Preface

Stein’s method is a popular tool for proving distributional convergence of a sequence of random variables. One of its main advantages over other techniques is that it usually gives concrete error bounds on the distance between the involved distributions in several metrics and, hence, automatically yields rates of convergence. Being first developed for the univariate standard normal distribution in [Ste72], [Ste86] and within much subsequent work, it was quickly recognized that Stein’s technique of a characterization through an appropriate differential equation is by no means restricted to the normal distribution. In fact, already in [Che75] the method was used to bound the distance to the Poisson distribution for a sum of dependent indicator random variables by replacing the differential equation by a suitable difference equation. When applying Stein’s method to a given random variable $W$, whose distribution is supposed to be close to a given distribution $\mu$, whose Stein characterization is at hand, one usually needs one more tool, which might for instance be a certain coupling $(W,W')$ for $W$ or a Stein characterization for $L(W)$, the distribution of $W$. One such classical coupling is the exchangeable pairs coupling, whose usefulness for univariate normal approximation was highlighted in the monograph [Ste86] and which was further extended in many following articles (see, e.g. [RR97], [SS06] and [Röl08]). In the recent papers [EL10] and [CS11], the exchangeable pairs approach for a given univariate, absolutely continuous distribution $\mu$ was developed in the context of the so-called density approach, which is a universal method of finding a Stein characterization for such distributions $\mu$. In [Ste86] and [RR97] it was shown that the exchangeable pairs approach for approximation by the standard normal distribution may be successfully applied given the exchangeable pair $(W,W')$ satisfies the linear regression property $E[W' - W | W] = -\lambda W + R$, where $\lambda > 0$ is constant and $R$ is a negligible remainder term. The question, what regression property to demand of an exchangeable pair for a general distribution $\mu$ within the density approach was independently answered in [EL10] and [CS11]. They pointed out that the right condition is $E[W' - W | W] = -\lambda \psi(W) + R$, where $\lambda$ and $R$ are as before and where $\psi$ is the logarithmic derivative of the density of $\mu$.

Having reviewed Stein’s method for the standard normal distribution and having given an abstract account of Stein’s method in Chapter 1, we turn to absolutely continuous univariate distributions in Chapter 2. After presenting several approaches for finding a Stein characterization we propose a new such approach,
which is motivated by a regression property, which is satisfied by a given exchangeable pair, whose members are supposed to be close to the given distribution \( \mu \), see Section 2.4. The elaboration of this approach is the theoretical centerpiece of this thesis. Afterwards, the theory from Section 2.4 is specialised to the family of Beta distributions in Section 2.5 and is then applied to Pólya urn models, Wigner’s semi-circle law and the arcsine law in Sections 2.6, 2.7 and 2.8.

In Chapter 3 we turn to multivariate normal approximation by Stein’s method, as developed in [Göt91], [Bar90], [CM08], [RR09], [Mec09] and further articles. Having reviewed and extended the exchangeable pairs approach for multivariate normal approximation in Section 3.1, we turn to spectral properties of random Haar distributed elements from one of the classical compact, connected Lie groups in Section 3.2. There, we prove a quantitative version of a theorem by Diaconis and Shahshahani on asymptotic normality and independence of the vector of traces of various powers of a Haar distributed element from one of the classical compact Lie groups, see [DS94], and apply it to obtain rates of convergence for the Gaussian fluctuations of suitable linear statistics of the spectral measure of a Haar distributed unitary matrix.

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