	0.681
Belousov and Panov [250]	9	298.14 - 298.14	0.101325	0.001 - 0.095	0.655
Bertrand et al. [251]	11	298.13 - 298.13	0.101325	0.0335 - 0.875	1.304
Boyne and Williamson [252]	17	298.13 - 298.13	0.101325	0.077 - 0.894	1.683
Brandt et al. [253]	56	333.15-383.15		0.0079 - 0.961	0.400
Chand and Fenby [254]	10	298.14 - 298.14	0.101325	0.0596 - 0.845	1.288
Christensen et al. [255]	41	382.97 - 412.96	1	0.0584 - 0.7994	0.220
Christensen and Izatt [256]	15	412.96 - 412.96	1	0.1092 - 0.8243	0.165
Costigan et al. [257]	100	298.14 - 298.14	0.101325	0.00015 - 0.962	1.177
Fang et al. [258]	30	423.2 - 523.2	5 - 15	0.0329 - 0.8534	0.139
Friese et al. [259]	19	298.15 - 298.15	0.101325	0.05 - 0.95	0.888
Kharin <i>et al.</i> [260]	58	273.15 - 333.15	0.101325	0.01 - 0.887	1.051
Lama and Lu [261]	19	298.13 - 298.13	0.101325	0.0132 - 0.959	0.775

Author	NrPts	( <i>T</i> <sub>min</sub> - <i>T</i> <sub>max</sub> ) / K	$(p_{\min} - p_{\max}) / MPa$	$x_{\min}$ - $x_{\max}$	AAD / %
Landgren <i>et al.</i> [262]	24	298.14 - 298.14	0.101325	0.005 - 0.95	0.941
Larkin [263]	83	298.14 - 383.12	0.101325	0.0206 - 0.945	0.477
Marongiu <i>et al.</i> [264]	21	298.15 - 298.15	0.101325	0.0201 - 0.928	0.921
Mathonat et al. [265]	104	348.15 - 523.15	5 - 20	0.0518 - 0.943	0.159
Nagamachi and Francesconi [266]	27	298.15 - 323.15	0.101325	0.033 - 0.968	0.881
Ott et al. [247]	179	323.13 - 373.12	0.4 - 15	0.01 - 0.9735	0.393
Ott et al. [267]	94	298.14 - 298.14	0.4 - 15	0.0249 - 0.948	1.157
Ott <i>et al.</i> [268]	152	398.11 - 473.11	5 - 15	0.0504 - 0.948	0.153
Priest <i>et al.</i> [269]	22	298.14 - 303.14	0.101325	0.147 - 0.926	1.039
Wormald and Lloyd [248]	131	398 - 548	15 - 15	0.1 - 0.9	0.130
Zel'tser [270]	19	293.13 - 293.13	0.101325	0.05 - 0.95	1.852
Zhao and Tremaine [246]	78	298.15 - 298.15	0.4 - 10	0.0126 - 0.945	1.333

#### Table 6.10 continued.

# 4.4.4 Isobaric Heat Capacities

Figure 4.12 presents comparisons of the isobaric heat capacity calculated from the new mixture model to experimental data. The mixture model of Lemmon [4] is plotted for comparison. For experimental data that were published as excess heat capacities, the heat capacity was calculated with the equation of state for water of Wagner and Pruß [20] and the equation of state for ethanol by Schroeder *et al.* [3], and added to the excess value to obtain the full mixture property.

The experimental data of Westh and Hvidt [271] at low temperatures were in the end not included in the fitting. The experimental data at T = 245.15 K is represented by the new model with deviations higher than 40%. With increasing temperature, the deviations decrease significantly. There were preliminary equations during the fitting process that could describe these data better, at the cost of a worse description of other properties. Therefore, the focus was kept on the description of properties with T > 273.15 K. Unfortunately, the publication of Westh and Hvidt [271] does not include a discussion of the experimental uncertainty.



**Figure 4.12:** Relative deviations of the liquid experimental isobaric heat capacity data at atmospheric pressure from the new mixture model. Results from the model of Lemmon [4] (dashed curve) are plotted for comparison.

The mixture model of Lemmon [4] represents these data in a superior manner. However, the superiority of the new ethanol equation of state of Schroeder *et al.* [3] over the outdated equation of state of Dillon *et al.* [21], which is applied in the mixture model of Lemmon [4], becomes obvious. At these low temperatures, the equation of Dillon *et al.* [21] deviates by up to 20% from the experimental data for pure ethanol ( $x_{\text{Ethanol}} = 1$ ), whereas the equation of Schroeder *et al.* [3] agrees very well. For temperatures T > 280 K, the new model shows a systematical deviation from the experimental data with a maximum at a composition of  $x_{Ethanol} \approx 0.1$ . The new mixture model represents the experimental data in this temperature region within  $\pm 5\%$ , which corresponds to the claimed uncertainty of the experimental data of Löwen *et al.* [272]. Table 4.11 presents the average absolute relative deviations of the new mixture model from the experimental data. Leaving out the data in the low temperature region with T < 280 K, the data sets are represented with AADs of  $\approx 2\%$ .

**Table 4.11:** Absolute average deviation of the new mixture model from the experimental liquid isobaric-heat capacity data.

Author	Nr of Pts	( <i>T</i> <sub>min</sub> - <i>T</i> <sub>max</sub> ) / K	<i>p /</i> MPa	X <sub>min</sub> - X <sub>max</sub>	AAD / %
Benson andD'Arcy [273]	98	288.15 - 308.14	0.1	0.0102 - 0.9498	2.088
Grolier and Wilhelm [218]	20	298.14 - 298.14	0.1	0.0189 - 0.9201	2.017
Löwen <i>et al.</i> [272]	8	298.16 - 298.16	0.1	0.056 - 0.746	2.18
Ogawa and Murakami [274]	22	298.14 - 298.14	0.1	0.025 - 0.95	1.483
Westh and Hvidt [271]	276	241.15 - 293.15	0.1	0.0218 - 1	11.394

# 4.4.5 Speed of Sound

There are only a few data sets with limited data points that published speed-of-sound data for ethanol-water mixtures. Figure 4.13 illustrates the course of the speed of sound of ethanol-water mixtures at T = 298.15 K and p = 0.1 MPa.

The new mixture model follows the data, whereas the model of Lemmon [4] deviates significantly and shows a different trend. In the beginning of the fitting process, the slope of the speed of sound at this temperature and pressure was constrained to follow the course of the experimental data. This constraint was no longer needed towards the end of the fitting process to ensure this behavior.

Figure 4.14 presents relative deviations of the experimental liquid speed-of-sound data at atmospheric pressure from the new mixture model. The mixture model of Lemmon [4] is again included for comparison. It deviates strongly from the experimental data with absolute deviations up to 25%. The new mixture model agrees well with the experimental data over the whole composition range.



**Figure 4.13:** Experimental speed-of-sound data and values calculated from the new mixture model and from the model of Lemmon [4] at T = 298.15 K and p = 0.1 MPa.



**Figure 4.14:** Relative deviations of the experimental liquid speed of sound data at atmospheric pressure from the new mixture model. Results from the model of Lemmon [4] (dashed curve) are plotted for comparison.

Arce *et al.* [202] claim only individual uncertainties for their experimental setup, which correspond to an approximated combined uncertainty of  $\pm 0.29\%$ . The remaining authors also claim an uncertainty of this magnitude, which cannot be maintained by the new model. Still, the new mixture model represents the experimental data within 4% and follows the data in a superior manner compared to the model of Lemmon [4]. Table 4.12 presents the average absolute relative deviations of the experimental data from the new mixture model. All AADs are below 2.5%.

**Table 4.12:** Absolute average deviation of the new mixture model from the experimental liquid speed-of-sounddata.

Author	Nr of Pts	( <i>T</i> <sub>min</sub> - <i>T</i> <sub>max</sub> ) / K	p / MPa	X <sub>min</sub> - X <sub>max</sub>	AAD / %
Arce <i>et al.</i> [202]	8	298.15 - 298.15	0.1	0.2018 - 0.8959	2.408
Bruun <i>et al.</i> [209]	14	298.14 - 298.14	0.1	0 - 1	1.524
García-Abuín <i>et al.</i> [275]	8	288.15 - 323.15	0.1	0.2336 - 0.2336	1.023
Gurung and Roy [276]	1	298.15 - 298.15	0.1	0.0416 - 0.0416	1.442
Kushare <i>et al.</i> [229]	12	298.15 - 298.15	0.1	0.9552 - 1	0.764
Larionov [277]	13	293.14 - 333.13	0.1	0.0506 - 1	1.84
Nikam and Hasan [234]	20	298.15 - 313.15	0.1	0.0329 - 0.3155	1.372

## 4.5 Summary

A new model for ethanol-water mixtures was presented, which is superior to the model of Lemmon [4] with respect to the pressure dependence of the density, and, consequently, the description of the speed of sound. The representation of vapor-liquid equilibria at high temperatures could be improved as well. For a further improvement, supplementary measurements investigating high-temperature phase equilibria, density and speed-of-sound measurements in the gaseous phase, and isobaric heat capacity measurements at elevated pressures are required.

However, both mixture models exhibit shortcomings in the representation of the strong mixing effects for low ethanol fractions at low temperatures (see Figure 4.11). Systematical deviations were found for the excess enthalpy as well as for the isobaric heat capacity. A new functional form for the reducing functions considering a stronger impact of the mole fraction is assumed to be beneficial. In 1996, Lemmon proposed a different form of equation in his doctoral thesis [75] where the composition was considered with a power in the temperature reducing function. Since the historical

development of these mixture models concentrated on the description of natural gases (GERG-2004 [79], GERG-2008 [80]) and combustion-like gases (EOS-CG [81]), which behave considerably different than the ethanol-water system, there was no need to enhance this approach. Nevertheless, for non-ideal mixtures such as the ethanol-water system, this extended corresponding states approach might improve the description of mixing effects in the liquid phase.

# 5 A New Generalized EOS for a PAO Lubricant and its Mixture with Ethanol

In this chapter, the development of a pseudo-pure fluid equation of state for the PAO lubricant employed in the proposed EHR process is presented. Based on the given theoretical background in sections 3.3 and 3.5, the fitting process is briefly discussed, and results are presented.

Applying the new pure fluid equation of state for PAO and the equation of state for ethanol by Schroeder *et al.* [3], a mixture model based on the Helmholtz energy as presented in section 3.6 was developed and results are shown.

# 5.1 Pseudo Pure Fluid Equation for PAO

A property model for the newly designed lubricant of the family PAO is needed. Unfortunately, there is only very limited experimental data available. Furthermore, the lubricant is not a pure fluid but a mixture of different PAO base oils and further additives. Since there was no information provided about the composition of the lubricant, and, more importantly, there were no experimental data of these components and their binary combinations available, it was impossible to model this lubricant as an actual mixture. Therefore, the PAO is assumed to behave like a pure fluid. The molar mass is given with M = 1773 g/mol.

# **Experimental data**

The company Fuchs Europe Schmierstoffe GmbH provided 13 density data points in a temperature interval from T = 293 K to T = 353 K. Furthermore, the isobaric heat capacity  $c_p$  was determined for 11 temperatures from T = 293 K to T = 393 K. Both data sets were measured at atmospheric pressure. [278]

Additionally, 15 experimental vapor pressures in the temperature range from T = 283 K to T = 423 K were provided by Fuchs Europe Schmierstoffe GmbH [279]. This data set was determined for a precursor lubricant in the development process. Since these lubricants are considered to be very similar, the dataset was applied to the final PAO as well. There was no information given about the uncertainty of any of the measurements.

Consequently, this limited data base is not sufficient to enable the development of a substance-specific Helmholtz energy equation of state as introduced in section 3.4. Therefore, a generalized approach is favored.

# 5.1.1 PAO: SRK

In a first approach, the SRK [34] equation was applied, which is widely accepted in industry. Section 3.3 contains a detailed description of this three-parameter corresponding states approach. Usually, the SRK is used predictively based on the critical temperature  $T_c$ , the critical pressure  $p_c$ , and the acentric factor  $\omega$ . Since all these properties are unknown in case of the PAO, they are used as adjustable parameters here.

The software Excel was used as a simple fitting environment. The software tool TREND [280] was embedded in a VBA macro to make the thermodynamic property routines available. The Excel solver was then used to minimize the SSQ, which was calculated from the deviation of the experimental data from the calculated densities and vapor pressures. In order to calculate isobaric heat capacities, an equation describing the ideal gas behavior is needed. Since the molecular structure is unknown, a group contribution method cannot be applied to estimate the ideal gas heat capacity. Therefore, the experimental isobaric heat capacity data is not considered in the adjustment. Table 5.1 presents the results of the fitting process of the corresponding states parameters.

Т <sub>с</sub> / К	p <sub>c</sub> / bar	ω/-
1821.5	6.45	-0.175

Table 5.1: Adjusted corresponding states parameters for the PAO described by the SRK [34].

A rather high critical temperature of  $T_c = 1821.5$  K was found. It is important to keep in mind, that this temperature is used as an adjustable reducing parameter and therefore loses its physical meaning as the critical temperature of the fluid. However, complex molecules like those this lubricant consists of have a limited thermal stability, which is lower than the true critical temperature. Additionally, the adjusted acentric factor  $\omega$  is unexpectedly small, but again, used as a freely adjustable parameter here.



**Figure 5.1:** Relative deviations of the experimental density data [278] from the adjusted SRK equation for the PAO. The representation of the experimental density data by the adjusted SRK equation can be seen in Figure 5.1. The equation shows linear deviations in temperature from the experimental data which leads to a deviation of +2% to -1.5%.

Figure 5.2 presents the relative deviation of the calculated vapor pressure from the experimental data. The deviations range from -30% to +50%.



**Figure 5.2:** Relative deviations of the experimental vapor pressure data [279] from the adjusted SRK equation for the PAO.

Evidently, the SRK is not suitable to model this PAO. Therefore, a generalized Helmholtz energy equation of state was chosen next to describe the experimental data.

# 5.1.2 PAO: Generalized Helmholtz Equation

The generalized equation of state proposed by Alexandrov *et al.* [63] was chosen due to its development especially for long-chained hydrocarbons. For a description of the underlying model, see section 3.5.

The corresponding state parameters of this model have been adjusted using the fitter provided by NIST [55], which was presented in section 4.2. Table 5.2 shows the results of the adjustment.

**Table 5.2:** Adjusted corresponding states parameters for the PAO described by the generalized equation of Alexandrov *et al.* [63].

Т <sub>с</sub> / К	$ ho_{ m c}$ / mol $\cdot$ m $^{-3}$	ω/-
1396.3	140.0	0.0207

The parameters according to equation (3.102) evaluated with the new adjusted corresponding states parameters can be taken from Appendix B.

The critical temperature found by the adjustment of this model is significantly lower than the one found for the SRK ( $T_{c,SRK} = 1821.5$  K). The acentric factor is again very small.

In order to use the isobaric heat capacity data in the adjustment, a  $c_p^{o}$ -equation has been simultaneously fitted using the nonlinear fitting technique provided by NIST [55]. Table 5.3 shows the adjusted parameters according to equation (3.66).

**Table 5.3:** Parameters of the adjusted  $c_p^0$ -equation for PAO with one polynomial and one Planck-Einstein term according to Eq. (3.66).

n <sub>i</sub> / -	$\boldsymbol{\theta}_i$ / K	Туре
414.79	-	Pol
878.39	2281.407	PE

Figure 5.3 presents the deviation of the model from the experimental density data. The experimental data can be represented within +0.15 % and -0.3%.


**Figure 5.3:** Relative deviations of the experimental density data [278] from the adjusted generalized Helmholtz equation for the PAO.

Figure 5.4 shows the relative deviation in terms of vapor pressure, which is significantly improved by this model. Now, the equation deviates from the experimental data within +4 % and -5%. Considering the very low vapor pressures of the PAO, this is a good representation. Absolute deviations are less than  $2.1 \cdot 10^{-8}$  MPa.



**Figure 5.4:** Relative deviations of the experimental vapor pressure data [279] from the adjusted generalized Helmholtz equation for the PAO.

Figure 5.5 illustrates the representation of the isobaric heat capacity data by the new generalized equation of state. The equation is in good agreement with the experimental data and deviates by only up to 2.4%.



**Figure 5.5:** Relative deviations of the experimental isobaric heat capacity data [278] from the adjusted generalized equation for the PAO

Table 5.4 summarizes average absolute relative deviations of the presented experimental data from the new generalized equation of state.

**Table 5.4:** Average absolute relative deviations (AAD / %) of the experimental data from the generalized equation of state for PAO based on the model of Alexandrov *et al.* [63].

Property	No. of data	Temperature range / K	Pressure / MPa	Average absolute relative deviations AAD / %
рρТ	13	293 - 354	0.1	0.1
$p_{ m v}$	15	283 - 424	$1.7 \cdot 10^{-11}$ - 6.6 $\cdot 10^{-7}$	2.6
$c_p$	11	293 - 394	0.1	1.8

Applying the generalized equation of Alexandrov *et al.* [63] was beneficial for the representation of the experimental data.

# 5.2 Mixture Model for Ethanol-PAO

For the development of the mixture model, the pure fluid equation for ethanol by Schroeder *et al.* [3] was used as well as the generalized equation presented in section 5.1.2.

#### 5.2.1 Experimental Data

There are no experimental data available in the literature describing this binary mixture. Therefore, it was decided by all project partners to investigate the densities, phase equilibrium, and rheological behavior in the framework of this research project. The measurements were carried out by the Chair of Process Technology of RUB [281] and are not published, yet. Since the rheological behavior is not relevant to this work, these data will not be discussed here. Only densities and phase equilibria are considered for the adjustment of the new model.

As desired, ethanol and the new PAO form an emulsion under ambient conditions. The long-term stability of an emulsion consisting of 20 vol.-% PAO was investigated and results are presented in Figure 5.6. One sample was emulsified by applying ultrasound, the other sample was shaken. Pictures were taken after 24 hours, and after 7 days. Already after 24 hours, there are significant differences between the samples. The lighter ethanol-rich phase on top loses its milky appearance in case of the shaken sample. Consequently, the ultrasound-emulsified sample is more stable than the sample that had been shaken. This becomes even more distinct after seven days.



**Figure 5.6:** Investigation of the long-term stability of a shaken PAO-Ethanol emulsion and an ultrasound-emulsified sample, both with 20 vol.-% PAO [282].

The light microscope pictures presented in Figure 5.7 show that the diameter of the oil droplets in the ethanol-rich phase is significantly decreased in the ultrasound-emulsi-fied sample. This characteristic leads to higher stability, which can be seen in Figure 5.6. Additionally, the light microscope pictures show that the oil-rich phase is an oil-ethanol-oil double emulsion, which means that the ethanol in the oil rich phase contains oil itself.

The emulsion of these two components is not a stable state, as was shown by the longterm stability investigation (Figure 5.6). It decomposed quickly into two phases, an ethanol-rich and an oil-rich phase. With time, the composition of the phases changed as the emulsion decomposed further. It is a challenge for the development of a mixture model to cope with this dynamic behavior, since by principle the model can only describe equilibrium states. In contrast to emulsions, liquid-liquid equilibria can be represented by this model.



**Figure 5.7:** Light microscope investigation of an ethanol-rich and an oil-rich phase of a shaken sample and an ultrasound-emulsified sample [282].

# Densities

Densities were measured in an autoclave mounted on a rocking rack with the help of a flexural resonator in form of a tuning fork. This experimental method determines a local density disregarding of a potential phase separation. For a homogeneous system, the overall density is measured, whereas for a heterogeneous system, only the density of one of the phases is determined. For details on this apparatus, see Brinkmann [283].



Figure 5.8: Relation between volumetric, mass, and mole fractions of the PAO in the mixture.

Only high volumetric concentrations of PAO have been measured in a temperature range from T = 310 K to T = 410 K. However, due to the very significant difference in the molar masses of the PAO ( $M_{PAO} = 1173$  g/mol) and ethanol ( $M_{ETO} = 46.07$  g/mol), the molar fraction deviates strongly from the respective volumetric or mass fractions. Figure 5.8 illustrates this effect and presents the compositions that have been investigated.

The samples were prepared under atmospheric conditions and filled in a sealed container, which was subsequently heated and shaken for 30 minutes. For higher temperatures, the samples were initially preheated before being sealed in the container. After shaking the sample for 30 minutes, the density was measured. Since the emulsion decomposes into two phases, and the composition of the phases change with the further decomposition of the emulsion, the measurement was repeated after 1, 2, 5, and 15 minutes to investigate the effect on the density and the simultaneously measured viscosity. In the case of two separated phases, the aim of the investigation was to measure the heavier oil-rich phase. The properties of the oil-rich phase were of special interest to the other project partners since this phase was to be used for the lubrication of the expander. For the adjustment of the new mixture model, only the initial measurement points were considered where the mixture was fully emulsified, and the measured density resembles the overall density of the emulsion.

Unfortunately, the pressure was not measured during the experiment. In order to be able to use this data, the pressure has been estimated to be slightly higher than the corresponding vapor pressure of the three-phase equilibrium, which means that a liquid-liquid equilibrium (LLE) is assumed for all measurement points. It is reasonable to assume that the effect of the error of the estimated pressure on the density is small, regarding the instability of the system. Evaluating the derivative of the density with respect to the pressure at constant temperature of the new model  $(\partial \rho / \partial p)_T$ , the error caused by an offset in the pressure estimation of for example 0.05 MPa can be approximated with 0.02% in molar density, which is reasonably small.

#### Phase equilibria

The measurements of the phase equilibrium were carried out in a high-pressure cell. The experimental setup has been described by Kukova *et al.* [284]. The samples were prepared and then filled in the initially evacuated measuring cell. The measuring points were determined by the overall composition of the sample and the employed temperature. Figure 5.9 and Figure 5.10 illustrate different equilibria of systems with 5, 10 and 20 vol.-% PAO.

Depending on the overall composition and temperature, a three-phase vapor-liquidliquid equilibrium (VLLE) could be observed, with an ethanol-rich vapor phase on top, an ethanol-rich liquid phase in the middle, and a PAO-rich liquid phase at the bottom.



Figure 5.9: Phase equilibria of samples of 5 vol.-% and 10 vol.-% PAO at different temperatures [281].

The volume of the heavy liquid phase decreases with rising temperature until only a vapor-liquid equilibrium is left.

20 vol.-% PAO



Figure 5.10: Phase equilibria of a sample of 20 vol.-% PAO at different temperatures [281].

Additionally, a relation between the overall composition and the phase fraction of the oil-rich phase can be observed. The more PAO is in the sample, the higher is the phase fraction of the PAO-rich phase.

During the experiment, a laser beam was used to examine the system for the occurrence of microemulsions. When a light beam passes through a colloidal system, the light is scattered at the microscopic particles or droplets and the beam is visible in the medium. This phenomenon is described by the Faraday-Tyndall effect [285]. Depending on the temperature and overall composition, a microemulsion could be determined in the ethanol-rich liquid phase, as can be seen in Figure 5.9 for T = 423 K and  $z_{PAO} = 10$  vol.-%. Figure 5.10 illustrates that this effect vanishes for T = 453 K and the mixture is in solution in all phases. The possible formation of a microemulsion has no influence on the interpretation of the experimental data with respect to the adjustment of an equation of state for the mixture. For more information about microemulsions see Kahlweit et *al.* [286].

When the ethanol-PAO system reached its equilibrium, the composition of the vapor phase, and of the lighter ethanol-rich liquid phase were determined for the three presented overall compositions at the presented temperatures. The composition of the oil-rich liquid phase could only be determined for the highest overall PAO composition of 20 vol.-% at T = 413 K. Otherwise, the liquid level of this phase was too low to allow for sampling, as can be seen in Figure 5.9 and Figure 5.10.

As already discussed, the PAO incorporated here consists of several base PAOs and an emulsifier. Therefore, the mixture of this oil with ethanol is a multi-component system. However, since the PAO is modeled as a pure fluid, the mixture with ethanol is binary by definition. Gibbs' phase rule indicates that for a mixture of N = 2 components with P = 3 phases in equilibrium, there is only F = 1 degree of freedom:

$$F = N - P + 2. (5.1)$$

Consequently, there is one three-phase line for a binary mixture, along which it decomposes into three phases. The compositions of the individual phases do not change with respect to a change in the overall composition of the mixture, given that the overall composition allows for the formation of three phases. However, the experimental results show a different behavior, which cannot be represented by the chosen binary model.

#### 5.2.2 The New Mixture Model for Ethanol-PAO

Based on the new generalized equation for PAO and the ethanol equation of state of Schroeder *et al.* [3], a new mixture model was developed. The software tool TREND [280] was embedded in a VBA macro to make the thermodynamic property routines available in Excel. The Excel solver was used to minimize the SSQ, which was calculated from the deviation of the experimental data from the respective properties calculated from the new model. TREND [280] was used in order to be able to calculate liquid-liquid and vapor-liquid-liquid equilibria. Due to the limited data base, only the reducing parameters determining the pseudocritical temperature and pseudocritical density of the mixture (see equations (3.113) and (3.114)) were adjusted. The final parameters can be taken from Table 5.5.

	β	γ
Tr	1.578	0.5212
$ ho_{ m r}$	1.571	0.9833

Table 5.5: Adjusted parameters of the reducing functions for ethanol-PAO mixtures.

Figure 5.11 presents relative deviations of densities calculated from the new mixture model for ethanol-PAO mixtures from the experimental values. As already discussed, the pressure was not measured and, therefore, estimated to be atmospheric or slightly above the corresponding vapor pressure. Thus, all calculated state points are in LLE, where the overall density is calculated as the sum of the phase densities weighed with the respective phase fraction as predicted by the new mixture model. As for the adjustment of the model, exclusively the initial measurements with the completely emulsified sample are considered for comparison. At temperatures from T = 310 K to T = 330 K, the deviations scatter and range from +6% to -3%. The new mixture model seems to overestimate the density by approximately 5% for temperatures above T = 350 K. The uncertainty in measurement of the density is claimed to be  $\pm 1 \text{ kg/m}^3$  [283] which corresponds to a maximum uncertainty of 0.14% in molar density with respect to this mixture. Nevertheless, the uncertainty in the composition of the mixture is not included. Since the boiling point of ethanol at atmospheric pressure is considerably low with  $T_{\rm h} \approx 350$  K, the preparation of the samples as conducted bears the risk of a significant evaporation of ethanol during the procedure at elevated temperatures. Assuming an overestimation of the ethanol fraction due to partial evaporation, the experimental

densities correspond to mixtures with less ethanol and are, therefore, lower than the "true" respective densities with respect to molar units. This effect might explain the change in the relative deviations of the experimental data from the new mixture model as discussed for temperatures above T = 350 K.



**Figure 5.11:** Relative deviations of the experimental densities [281] from the new mixture model for ethanol-PAO mixtures.

However, with respect to the challenge of modeling a multi-fluid mixture as a binary system, the new model is in good agreement with the experimental data with an overall average absolute relative deviation of AAD = 3.5 %.

Figure 5.12 presents the phase envelops of several overall compositions of ethanol-PAO mixtures, calculated from the new model with TREND [280]. The left line (dashed) represents the three-phase line, whereas the right line depicts the dew line.



**Figure 5.12:** Phase envelops of several ethanol-PAO systems in a *p*,*T*-diagram calculated from the new model with TREND [280].

As already discussed by means of the Gibbs phase rule, the location of the three-phase line does not depend on the overall composition. Left of this line, the mixture is in liquid-liquid equilibrium. At the three-phase line, the ethanol-rich vapor phase emerges, leading to the vapor-liquid region right of the three-phase line. Here, an ethanol-rich vapor phase is in equilibrium with an oil-rich liquid phase. Right of the dew line, the system is completely evaporated. Figure 5.12 also illustrates how the volumetric oil fraction influences the phase envelop. The vapor-liquid region expands with increasing overall oil fraction significantly. However, the corresponding mole fractions of the presented phase envelops vary only from  $x_{PAO} = 0.0014$  to  $x_{PAO} = 0.0604$  (see Figure 5.8).



**Figure 5.13:** Phase equilibria of ethanol-PAO mixtures in a *T*, *x*-diagram. (a): Vapor phase composition for all overall compositions. (b): Liquid phase (ethanol-rich) composition for all overall compositions.

For the binary model, the compositions of three coexisting phases in equilibrium do not vary with the overall mixture composition. Consequently, one of the three investigated ethanol-PAO systems had to be chosen for the adjustment. Since the system with  $z_{PAO} = 5$  vol.-% contains least of the pseudo-pure fluid PAO, these experimental values are considered the most suitable for the adjustment of the model and therefore the most suitable for comparison here. Figure 5.13 presents the compositions of the ethanol-rich vapor and ethanol-rich liquid phase found by the experiment for the three overall compositions as well as the compositions calculated from the new model for these two phases. The new model predicts a three-phase equilibrium in the complete temperature range presented. The model underestimates the oil fraction in the ethanol-rich vapor and liquid phase significantly. Given that the PAO contains an emulsifier to increase the solubility and induce an emulsion, this was to be expected. The phase equilibrium of an emulsion, as presented here, is not a stable state in a thermodynamic sense. Therefore, it cannot be represented by the chosen model. As already discussed, there is only one experimental point describing the composition of the oil-rich phase, which was determined at T = 414.2 K for an overall composition of  $z_{PAO} = 20$  vol.-% ( $\approx z_{PAO} = 0.68$  mol-%) to be  $x_{PAO} = 0.047$ . The value calculated by the new model deviates with  $x_{PAO} = 0.911$  strongly.

During the fitting process, it became evident, that the model cannot reproduce the compositions of the coexisting phases of the three-phase equilibrium ( $z_{PAO} = 5 \text{ vol.-\%}$ ) satisfactorily. Therefore, the heavy PAO-rich liquid phase was neglected and the ethanolrich liquid phase used as input for a simple dew point calculation. In this manner, all three overall compositions could be considered for the adjustment. Figure 5.14 presents three *T*, *x*-diagrams containing the experimental data and the vapor phase compositions calculated with the new model using the liquid phase composition as input for a dew point calculation. The calculated vapor phase composition is in good agreement with the experimental data for all three overall compositions.

The representation of the VLLE did not change significantly with regard to the different fitting method of interpreting the experimental data as simple vapor-liquid equilibria.





Figure 5.15 illustrates the relative deviation of the new model from the experimentally found vapor pressures. The model is in good agreement with the data and deviates from -1% to +2.3%.



**Figure 5.15:** Relative deviations of the experimental vapor pressures from the new mixture model for ethanol-PAO.

#### 5.2.3 Summary

A new model describing ethanol-PAO mixtures was found based on the new generalized equation of state for the PAO. The Helmholtz mixture approach was used for the description of this strongly asymmetric mixture. The new model reproduces the experimental densities satisfactorily. However, only the overall density of the emulsion or LLE could be compared because there were no experimental data available that describe the densities and compositions of the two coexisting liquid phases in equilibrium. The investigation of the long-term stability of the emulsion displayed in Figure 5.6 showed that the decomposition of the emulsion is a rather time-consuming process. Since the emulsifier increases the mutual solubility of ethanol and PAO significantly, it is assumed that the phase compositions predicted by the binary model would deviate rather strongly from the composition of the two stable phases of the fully decomposed emulsion. This theory is supported by the observation of the phase equilibrium calculations. When the model is used to freely predict the phases that a mixture of a given overall composition decomposes to, it finds three coexisting phases as found by experiment. However, the high solubility of both components due to the emulsifier cannot be represented. On the other hand, if the composition of the dominating ethanol-rich liquid phase is defined and the coexisting vapor phase is calculated, the model gives reliable results, as shown for the dew point calculations in Figure 5.14.

# 6 New Generalized EOS for two POE Lubricants and their Mixtures with CO<sub>2</sub>

In this chapter, two generalized Helmholtz equations of state for two base stock POEs, PEC5 and PEC7, are presented. These equations were adjusted to the experimental data available in the literature, which comprise only densities and vapor pressures. They are compared to fluid-specific equations of state, which were developed by Lemmon and Eckermann and which are still to be published [23].

In order to develop the fluid-specific equations of state, experimental measurements were carried out at NIST including densities, vapor pressures, isobaric heat capacities, and speeds of sound. To evaluate the predictive representation of caloric properties by the generalized equations, which were exclusively adjusted to thermal property data, the new caloric data were not used for the adjustment of the generalized equations presented here. The thermal data measured at NIST was not yet available at the time of the development of the generalized equations.

Subsequently, a mixture model for PEC5-CO<sub>2</sub> was developed and transferred to the system PEC7-CO<sub>2</sub>. The results are presented in sections 6.3 and 6.4.

# 6.1 Pure Fluid Equation for PEC5

# Experimental data used for the adjustment

There are four publications in the literature dealing with the  $p\rho T$  relationship of PEC5. Fandiño *et al.* [287], Fedele *et al.* [288], Wahlström and Vamling [289], and Shobha and Kishore [290] measured the homogeneous liquid density of this substance. Razzouk *et al.* [291] investigated vapor pressures of PEC5 and PEC7.

In addition to the vapor pressure data, Razzouk *et al.* [291] published PC-SAFT parameters, that were adjusted to their data set and are used for comparison here. The parameters are given in Table 6.1.

m	σ / Å	$\epsilon \cdot k_{ m B}^{-1}$ / K
10.5329	3.9721	261.81

### 6.1.1 The New Generalized Equation of State

The new generalized equation of state for PEC5 has been developed with the help of the fitter provided by NIST [4]. It is based on the generalized equation of Alexandrov *et al.* [63] presented in section 3.5 and uses the critical temperature  $T_c$ , the critical density  $\rho_c$ , and the acentric factor  $\omega$  as adjustable parameters. Table 6.2 presents the results of the adjustment.

**Table 6.2:** The corresponding states parameters for PEC5 adjusted in this work based on the generalized equation of Alexandrov *et al.* [63] and the corresponding states parameters found by Lemmon and Eckermann [23].

	This work	Lemmon and Eckermann [23]
<i>Т</i> <sub>с</sub> / К	875.19	890.00
$ ho_{\rm c}$ / mol $\cdot$ m <sup>-3</sup>	549.69	556.00
ω/-	0.98689	0.89

The critical parameters found in this work are compared to the critical parameters used in the equation of Lemmon and Eckermann [23]. The adjusted acentric factor is compared to the value calculated from the equation of Lemmon and Eckermann [23]. The agreement with the critical parameters and the acentric factor found by Lemmon and Eckermann [23] validates their order of magnitude. However, PEC5 molecules are thermally stable only up to  $T \approx 600$  K. For higher temperatures, the molecules begin to decompose significantly [292]. Therefore, these parameters are treated as mathematically adjustable parameters and their meaning is basically reduced to their function as reducing parameters in the respective equations.



**Figure 6.1:** *p*,  $\rho$ -diagram of PEC5 with the isotherm *T* = 793 K calculated from the new generalized equation of state with *T*<sub>c</sub> = 875.19 K.

Figure 6.1 shows a p,  $\rho$ -diagram of PEC5 with the isotherm T = 793 K calculated from the new generalized equation of state. This isotherm illustrates an unfavorable course and placement of the outer minimum. With increasing temperature, the outer extrema no longer overlap with respect to pressure. As a result, the flash calculation fails finding a vapor pressure by combining the outer extrema, leading to a discontinuity in the saturated vapor line at this temperature. This effect was also found for other fluids with comparable acentric factors as shown and discussed in section 3.5. However, this issue is not crucial for the description of the pure fluid PEC5, since the molecules already decompose in this temperature region [292]. Nevertheless, it might affect the application of the equation in a mixture model.

Since there was no data that could have been used for the adjustment of a  $c_p^{o}$ -equation, the isobaric heat capacity of the ideal gas has been estimated with the group contribution method by Joback and Reid [293]:

$$\frac{c_p^0}{J \cdot \text{mol}^{-1} \cdot \text{K}^{-1}} = \sum_{i=0}^3 n_i \cdot T^{t_i} = \left(\sum \Delta_A - 37.93\right) + \left(\sum \Delta_B - 0.21\right) \cdot \binom{T}{K} + \left(\sum \Delta_C - 3.91 \cdot 10^{-4}\right) \cdot \left(\frac{T}{K}\right)^2 + \left(\sum \Delta_D - 2.06 \cdot 10^{-7}\right) \cdot \left(\frac{T}{K}\right)^3.$$
(6.1)

The coefficients of this classical polynomial form are determined by the structure of the molecule, which is presented in Figure 6.2. The individual contributions of the functional groups to the constants  $\Delta_A$ ,  $\Delta_B$ ,  $\Delta_C$ , and  $\Delta_D$  can be taken from Joback and Reid [293]. The resulting coefficients for the  $c_p^{o}$ -equation (3.66) are listed in Table 6.3.



**Table 6.3:** Parameters in form of the  $c_p^{\text{o}}$ -equation (3.66) determined by the group contribution method of Joback and Reid [293] for PEC5, reduced with  $R = 8.3144598 \, \text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ .

$n_i$	t,
10.47885	0
0 257368	1
$-118973, 10^{-4}$	2
-1.16975.10	2
$1.236881 \cdot 10^{-8}$	3

 $\mathbf{R} = \mathbf{C}\mathbf{H}_2 - \mathbf{C}\mathbf{H}_2 - \mathbf{C}\mathbf{H}_2 - \mathbf{C}\mathbf{H}_3$ 

Figure 6.2: Chemical structure of a PEC5 molecule.

# 6.1.2 Comparison to $p\rho T$ Data

Figure 6.3 presents relative deviations of the experimental density data from the new generalized equation of state. The deviations from the fluid-specific equation of state of Lemmon and Eckermann [23] and from the PC-SAFT equation of state [91,291] are also included in all figures of this section. The experimental data points in red were exclusively used for the adjustment of the fluid-specific equation of state [23]. The data sets of Fandiño et al. [287], and Fedele et al. [288] deviate from each other by approximately 0.5% in the overlapping temperature range. Since the data of Fortin [294] and Outcalt [295] were not yet available, the data of Fandiño et al. [287] were preferred in the fitting process of the generalized equation due to the lower estimated uncertainty of 0.14 kg/m<sup>3</sup> compared to the uncertainty of the data set of Fedele *et al.* [288], which is estimated to be  $0.6 \text{ kg/m}^3$ . Nevertheless, the data of these authors can be reproduced within 0.5% by the new generalized equation of state. The data of Fandiño *et al.* [287] are represented with an AAD of 0.14%, the data of Fandiño et al. [287] with an AAD of 0.35%. The generalized equation reproduces the experimental data of Outcalt [295], which agree very well with the data of Fedele *et al.* [288], with -0.5% to +0% in the whole temperature and pressure range (AAD = 0.29%). Only for temperatures lower than T = 290 K, the deviations enlarge to -0.7%.

The deviation of the PC-SAFT equation of state [91,291] increases rapidly with increasing pressure, always overestimating the density. The density predicted by Lemmon and Eckermann [23] is lower than the predicted density of the new generalized equation of state in the whole temperature and pressure range presented and follows the data of Outcalt [295] and Fedele *et al.* [288] accurately with an AAD of 0.061% and 0.078%, respectively.

The representation of the ambient pressure data of Wahlström and Vamling [289], Shobha and Kishore [290], and Fortin [294] is illustrated in Figure 6.4. The new generalized equation of state deviates by -0.5% to +1% from the experimental data of Wahlström and Vamling [289] (AAD = 0.18%), and Shobha and Kishore [290] (AAD = 0.39%). The equation of Lemmon and Eckermann [23] deviates by up to -0.5% from the new generalized equation following the data of Fortin [294] accurately (AAD = 0.049%). The PC-SAFT equation of state [91,291] is in agreement with the new generalized equation exhibiting only a small constant offset from the new equation.



**Figure 6.3:** Relative deviations of experimental liquid densities from the new generalized equation of state for PEC5. Deviations from the PC-SAFT equation of state [91,291] (dotted curve), and from the equation of Lemmon and Eckermann [23] (dashed curve) are plotted for comparison.



**Figure 6.4:** Relative deviations of experimental liquid densities at ambient pressure from the new generalized equation of state for PEC5. Deviations from the PC-SAFT equation of state [91,291] (dotted curve), and from the equation of Lemmon and Eckermann [23] (dashed curve) are plotted for comparison.

Table 6.4 summarizes the average absolute relative deviations of the three compared equations of state from the experimental density data. The fluid-specific equation of

state has the lowest AADs except for the data sets of Fandiño *et al.* [287], Wahlström and Vamling [289], and Shobha and Kishore [290], which do not agree accurately with the data of Fortin [294] and Outcalt [295], which were measured at NIST and used for the adjustment of this equation [23]. These data sets are represented best by the new generalized equation of state. The PC-SAFT equation of state [91,291] shows the highest AADs.

**Table 6.4:** Average absolute relative deviations of the experimental densities from the new generalized equation of state for PEC5, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon and Eckermann [23].

Author	No. of data	Temperature range / K	Pressure range / MPa	Av relative	verage absol e deviations	ute AAD / %
				This work	PC-SAFT [91,291]	Lemmon and Ecker- mann [23]
Fandiño <i>et al.</i> [287]	99	278 - 354	0.1 - 46	0.14	1.2	0.45
Fedele <i>et al.</i> [288]	40	283 - 344	0.1 - 35	0.35	1.0	0.078
Wahlström and Vamling [289]	13	303 - 364	0.101325	0.18	0.28	0.24
Shobha and Kishore [290]	12	303 - 414	0.101325	0.39	0.28	0.74
Fortin [294]	14	278 - 344	0.083	0.47	0.57	0.049
Outcalt [295]	165	269 - 471	<0.1 - 51	0.29	1.4	0.061

#### 6.1.3 Comparison to Vapor Pressure Data

There is only one data set describing the vapor pressures of PEC5 published in the literature so far. Razzouk *et al.* [291] measured five state points in a temperature range from T = 334 K to T = 414 K. Relative deviations of these experimental data points and the data measured at NIST by Widegren [296] from the new generalized equation of state are illustrated in Figure 6.5. The new generalized equation of state reproduces the data of Razzouk *et al.* [291] within -8% to +3% (AAD = 3.5\%). The model of Lemmon and Eckermann [23] shows a different course over the temperature by following the data of Widegren [296], which it was adjusted to. The PC-SAFT parameters used here were adjusted to the experimental data of Razzouk *et al.* [291]. Therefore, the PC-SAFT equation of state [91,291] reproduces these data with deviations similar to those observed for the new generalized equation of state (AAD = 3.1%). The new generalized equation of state has an offset of approximately -30% from the experimental data of Widegren [296], since these data differ significantly from the vapor pressures investigated by Razzouk *et al.* [291] that were used for the development of the new equation. Despite this discrepancy, both data sets agree with respect to the estimated uncertainty

of both measurements acknowledging the effect of very small absolute values of the vapor pressure ( $2.55 \cdot 10^{-10}$  MPa to  $9.35 \cdot 10^{-7}$  MPa). Razzouk *et al.* [291] claim an uncertainty of ±4%, whereas Widegren [296] proposes ±24%.



**Figure 6.5:** Relative deviations of the experimental vapor pressure data used for the development of the present generalized equation of state for PEC5 from the new equation of state. Results from the PC-SAFT equation of state [91,291] (dotted curve), and from the equation of Lemmon and Eckermann [23] (dashed curve) are plotted for comparison.

Table 6.5 presents the average absolute relative deviations of the experimental vapor pressures from the new generalized equation of state for PEC5, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon and Eckermann [23].

<b>Γable 6.5:</b> Average absolute relative deviations of the experimental vapor pressures used for the development of
the present generalized equation of state for PEC5 from the present generalized equation of state, from the PC-SAFT
equation of state [91,291], and from the equation of Lemmon and Eckermann [23].

Author	No. of data	Temperature range / K	Average absolute relative deviations AAD / %			
			This work	PC-SAFT [91,291]	Lemmon and Eckermann [23]	
Razzouk <i>et al.</i> [291]	5	334 - 414	3.5	3.1	5.3	
Widegren [296]	5	353 - 363	29	25	19	

In Figure 6.6, the vapor pressure curves calculated from the three equations of state as well as both experimental data sets for the vapor pressure are presented in a p, T-diagram. For temperatures above T = 370 K, all models agree rather well with respect to the very low vapor pressures of PEC5. For lower temperatures, the course of the curve calculated from the equation of Lemmon and Eckermann [23] deviates from the others

following the data of Widegren [296]. It predicts lower vapor pressures than the new generalized equation of state and the PC-SAFT equation of state [91,291].



**Figure 6.6:** *p*, *T*-diagram of PEC5 with the vapor pressure curve calculated from the new generalized equation of state, from the PC-SAFT equation of state [91,291] (dotted curve), and from the equation of Lemmon and Eckermann [23] (dashed curve).

# 6.1.4 Comparison to Caloric Data

There were no experimental data sets containing caloric properties published in the literature. Therefore, no such data was used for the adjustment of the generalized equation of state. However, the investigation at NIST included isobaric heat-capacity measurements in the liquid phase, which were conducted with a commercial differential scanning calorimeter, as well as speed-of-sound measurements in the liquid phase, which were conducted with a commercial differential scanning calorimeter, as well as speed-of-sound measurements in the liquid phase, which were conducted with a dual-path, pulse-echo-type instrument. These data sets have not yet been published, but were used for the development of the fluid-specific equation of state of Lemmon and Eckermann [23]. In order to analyze the ability of the generalized equation of state to predict caloric properties, the group contribution method of Joback and Reid [293] has been used to provide a  $c_p^{o}$ -equation, which is required for such calculations. For comparison, the PC-SAFT equation of state [91,291] is also combined with this  $c_p^{o}$ -equation.



**Figure 6.7:** Relative deviations of the experimental liquid isobaric heat capacities at p = 0.083 MPa used for the development of the fluid-specific equation of state for PEC5 [23] from the new generalized equation of state. Results from the PC-SAFT equation of state [91,291] (dotted curve), and from the equation of Lemmon and Eckermann [23] (dashed curve) are plotted for comparison.

Relative deviations of the experimental liquid isobaric heat capacities at p = 0.083 MPa used for the development of the fluid-specific equation of state for PEC5 [23] from the new generalized equation of state, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon and Eckermann [23] are presented in Figure 6.7. The new generalized equation of state is able to predict the experimental data within -1% to +1.8%.

**Table 6.6:** Average absolute relative deviations of the experimental isobaric heat capacity data used for the development of the fluid-specific Helmholtz equation of state for PEC5 of Lemmon and Eckermann [23] from the present generalized equation of state, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon and Eckermann [23].

Author	No. of data	Temperature range / K	Pressure range / MPa	Aver de	age absolute r eviations AAD	relative / %
				This work	PC-SAFT [91,291]	Lemmon and Eckermann [23]
Fortin [294]	170	263 - 433	0.083	0.79	1.4	0.096

Table 6.6 lists the AAD of the new equation as 0.79%. This is a remarkably good agreement considering the application of a group contribution  $c_p^{o}$ -equation and the missing adjustment to any caloric property data. The PC-SAFT equation of state [91,291] reproduces the data within +1% to -3% (AAD = 1.4%). The fluid-specific Helmholtz equation of state by Lemmon and Eckermann [23] represents the data with an AAD of 0.096%.



**Figure 6.8:** Relative deviations of the experimental liquid speed-of-sound data used for the development of the fluid-specific equation of state for PEC5 [23] from the new generalized equation of state. Results from the equation of Lemmon and Eckermann [23] (dashed curve) are plotted for comparison.



Figure 6.8 continued.

**Table 6.7:** Average absolute relative deviations of the experimental speed-of-sound data used for the development of the fluid-specific Helmholtz equation of state for PEC5 of Lemmon and Eckermann [23] from the present generalized equation of state, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon and Eckermann [23].

Author	No. of data	Temperature range / K	Pressure range / MPa	Average ab	solute relative AAD / %	e deviations
				This work	PC-SAFT [91,291]	Lemmon and Eckermann [23]
Fortin [294]	14	278 - 344	0.083	5.1	25	0.027
Perkins [297]	66	283 - 424	0.0 - 63	3.1	23	0.050

In Figure 6.8, the experimental speed-of-sound data used for the development of the fluid-specific equation of state [23] and their reproduction by the fluid-specific and generalized equation of state are shown. The PC-SAFT equation of state [91,291] is not included here, since the relative deviations of this model lie outside the plotted range. The deviations of the new generalized equation are almost independent of

temperature. They decrease with increasing pressure, starting from -5% at atmospheric pressures, ending at -1% for p = 60 MPa in the complete temperature range. The equation of Lemmon and Eckermann [23] follows the data in all presented regions. Table 6.7 lists the average absolute relative deviations of the experimental speed-of-sound data from the three equations of state. The AAD of the PC-SAFT equation of state [91,291] is 23% with respect to the experimental speed-of-sound data of Perkins [297]. Significantly higher deviations compared to the generalized equation were expected due to the unfavorable pressure dependency of the model which could already be observed in the representation of densities at increasing pressures (Figure 6.3). As expected, all thermodynamic properties are best described by the fluid-specific Helmholtz equation of state, which has numerous fluid-specific adjustable parameters. In contrast to the PC-SAFT equation of state [91,291], which is based on a three-parameter corresponding states approach as well, the new generalized equation of state shows a correct pressure dependence of densities in the liquid. Thereby, the new equation satisfactorily predicts liquid heat capacities and liquid speeds of sound more than. All AADs of the new equation are within the range stated by Alexandrov et al. [63] (see section 3.5).

# 6.2 Pure Fluid Equation for PEC7

#### Experimental data used for the adjustment

There are three publications in the literature dealing with the  $p\rho T$  relationship of PEC7. Fandiño *et al.* [18], Fedele *et al.* [288], as well as Shobha and Kishore [290] measured the liquid density of this substance. Razzouk *et al.* [291] investigated vapor pressures of PEC5 and PEC7. There are PC-SAFT parameters published by Razzouk *et al.* [291], which are used for comparison.

т	σ / Å	$\epsilon \cdot k_{ m B}^{-1}$ / K
12.1041	4.1405	269.99

Table 6.8: PC-SAFT parameters for PEC7 [291].

# 6.2.1 The New Generalized Equation of State

Like PEC5, the new generalized equation of state for PEC7 has been developed using of the fitter provided by NIST, and is based on the generalized equation of state of Alexandrov *et al.* [63], which is presented in section 3.5. The critical temperature  $T_c$ , the critical density  $\rho_c$ , and the acentric factor  $\omega$  are again used as adjustable parameters. Table 6.9 gives the results of the adjustment.

**Table 6.9:** The corresponding states parameters for PEC7 adjusted in this work based on the generalized equation of Alexandrov *et al.* [63], and the corresponding states parameters found by Lemmon and Eckermann [23].

	This work	Lemmon and Eckermann [23]		
<i>T</i> <sub>c</sub> / K	916.03	940.00		
$ ho_{\rm c}$ / mol $\cdot$ m <sup>-3</sup>	408.99	412.00		
ω/-	1.2011	1.06		

The critical parameters found in this work are compared to the critical parameters used in the equation of Lemmon and Eckermann [23]. The adjusted acentric factor is compared to the acentric factor calculated from the equation of Lemmon and Eckermann [23] according to its definition (see equation (3.7)). Again, the good agreement with the corresponding states parameters found by Lemmon and Eckermann [23] validates their order of magnitude. Like PEC5, PEC7 molecules are thermally stable until T = 600 K. For higher temperatures, the molecules begin to decompose significantly [292]. Consequently, the meaning of the critical parameters is basically reduced to their function as reducing parameters in the respective equations.

Figure 6.9 shows a p,  $\rho$ -diagram of PEC7 with the isotherm T = 805 K calculated from the generalized equation of state. Again, this isotherm features an unfavorable course and placement of the outer minimum. Consequently, the flash calculation cannot find a correct solution, which leads to a discontinuity in the saturated vapor line at this temperature. This effect was also found for PEC5 and for other fluids with comparable acentric factors calculated with the model of Alexandrov *et al.* [63] as shown and discussed in section 3.5.



**Figure 6.9:** p,  $\rho$ -diagram of PEC7 with the isotherm T = 805 K calculated from the new generalized equation of state.

Again, this effect is not crucial for the description of the pure fluid PEC7, since the molecules already decompose in this temperature region [292]. However, it might affect the use of this equation in a mixture model.

$$R = 0$$

$$C = 0$$

$$R = C = 0 - CH_2 - C = CH_2 - 0 - CH_2 - 0 - CH_2 - R$$

$$CH_2 = 0$$

$$CH_2 = 0$$

$$R = 0$$

$$R = 0$$

$$R = 0$$

**Table 6.10:** Parameters in form of the  $c_p^{\text{o}}$ -equation (3.66) determined by the group contribution method of Joback and Reid [293] for PEC7, reduced with  $R = 8.3144598 \text{ J/(mol \cdot K)}$ .

$n_i$	t <sub>i</sub>
9.60423	0
0.348775	1
$-1.71316 \cdot 10^{-4}$	2
$2.381874 \cdot 10^{-8}$	3

Figure 6.10: Chemical structure of a PEC7 molecule.

 $\mathbf{R} = \mathbf{CH}_2 - \mathbf{CH}_2 - \mathbf{CH}_2 - \mathbf{CH}_2 - \mathbf{CH}_2 - \mathbf{CH}_3$ 

As with PEC5, the isobaric heat capacity has been estimated for PEC7 with the group contribution method by Joback and Reid [293]. In Figure 6.10, the structure of a PEC7 molecule is illustrated. The resulting coefficients according to the  $c_p^{o}$ -equation (3.66) are given in Table 6.10.

# 6.2.2 Comparison to $p\rho T$ Data

Relative deviations of the experimental density data from the new generalized equation of state are illustrated in Figure 6.11. The data sets of Fandiño et al. [18] and Fedele et al. [288] deviate from each other by approximately 0.5% in the overlapping temperature range, as they did for PEC5. Again, the data set of Fandiño et al. [18] was favored in the adjustment due to a lower estimated uncertainty  $(0.14 \text{ kg/m}^3)$  compared to the data set of Fedele et al. [288]. The new generalized equation reproduces the data of these authors within -1% to +0.25%, representing the data of Fandiño *et al.* [18] with an AAD of 0.19% and the data of Fedele et al. [288] with an AAD of 0.69%. The equation of Lemmon and Eckermann [23] follows the data of Fedele et al. [288] and Outcalt [295], which agree very well, accurately with an AAD of 0.58% and 0.63%, respectively. The offset between the new generalized equation of state and the fluid-specific equation of state is 1% for low temperatures and decreases to 0.5% for elevated temperatures. The experimental data of Outcalt [295] cover a broader temperature range than the data available in the literature. The offset between the new generalized equation and the fluid-specific Helmholtz equation [23] as well as the offset between the new generalized equation and the data of Outcalt [295] almost vanishes at T = 450 K. Deviations of the PC-SAFT equation of state [91,291] from the new model and from the experimental data increase rapidly with increasing pressure.

Figure 6.12 illustrates relative deviations of the experimental values at ambient pressure from the new generalized equation of state for PEC7. The course of the data of Shoba and Kishore [290] is discontinuous with a sudden jump at T = 360 K. Up to that temperature, the offset of the new equation increases with increasing temperature from approximately +0.3% to +1%. The deviations do not exceed +0.2% for elevated temperatures (AAD = 0.39%). At ambient pressure, the PC-SAFT equation of state [91,291] is in good agreement with the experimental data of Shoba and Kishore [290] as well (AAD = 0.49%). The fluid-specific equation of Lemmon and Eckermann [23] follows the data of Fortin [294] closely. The offset of the new generalized equation of state from this data set decreases from -1% to -0.5% with increasing temperature (AAD = 0.75%).



**Figure 6.11:** Relative deviations of the experimental liquid density data from the new generalized equation of state for PEC7. Results from the PC-SAFT equation of state [91,291] (dotted curve) and from the equation of Lemmon and Eckermann [239] (dashed curve) are plotted for comparison.

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Figure 6.11 continued.



**Figure 6.12:** Relative deviations of the experimental densities at ambient pressure from the new generalized equation of state for PEC7. Results from the PC-SAFT equation of state [91,291] (dotted curve), and from the equation of Lemmon and Eckermann [239] (dashed curve) are plotted for comparison.

Table 6.11 gives the average absolute relative deviations of the experimental densities from the three compared equations of state. All density data sets can be represented by the new generalized equation of state with an AAD lower than 0.75%.

Author	No. of data	Temperature range / K	Pressure range / MPa	Average absolute relative deviations AAD / %		
				This work	PC-SAFT [91,291]	Lemmon and Eckermann [23]
Fandiño <i>et al.</i> [18]	98	278 - 354	<0.1 - 46	0.19	1.1	0.61
Fedele <i>et al.</i> [288]	40	283 - 344	0.1 - 35	0.69	1.5	0.058
Shobha and Kishore [290]	12	303 - 414	0.101325	0.39	0.49	0.86
Fortin [294]	14	278 - 344	0.083	0.75	0.81	0.053
Outcalt [295]	161	269 - 471	0.5 - 51	0.41	1.691	0.063

**Table 6.11:** Average absolute relative deviations of the experimental densities from the new generalized equation of state for PEC7, from the PC-SAFT equation of state [91,291], and from the model of Lemmon and Eckermann [23].

#### 6.2.3 Comparison to Vapor Pressure Data

Figure 6.13 illustrates relative deviations of the experimental vapor pressures used for the adjustment of the new generalized equation of state for PEC7 from the new equation, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon



**Figure 6.13:** Relative deviations of the experimental vapor pressure data from the new generalized equation of state for PEC7. Results from the PC-SAFT equation of state [91,291] (dotted curve), and from the equation of Lemmon and Eckermann [239] (dashed curve) are plotted for comparison.

and Eckermann [239]. The average absolute relative deviations of the experimental vapor pressures presented in Figure 6.13 from the three equations of state are given in Table 6.12.

The data scatter between -3% and +4% around the generalized equation of state. With an AAD of 1.9%, the new equation represents the data best, although the PC-SAFT parameters were adjusted to these data. The PC-SAFT equation of state [91,291] (AAD = 6.1%) and the equation of state of Lemmon and Eckermann [23] (AAD = 5.7%) exhibit opposite trends.

**Table 6.12:** Average absolute relative deviations of the experimental vapor pressures used for the development of the present generalized equation of state for PEC7 from the present generalized equation, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon and Eckermann [23].

Author	No. of data	Temperature range / K	Average absolute relative deviations AAD / %		
			This work	PC-SAFT [91,291]	Lemmon and Eckermann [23]
Razzouk <i>et al.</i> [291]	5	373 - 434	1.925	6.114	5.679

#### 6.2.4 Comparison to Caloric Data

Since there are no experimental data sets containing caloric properties published in the literature, no such data were used for the adjustment of the generalized equation of state. However, the investigation at NIST included isobaric heat-capacity measurements in the liquid phase, which were conducted with a commercial differential scanning calorimeter, as well as speed-of-sound measurements in the liquid phase, which were conducted with a dual-path, pulse-echo-type instrument. These data sets were used for the adjustment of the fluid-specific equation of state of Lemmon and Eckermann [23] and are still to be published. In order to see how the new generalized equation of state for PEC7 reproduces these data sets without being adjusted to any caloric property, the group contribution method of Joback and Reid [293] has been used to develop a  $c_p^0$ -equation, which is required for such calculations. For comparison, the PC-SAFT equation of state [91,291] is also combined with this  $c_p^0$ -equation.

Relative deviations of the experimental isobaric heat capacities at p = 0.083 MPa from the new generalized equation of state are presented in Figure 6.14. Deviations of the PC-SAFT equation of state [91,291] and of the equation of Lemmon and Eckermann [23], which was fitted to these data, are included for comparison. The fluid-specific Helmholtz equation of state follows the data of Fortin [294] closely (AAD = 0.088%). The new generalized equation of state deviates by -2% from the experimental data at the lowest investigated temperature. This deviation decreases with increasing temperature to -0.5% (AAD = 1.3%).



**Figure 6.14**: Relative deviations of the experimental liquid isobaric heat capacity data at p = 0.083 MPa used for the development of the fluid-specific equation of state for PEC7 [239] from the new generalized equation of state. Results from the PC-SAFT equation of state [91,291] (dotted curve), and from the equation of Lemmon and Eckermann [23] (dashed curve) are plotted for comparison.

The PC-SAFT equation of state [91,291] shows an opposite trend reproducing the experimental data better at low temperatures with increasing deviations at elevated temperatures (AAD = 1.1%).

Table 6.13 contains the average absolute relative deviations of the experimental isobaric heat capacities used for the development of the fluid-specific Helmholtz equation of state for PEC7 of Lemmon and Eckermann [23] from the present generalized equation, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon and Eckermann [23].

**Table 6.13:** Average absolute relative deviations of the experimental liquid isobaric heat capacity data at p = 0.083 MPa used for the development of the fluid-specific Helmholtz equation of state for PEC7 of Lemmon and Eckermann [23] from the present generalized equation of state, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon and Eckermann [23].

Author	No. of data	Temperature range / K	Pressure / MPa	Average absolute relative deviations AAD / %			
				This work	PC-SAFT [91,291]	Lemmon and Eckermann [23]	
Fortin [294]	170	263 - 433	0.083	1.3	1.1	0.088	



**Figure 6.15:** Relative deviations of the experimental liquid speed-of-sound data used for the development of the fluid-specific equation of state for PEC7 [239] from the new generalized equation of state. Results from the equation of Lemmon and Eckermann [239] (dashed curve) are plotted for comparison.


Figure 6.15 continued.

**Table 6.14:** Average absolute relative deviations of the experimental sound speeds used for the development of the fluid-specific Helmholtz equation of state for PEC7 [23] from the present generalized equation, from the PC-SAFT equation of state [91,291], and from the equation of Lemmon and Eckermann [23]

Author	No. of data	Temperature range / K	Pressure range / MPa	Average absolute relative deviations AAD / %		
				This work	PC-SAFT [91,291]	Lemmon and Eckermann [23]
Fortin [294]	14	278 - 344	0.083	2.2	27	0.077
Perkins [297]	66	283 - 424	0.1 - 70	2.1	23	0.081

In Figure 6.15, the relative deviations of the experimental liquid speed-of-sound data used for the development of the fluid-specific equation of state for PEC7 [239] from the new generalized equation of state are illustrated. Only the equation of state of Lemmon and Eckermann [23] is plotted for comparison, since the deviations of the PC-SAFT equation of state [91,291] exceed the limits of the presented graph (AAD > 23%). The fluid-specific Helmholtz equation of state [23] follows the experimental data closely,

whereas the generalized equation shows a linear offset in the whole presented temperature range. This offset starts at low temperatures from -2.5% at the lowest pressure going to -0.5% with increasing pressure, shifting to -2% going to +1% at higher temperatures.

Table 6.14 shows the average absolute relative deviations of the experimental speedof-sound data from the present generalized equation, and from the equation of Lemmon and Eckermann [23]. The new generalized equation of state reproduces both experimental data sets with an AAD of about 2%. This result is remarkably positive considering the strictly predictive character of the new equation with respect to the speed of sound.

As shown for PEC5 in section 6.1, the experimental data of all thermodynamic properties except for the vapor pressure is best described by the fluid-specific Helmholtz equation of state [23] with numerous fluid-specific adjustable parameters. In contrast to the PC-SAFT equation of state [91,291], which is based on a three-parameter corresponding states approach as well, the new generalized equation of state for PEC7 presents a correct pressure dependence. Thereby, the model predicts liquid speeds of sound more than satisfactorily. All presented AADs of the new equation for PEC7 are within the stated range of Alexandrov *et al.* [63] (see section 3.5).

#### 6.3 Mixture Model for CO<sub>2</sub>-PEC5

A new mixture model incorporating the reference equation of state for CO<sub>2</sub> of Span and Wagner [22] and the new generalized equation of state for PEC5 introduced in section 6.1 is presented here.

In total, there are three data sets available, which contain experimental data of CO<sub>2</sub>-PEC5 mixtures. Pensado *et al.* [26] measured liquid densities for two compositions with a very low lubricant content,  $x_{PEC5} = 8$  mass-% and  $x_{PEC5} = 15$  mass-%, in the temperature range from T = 303.15 K to T = 353.15 K and pressures from p = 10 MPa to p = 60 MPa. Bobbo *et al.* [24] published VLE data (pTx') for the isotherm T = 283.15 K. Fandiño *et al.* [25] reported VLE data (pTx') between T = 283.15 K and T = 333.15 K. A binary interaction parameter for the PC-SAFT equation of state adjusted to the experimental data presented in [25] was proposed by Fandiño *et al.* [298], which is used for comparison here.

Due to the limited data base, only the reducing parameters have been fitted. The results are listed in Table 6.15.

 Table 6.15: Adjusted reducing parameters for CO<sub>2</sub>-PEC5 mixtures.

$\beta_T$	$\gamma_T$	$\beta_v$	$\gamma_v$
0.90106	1.6247	2.4339	3.1587

The values of the binary parameters of the density reducing function have a rather high value compared to typical values of reducing parameters of the GERG-2008 [80]. This is probably caused by the large difference in molecular weight and critical density of the two mixture components.

In order to investigate the effect of the numerical problems found for near-critical temperatures for PEC5 (see section 6.1.1), Figure 6.16 displays the temperatures at which the pure fluid equations are evaluated for an exemplary mixture calculation at  $T_{\text{mix}} = 293$  K with the new mixture model.



**Figure 6.16:** Temperatures at which the pure fluid equations are evaluated for an exemplary mixture calculation with T = 293 K with the new mixture model.

The pure fluid temperatures are calculated as shown for PEC5:

$$T_{\text{PEC5}} = \frac{T_{\text{c,PEC5}}}{\tau_{\text{mix}}}, \text{ with } \tau_{\text{mix}} = \frac{T_{\text{r}}(\boldsymbol{x})}{T_{\text{mix}}}.$$
 (6.2)

For T = 293, the PEC5 equation is evaluated in the problematic region in the composition range with  $0.54 < x_{CO_2} < 0.61$ . At T = 313.15 K, this region is shifted to  $0.47 < x_{CO_2} < 0.53$ . The following discussion of the experimental data shows that the short-comings of the pure fluid equation do not cause a failure in the iterative mixture calculations.

Figure 6.17 illustrates the experimental phase equilibrium data as well as calculated values from the new model, and from the PC-SAFT equation of state [91,291,298]. The solubility of CO<sub>2</sub> in PEC5 increases with pressure and decreases with temperature. The PC-SAFT equation of state [91,291,298] follows the experimental data closely throughout the entire presented range. Since the bubble line calculated from the new model oscillates slightly, the new model presented here follows the data but deviates significantly more than the PC-SAFT equation of state [91,291,298]. The deviations between the new model and the experimental data increase with increasing temperature. The oscillation does not result from unphysical contributions from the generalized pure fluid equation. Figure 6.18 exemplarily illustrates the bubble line of the isotherm T = 313.15 K calculated from the original setup of the new mixture model and calculated from the new mixture model using the fluid-specific equation features an even more pronounced bump at high CO<sub>2</sub> fractions. Therefore, the mixture model itself seems responsible for the oscillations.

For temperatures above the critical temperature of  $CO_2$  ( $T_{c,CO_2} = 304.13$  K), both models predict liquid-liquid equilibria at high  $CO_2$  fractions. The vicinity of the change from VLE to LLE is marked in the respective plots. The slope of the "bubble" line (LLE) predicted with the PC-SAFT equation of state [91,291,298] is much steeper than the slope predicted by the new model. Consequently, the two-phase region predicted by the PC-SAFT equation of state [91,291,298] extends to significantly higher pressures.



**Figure 6.17:** Experimental and calculated values for the pressure on the bubble line of CO<sub>2</sub>-PEC5 mixtures for several isotherms. The PC-SAFT equation of state [91,291,298] (dotted curve) is plotted for comparison.



**Figure 6.18:** Experimental and calculated values for the pressure on the bubble line of CO<sub>2</sub>-PEC5 mixtures for T = 313.15 K. The new mixture model was additionally evaluated based on the fluid-specific equation of state of Lemmon and Eckermann [23] for PEC5.

**Table 6.16:** Average absolute relative deviations of the experimental bubble line data with respect to the CO<sub>2</sub> fraction, and with respect to pressure from the present mixture model for CO<sub>2</sub>-PEC5, and from the PC-SAFT equation of state [91,291,298].

Author	No. of data	Temperature range / K	Pressure range / MPa	Average absolute relative deviations AAD / %			1
				This work		PC-SAFT [91,291,298]	
				$x_{\rm CO_2}$	р	$x_{\rm CO_2}$	р
Bobbo <i>et al</i> . [24]	9	283.16	0.50 - 4.41	8.9	14	7.2	10
Fandiño <i>et al.</i> [25]	51	283.12 - 333.16	0.74 - 6.39	11	14	3.4	5.0

In Table 6.16, the average absolute relative deviations of the experimental phase equilibrium data from the new mixture model for CO<sub>2</sub>-PEC5, and from the PC-SAFT equation of state [91,291,298] are given. It is calculated as a deviation in the composition of CO<sub>2</sub> in the liquid phase as function of temperature and pressure, and as a deviation in bubble pressure as a function of temperature and composition. Fandiño *et al.* [25] claim an uncertainty of 4% in the CO<sub>2</sub> fraction. The new mixture model represents this data set with an AAD of 11%. The data of Fandiño *et al.* [25] is best reproduced by the PC-SAFT equation of state [91,291,298] (AAD = 3.4%). The new model represents the data of Bobbo *et al.* [24] [24] with an AAD of 8.9%, the PC-SAFT equation of state [91,291,298] with an AAD of 7.2%. The authors did not discuss the experimental uncertainty. Relative deviations of the experimental liquid densities of a CO<sub>2</sub>-PEC5 mixture consisting of  $x_{CO_2} = 0.9831$  from the present mixture model are presented in Figure 6.19. Results from the PC-SAFT equation of state [91,291,298] are plotted for comparison. The average absolute relative deviations of the experimental density data from the new mixture model for CO<sub>2</sub>-PEC5, and from the PC-SAFT equation of state [91,291,298] are presented in Table 6.17. Pensado *et al.* [26] estimate the uncertainty of their experimental data to be ±0.4%.



**Figure 6.19:** Relative deviations of the experimental liquid densities of a CO<sub>2</sub>-PEC5 mixture consisting of  $x_{CO_2} = 0.9831$  from the present mixture model. The PC-SAFT equation of state [91,291,298] (dotted curve) is plotted for comparison. The dashed line represents the pressure below which LLE occur calculated from the new model, the dashed-dotted line indicates the pressure below which LLE occur calculated from the PC-SAFT equation of state [91,291,298].

Except for the isotherm T = 353.15 K, the data of Pensado *et al.* [26] have a negative offset from the new model for low pressures, turning into a positive offset for high pressures. The deviations range from -1% to +1% with an AAD of 0.67%. Close two the phase boundary, the deviations go up to -2% For lower temperatures, the PC-SAFT equation of state [91,291,298] describes the data better (AAD = 0.47%). The rapid change in course of the PC-SAFT equation of state [91,291,298] is caused by a shift of the liquid-liquid equilibrium of the two models, which results in a change of the calculated densities and consequently a jump in the calculated relative deviations. In contrast to the new equation and except for the two lowest isotherms, the PC-SAFT equation of state [91,291,298] describes the single-phase experimental data points at the lowest pressure as two-phase states.

**Table 6.17:** Average absolute relative deviations of the experimental phase equilibrium data from the present mix-<br/>ture model for  $CO_2$ -PEC5, and from the PC-SAFT equation of state [91,291,298].

Author	No. of data	Temperature range / K	Pressure range / MPa	<i>x</i> <sub>CO2</sub>	Average absolute relativ deviations AAD / %	
					This work	PC-SAFT [91,291,298]
Pensado <i>et al.</i> [26]	57	303 - 353	10 - 60	0.9831	0.67	0.47
Pensado <i>et al.</i> [26]	55	303 - 353	10 - 60	0.9919	0.28	0.45
Pensado <i>et al.</i> [26]	112	303 - 353	10 - 60	0. 9831 - 0. 9919	0.477	0.473

Figure 6.20 shows relative deviations of experimental density data for a CO<sub>2</sub>-PEC5 mixture consisting of  $x_{CO_2} = 0.9919$  from the present mixture model. The PC-SAFT equation of state [91,291,298] is plotted for comparison. The new model represents these data well within the estimated uncertainty (±0.4%). Only the points near the phase boundary or at high temperatures and high pressures deviate stronger. Again, the effect of the shifted LLE of this model and the PC-SAFT equation of state [91,291,298] is distinct. At this composition, the experimental data are better described by the new model (AAD = 0.28%), than by the PC-SAFT equation of state [91,291,298] (AAD = 0.45%). Again, the PC-SAFT equation of state [91,291,298] describes some of the the single-phase experimental data points at the lowest pressure as two-phase states.



**Figure 6.20:** Relative deviations of the experimental liquid densities of a CO<sub>2</sub>-PEC5 mixture consisting of  $x_{CO_2} = 0.9919$  from the present mixture model. Results from the PC-SAFT equation of state [86,244,249] (dotted curve) are plotted for comparison. The dashed line represents the pressure below which LLE occur calculated from the new model, the dashed-dotted line indicates the pressure below which LLE occur calculated from the PC-SAFT equation of state [91,291,298].

### 6.4 Mixture Model for CO<sub>2</sub>-PEC7

A new mixture model based on the reference equation of state for CO<sub>2</sub> of Span and Wagner [22] and the new generalized equation of state for PEC7 introduced in section 6.2 is presented here.

As for CO<sub>2</sub>-PEC5, three data sets are available in the literature for the CO<sub>2</sub>-PEC7 system, which are published by the same authors. Bobbo *et al.* [24] published phase equilibrium data (pTx') for the isotherm T = 283.15 K, whereas Fandiño *et al.* [298] reported phase equilibrium data (pTx') from T = 283.15 K to T = 343.15 K. Pensado *et al.* [26] investigated the density for two overall compositions with a very low lubricant content ( $x_{PEC7} = 8$  mass-% and  $x_{PEC7} = 15$  mass-%), in the temperature range from T = 303.15 K to T = 353.15 K with pressures from p = 10 MPa to p = 60 MPa.

A binary interaction parameter for the PC-SAFT equation of state adjusted to the experimental data presented in [298] was proposed by Fandiño *et al*. [298], which is used for comparison here.

The mixture model for CO<sub>2</sub>-PEC7 developed in this work was not adjusted to the experimental data but was found by scaling of the mixture model for CO<sub>2</sub>-PEC5. Thus, the experimental data is used only for the validation of this method. The aim was to show the applicability of this principle, in order to gain mixture models for CO<sub>2</sub> with similar POEs without any adjustment. The binary parameters of the functions describing the reducing temperature are scaled with the ratio of the critical temperatures of the two POEs:

$$\beta_{T,\text{CO}_2\text{PEC7}} = \beta_{T,\text{CO}_2\text{PEC5}} \cdot \frac{T_{\text{c},\text{PEC5}}}{T_{\text{c},\text{PEC7}}}, \text{ and}$$
(6.3)

$$\gamma_{T,CO_2PEC7} = \gamma_{T,CO_2PEC5} \cdot \frac{T_{c,PEC7}}{T_{c,PEC5}}.$$
(6.4)

The binary parameters of the functions describing the reducing density are scaled with the ratio of the critical densities of the two POEs:

$$\beta_{\nu,\text{CO}_2\text{PEC7}} = \beta_{\nu,\text{CO}_2\text{PEC5}} \cdot \frac{\rho_{\text{c},\text{PEC7}}}{\rho_{\text{c},\text{PEC5}}}, \text{ and}$$
(6.5)

$$\gamma_{\nu,\text{CO}_2\text{PEC7}} = \gamma_{\nu,\text{CO}_2\text{PEC5}} \cdot \frac{\rho_{\text{c},\text{PEC7}}}{\rho_{\text{c},\text{PEC5}}}.$$
(6.6)

Different approaches for the conversion of the reducing parameters were investigated and these relations were found to be most beneficial.

Table 6.18 gives the reducing parameters for CO<sub>2</sub>-PEC7 mixtures found by this method.

Table 6.18: Scaled reducing parameters for CO<sub>2</sub>-PEC7 mixtures.

$\beta_T$	$\gamma_T$	$\beta_v$	$\gamma_v$	
0.8621	1.6981	3.2724	4.2468	

Experimental and calculated values for the phase equilibrium of CO<sub>2</sub>-PEC7 mixtures are presented in Figure 6.21. The scaled model shows reliable results for compositions with CO<sub>2</sub> contents of  $x_{CO_2} > 0.4$ . At lower CO<sub>2</sub> fractions, the model shows an unphysical negative slope of the bubble line. This behavior was observed during the fitting process of the CO<sub>2</sub>-PEC5 system as well. The numerical stability of such asymmetric models is very sensitive with respect to the values of the reducing parameters. For very high CO<sub>2</sub> contents, the new model predicts liquid-liquid equilibria for temperatures above the critical temperature of CO<sub>2</sub> ( $T_{c,CO_2} = 304.13$  K). With the PC-SAFT equation of state [91,291,298], liquid-liquid equilibria can already be found at T = 298.15 K. Although the bubble line calculated from the new model oscillates slightly, there is no significant difference in the description of the experimental data by both models for CO<sub>2</sub> fractions of  $x_{CO_2} > 0.4$ .

**Table 6.19:** Average absolute relative deviations of the experimental bubble-line data with respect to the CO<sub>2</sub> fraction, and with respect to pressure from the present mixture model for CO<sub>2</sub>-PEC7, and from the PC-SAFT equation of state [91,291,298].

Author	No. of data	Temperature range / K	Pressure range / MPa	Average absolute relative deviations AAD / %			)
				This w	vork	PC-SAFT [91,	291,298]
				<i>x</i> <sub>CO<sub>2</sub></sub>	р	$x_{\rm CO_2}$	p
Bobbo <i>et al</i> . [24]	9	283	0.50 - 4.41	7.5	14	5.8	10
Fandiño <i>et al</i> . [298]	51	283 - 333	0.74 - 6.39	18(4.6)*	14	6.8	5.0

\*: For  $x_{CO_2} > 0.4$ .

Figure 6.22 illustrates relative deviations of experimental densities of a CO<sub>2</sub>-PEC7 mixture consisting of  $x_{CO_2} = 0.9872$  from the present mixture model. The PC-SAFT equation of state [91,291,298] is again plotted for comparison. The new model reproduces the experimental data with a maximum offset of -2% to +1.5%. A deviation of -4.5%is reached only for the lowest pressure of the highest isotherm. For temperatures T < 333.15 K, the experimental data is best described by the new model within -0.5% to +1%. For higher temperatures, the PC-SAFT equation of state [91,291,298] follows the trend of the experimental data better. The new model reproduces the data with an average absolute relative deviation of AAD = 0.69\%, whereas the PC-SAFT equation of



**Figure 6.21:** Experimental and calculated values for the bubble-line pressure of CO<sub>2</sub>-PEC7 mixtures for several isotherms. The PC-SAFT equation of state [91,291,298] (dotted curve) is plotted for comparison.

state [91,291,298] reproduces the data with an AAD of 1.7%. Density calculations from the PC-SAFT equation of state did not converge for all pressures at this composition, resulting in an "edgy" appearance in the plot.



**Figure 6.22:** Relative deviations of the experimental liquid density data of a CO<sub>2</sub>-PEC7 mixture consisting of  $x_{CO_2} = 0.9872$  from the present mixture model. Results from the PC-SAFT equation of state [91,291,298] (dotted curve) are plotted for comparison.

Relative deviations of the experimental densities of a CO<sub>2</sub>-PEC7 mixture consisting of  $x_{CO_2} = 0.9934$  from the present mixture model are shown in Figure 6.23. For temperatures T < 323.15 K, the new mixture model deviates from the data by -1% to 0%. For higher temperatures, the PC-SAFT equation of state [91,291,298] follows the trend of the data more closely. The average absolute relative deviation of the new model from these data is AAD = 0.59%, whereas the PC-SAFT equation of state [91,291,298] reproduces the data with AAD = 0.83%.



**Figure 6.23:** Relative deviations of the experimental liquid density data of a CO<sub>2</sub>-PEC7 mixture consisting of  $x_{CO_2} = 0.9934$  from the present mixture model. Results from the PC-SAFT equation of state [91,291,298] (dotted curve) are plotted for comparison.

**Table 6.20:** Average absolute relative deviations of the experimental phase equilibrium data from the present mixture model for CO<sub>2</sub>-PEC7, and from the PC-SAFT equation of state [91,291,298].

Author	No. of data	Temperature range / K	Pressure range / MPa	<i>x</i> <sub>CO2</sub>	Average absolute relativ deviations AAD / %	
					This work	PC-SAFT [91,291,298]
Pensado <i>et al.</i> [26]	55	303 - 353	10 - 60	0.9872	0.69	1.7
Pensado <i>et al.</i> [26]	55	303 - 353	10 - 60	0.9934	0.59	0.83
Pensado <i>et al.</i> [26]	110	303 - 353	10 - 60	0. 9872 - 0. 9934	0.64	1.27

#### 6.5 Summary

Generalized equations of state for the synthetic lubricants PEC5 and PEC7 were developed, which were exclusively adjusted to thermal property data. Nevertheless, caloric properties such as the isobaric heat capacity and speed of sound are remarkably well predicted by the present equations. The new data measured at NIST [294,297] as well as the fluid-specific equations of Lemmon and Eckermann [23] were used to validate this feature. In contrast, the PC-SAFT equation of state [91,291] deviates strongly from the fluid-specific equation [23] and from experimental data with respect to the caloric properties. In particular, speeds of sound in the liquid phase are not well represented. Subsequently, the new generalized equation of state for PEC5 was applied to establish a new Helmholtz mixture model for the system CO<sub>2</sub>-PEC5, which is based on the extended corresponding states approach. It was compared to the PC-SAFT equation of state [91,291,298] which yields better results in the context of such an asymmetric mixture, especially with regard to the description of the phase behavior. Figure 6.18 shows that these shortcomings are not caused by the generalized equation of state developed in this work. It rather seems that the extended corresponding states approach is limited in the description of such asymmetric mixtures. Consequently, different mixture models are required for future improvements. However, the new model is capable of a reasonable estimation of phase equilibria and homogeneous liquid densities of CO<sub>2</sub>-PEC5 mixtures and is still slightly superior with regard to the representation of liquid densities.

Finally, a converted CO<sub>2</sub>-PEC7 mixture model based on the new mixture model for the system CO<sub>2</sub>-PEC5 was developed. It was shown that the model predicts the available experimental data with a similar accuracy as the base model for CO<sub>2</sub>-PEC5, except for some issues in the representation of the phase behavior for low CO<sub>2</sub> fractions. Although the new model was not adjusted to these data, the experimental densities are better described by the new scaled equation than by the PC-SAFT equation of state [91,291,298]. Thus, general applicability of the proposed conversion approach is assumed, with limitations for the prediction of phase equilibria at low CO<sub>2</sub> fractions.

# 7 Conclusions

In a joint research project, the potential of (Organic) Rankine Cycles for Exhaust Heat Recovery (EHR) in heavy-duty diesel engines for mobile applications was investigated. For a safe process design, the knowledge of thermophysical properties of the involved fluids or fluid mixtures is crucial. The EHR process design features ethanol as working fluid. The potential benefit of ethanol-water mixtures as working fluid was to be investigated based on a parameter study. A preliminary ethanol-water mixture model is available in REFPROP [4], which is based on an outdated ethanol pure fluid equation [21]. In order to incorporate the more current ethanol equation of Schroeder *et al.* [3], which is based on a broader experimental data base, a new Helmholtz mixture model was developed in this work.

The uncertainty of the new mixture model for ethanol-water with respect to the speed of sound is estimated to be  $\pm 4\%$ , which is a significant improvement in contrast to the model of Lemmon [4], which deviates by up to 25% from the experimental data. For temperatures T > 280 K, the isobaric heat capacity is represented within 5%. The uncertainty in terms of density is estimated to be 0.5% for temperatures T < 450 K, increasing to 1% for elevated pressures. For higher temperatures, the uncertainty is estimated to be 2% in the homogeneous phase up to 5% approaching the phase boundary. The relative deviations in bubble and dew pressure increase from 1% at T = 313 K to 3% at T = 623 K.

The design of the ORC application includes direct lubricant injection into the expansion machine. Hence, the knowledge of the thermodynamic properties of the lubricant, and the thermodynamic properties of its mixture with ethanol are required. A poly-alphaolefin (PAO)-based synthetic lubricant containing several PAO base oils and an emulsifier was specially designed for this EHR process and is commercially available [278]. Despite the different compounds, the lubricant is treated as a pure fluid in the models developed in this work. Due to an insufficient data base, a generalized equation of state was developed for the PAO, which is based on the model of Alexandrov *et al* [63]. For the adjustment of such a three-parameter corresponding states model, less experimental data are required than for a fully optimized equation of state. With the new generalized equation of state, the experimental data describing the PAO can be reproduced reasonably well.

A new Helmholtz model describing ethanol-PAO mixtures was subsequently developed, which is based on the new generalized pure fluid equation of state for the PAO and the current ethanol equation of state of Schroeder *et al.* [3]. General application of this method for the estimation of fluid properties was shown. The mixture model developed in this work reproduces the experimental densities reasonably and predicts a three-phase equilibrium as found by experiment in the investigated temperature range. In flash calculations, the compositions of the phases deviate strongly from experimentally determined compositions. However, the vapor composition can be reproduced, if the lubricant-rich liquid phase is neglected and the ethanol-rich liquid phase composition is used as input for a flash calculation.

To validate the new approach of using a generalized equation of a state in a mixture model with challenging, asymmetric components, the systems CO<sub>2</sub>-PEC5 and CO<sub>2</sub>-PEC7 were investigated. Therefore, new generalized equations for the POE base lubricants PEC5 and PEC7 were fitted and mixture models for these lubricants with CO<sub>2</sub>, a promising refrigerant in ORC processes, were developed. For the new generalized equations of state, exclusively the thermal property data available in the literature were included in the adjustment. In a collaborative work with NIST, which is not part of this thesis, two substance-specific Helmholtz equations of state for PEC5 and PEC7 were adjusted to new experimental data measured at NIST containing thermal and caloric properties. The intention was to use these fluid-specific equations for the validation of the new generalized equations of state, especially with regard to the prediction of caloric properties that were not included in the adjustment of the generalized equation of state. It was shown that the generalized equations of state reproduce the available experimental data within the AAD uncertainties range claimed by Alexandrov et al. [63]. Comparisons with the PC-SAFT equation of state [91,291] showed that although both models are three-parameter corresponding states approaches, the generalized equations adjusted in this work predict the caloric properties in a superior manner.

The new mixture model for CO<sub>2</sub>-PEC5 represents the available phase-equilibrium and density data reasonably. Nevertheless, bubble lines calculated from the new model exhibit an unphysical oscillation that is not caused by the shortcomings of the generalized equation of state but by the mixture model itself. New mixture models are required for a better description of such asymmetric mixtures.

The new mixture model for CO<sub>2</sub>-PEC5 was converted to the CO<sub>2</sub>-PEC7 system by a simple empirical conversion developed in this work. This conversion was accomplished through the use of the critical pure-fluid parameters and the molar masses only. General applicability was shown with the restriction of  $x_{CO_2} > 0.4$ . For smaller CO<sub>2</sub> fractions, the bubble lines calculated from this model show an unphysical negative slope. In this work, the extended corresponding states-based mixture approach was challenged by the strongly non-ideal ethanol-water system as well as by mixtures of significantly different molecular size and weight. Shortcomings were found that demand for a further development of new mixture approaches. The current status of mixture models is based on a development for natural gases and for mixtures of refrigerants, which contain mostly similar components with significantly less intermolecular forces. Therefore, future research should be focused on the investigation of new mixture models that improve the description of complex mixtures.

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i	t <sub>i</sub>	d <sub>i</sub>	$p_i$	Υ <sub>i</sub>
1	0.686	1	0	0
2	1.118	1	0	0
3	0.857	1	0	0
4	0.559	3	0	0
5	0.442	7	0	0
6	0.831	2	0	0
7	0.484	1	1	1
8	2.527	1	1	1
9	1.549	2	1	1
10	0.757	5	1	1
11	3.355	1	2	1
12	1.905	1	2	1
13	4.941	4	2	1
14	12.805	2	3	1

Appendix A – Parameters of the Basic Generalized Equation of State

**Table A 1:** Temperature and density exponents of the generalized model by Alexandrov *et al.* [63].

Table A 2: Values of the parameters for the determination of the generalized coefficients.

i	<i>c</i> <sub><i>i</i>,1</sub>	<i>C</i> <sub><i>i</i>,2</sub>	<i>c</i> <sub><i>i</i>,3</sub>	C <sub><i>i</i>,4</sub>
1	$0.53410734 \cdot 10^{+1}$	$0.66819473 \cdot 10^{+1}$	$0.16692414 \cdot 10^{+1}$	0.29446922 · 10+1
2	$-0.22778189 \cdot 10^{+1}$	$-0.12846893 \cdot 10^{+1}$	$0.13795302 \cdot 10^{+1}$	0.23284396 · 10+1
3	$-0.38785499 \cdot 10^{+1}$	-0.86095696 · 10+1	$-0.26707821 \cdot 10^{+1}$	$0.27960114 \cdot 10^{+1}$
4	$-0.12190959 \cdot 10^{-1}$	0.36869492 · 10+0	$-0.20627285 \cdot 10^{+0}$	$0.63731470 \cdot 10^{+0}$
5	0.92942159 · 10-3	0.80731074 · 10-1	-0.81358186 · 10-1	0.99619992 · 10+0
6	$-0.16631229 \cdot 10^{-1}$	$-0.80314182 \cdot 10^{-1}$	$-0.35343719 \cdot 10^{+0}$	$0.11870929 \cdot 10^{+1}$
7	$-0.16572887 \cdot 10^{+1}$	0.21646346 · 10+2	$-0.1618967 \cdot 10^{+2}$	$0.10375103 \cdot 10^{+1}$
8	0.12642606 · 10+1	$0.21645843 \cdot 10^{+1}$	$-0.25726222 \cdot 10^{+1}$	$0.13733437 \cdot 10^{+1}$
9	0.96008662 · 10-1	$0.44221976 \cdot 10^{+1}$	$0.11591367 \cdot 10^{+1}$	$0.11168557 \cdot 10^{+1}$
10	0.92950830 · 10 <sup>-1</sup>	$-0.57463893 \cdot 10^{-1}$	0.44419682 <sup>.</sup> 10 <sup>+0</sup>	0.76390402 · 10+0
11	$-0.38271299 \cdot 10^{+0}$	$-0.20429713 \cdot 10^{+1}$	$0.11751452 \cdot 10^{+1}$	$0.14829049 \cdot 10^{+1}$
12	0.34936066 · 10+0	0.6455642 · 10+1	$-0.83598749 \cdot 10^{+1}$	$0.10080516 \cdot 10^{+1}$
13	$0.4171879 \cdot 10^{-1}$	$-0.90287649 \cdot 10^{+0}$	0.23069811 · 10+0	$0.13320474 \cdot 10^{+1}$
14	$-0.12149915 \cdot 10^{-1}$	$-0.15474203 \cdot 10^{+0}$	0.23233099 · 10+0	0.12062411 · 10+1

# Appendix B – Parameters of the adjusted generalized equation of state

i	N <sub>i</sub>	t <sub>i</sub>	$d_i$	$p_i$
1	5.479408056325	0.686	1	0
2	-2.304246555419	1.118	1	0
3	-4.056820239042	0.857	1	0
4	$-2.198447281716 \cdot 10^{-2}$	0.559	3	0
5	8.914407509800 · 10 <sup>-4</sup>	0.442	7	0
6	-2.183543931921 · 10-2	0.831	2	0
7	-1.495915082277	0.484	1	1
8	1.296546913602	2.527	1	1
9	2.027997920456 · 10 <sup>-1</sup>	1.549	2	1
10	1.147301175206 · 10-1	0.757	5	1
11	-4.212628050360 · 10-1	3.355	1	2
12	3.142257447543 · 10-1	1.905	1	2
13	2.434692684979 · 10-2	4.941	4	2
14	-1.319154456018 · 10 <sup>-2</sup>	12.805	2	3

**Table B 1:** Generalized equation parameters for PAO with coefficients evaluated from the adjusted corresponding states parameters.



**Figure C 1:** Relative deviations of the high temperature experimental densities along isochores over pressure from the new mixture model with  $x_{\text{Ethanol}} = 0.2$ .



Figure C 1 continued.



**Figure C 2:** Relative deviations of the high temperature experimental densities along isochores over pressure from the mixture model of Lemmon [4] with  $x_{\text{Ethanol}} = 0.2$ .



Figure C 2 continued.



**Figure C 3:** Relative deviations of the high temperature experimental densities along isochores over pressure from the new mixture model with  $x_{\text{Ethanol}} = 0.8$ .



Figure C 3 continued.



**Figure C 4:** Relative deviations of the high temperature experimental densities along isochores over pressure from the mixture model of Lemmon [4] with  $x_{\text{Ethanol}} = 0.8$ .



Figure C 4 continued.

### Lebenslauf

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